A novel variable-lag probability hypothesis density smoother for multi-target tracking

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Abstract It is understood that the forward–backward probability hypothesis density (PHD) smoothing algorithms proposed recently can significantly improve state estimation of targets. However, our analyses in this paper show that they cannot give a good cardinality (i.e., the number of targets) estimate. This is because backward smoothing ignores the effect of temporary track dropping caused by forward filtering and/or anomalous smoothing resulted from deaths of targets. To cope with such a problem, a novel PHD smoothing algorithm, called the variable-lag PHD smoother, in which a detection process used to identify whether the filtered cardinality varies within the smooth lag is added before backward smoothing, is developed here. The analytical results show that the proposed smoother can almost eliminate the influences of temporary track dropping and anomalous smoothing, while both the cardinality and the state estimations can significantly be improved. Simulation results on two multi-target tracking scenarios verify the effectiveness of the proposed smoother.

1. Introduction

Estimating the number of targets and their states from a sequence of noisy and cluttered observation sets is the major objective in multi-target tracking (MTT) applications. A challenging problem in these applications is the unknown association of measurements from the sensor with the appropriate targets.1–4 Most traditional MTT formulations for coping with this problem involve the explicit associations between measurements and targets such as the multiple hypotheses tracking (MHT) and its variations,4 the joint probabilistic data association filter (JPDAF),5 and the probabilistic MHT (PMHT).6 These approaches are mainly based on some subsets of the set of all possible associations, which will result in an intensive and computationally complex algorithm in the application to collections of many targets in the same validation gate.7 A promising alternative way that avoids the explicit associations between the measurements and the targets is random finite sets (RFS).8–10

In RFS tracking, the states of the targets and the measurements, at each time step, can be represented as finite sets. Under this theoretical contribution, the probability hypothesis density (PHD) filter was proposed as an approximation to RFS solution for MTT.8,9 This filter propagates only the first-order statistical moment of a multi-target posterior instead of the full multi-target posterior or the intensity of the multi-target evolving point process.10

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Two kinds of implementations of the PHD filter have been proposed up to now. One is the sequential Monte Carlo PHD (SMC-PHD) filter for general dynamic models, which estimates the PHD density using weighted sum of particles. Another is the Gaussian Mixture PHD (GM-PHD) filter for linear Gaussian models, which models the PHD distribution as a mixture of Gaussian densities that renders a closed-form solution. As it is well known, the objective of the filtering is to recursively estimate the current state given the observation history up to the current time, while smoothing can yield significantly better estimates than filtering by delaying the decision time and using data at a later time. Following the great achievement of the PHD filter in MTT scenarios, PHD-filter-based smoothing algorithms for PHD-based systems arouse enormous interest among researchers. Recently, forward–backward fixed-lag PHD (FPHD) smoothing algorithms were proposed to improve the performance of PHD-based tracking systems. A PHD smoothing approach was derived using a physical-space approach, which involved forward multi-target filtering using standard PHD filter recursion followed by backward smoothing recursion using a novel recursive formula. At the meantime, it was extended to the multiple models PHD (MM-PHD) smoothing for tracking maneuvering targets. Mahler et al. also derived the same PHD smoothing algorithm using finite set statistics (FISST) and standard point process theory. In Ref. [19], a closed-form Gaussian sum smoother was proposed for the GM-PHD filter, called the GM-PHD smoother, which was used to decrease the computational complexity of the SMC implementation of the PHD smoother.

It is notable that the key point of the current forward–backward PHD smoothing approaches is the backward smoothing process. Essentially, it is the adjustment of the filtered PHD distribution at present time using the later adjusted PHD distribution, which involves the innovation from the forward filtering using measurements beyond the present time. Therefore, if the forward filtering is accurate (i.e., no track dropping), the backward smoothing can propagate the innovation perfectly to improve the filtered cardinality and states. Unfortunately, the current implementations of the PHD filter (i.e., SMC-PHD filter or GM-PHD filter.) are all to some extent of approximation, so the cardinality bias resulted from the practical forward filtering cannot be reduced in the following backward smoothing, because the track lost can destroy innovation backward propagation. In addition, if some targets die at some time instants, it can also affect innovation backward propagation at the proximity time of the death of targets (relating to the smooth lag), which we refer as anomalous smoothing henceforth. For these reasons, the performance of the current FPHD smoothing algorithm dramatically decreases with an incorrect cardinality estimation and is sensitive to the death of targets.

In this paper, a novel approach called variable-lag PHD (VPHD) smoother is proposed to overcome the shortcomings of the existing FPHD smoother. The suggested method considers the effect of the temporary track dropping caused by forward filtering and/or anomalous smoothing resulted from the death of targets. It adjusts the smooth lag according to the forward filtered number of targets, which can almost eliminate the negative effect of temporary track dropping and anomalous smoothing.

The remainder of the paper is organized as follows. In Section 2, a necessary background of RFS-based MTT is given. Additionally, the PHD filter and the prevailing FPHD smoother are reviewed. In Section 3, the drawback of the FPHD smoother is illustrated in detail and the novel approach is proposed followed by the simulation results discussed in Section 4. Finally, conclusions are presented in Section 5.

2. Background

2.1. Multi-target tracking and random finite sets

In MTT, the collections of target states and measurements at time \( k \) can be represented as finite sets, i.e.,

\[
X_k = \{x_{k,1}, x_{k,2}, \ldots, x_{k,M(k)}\} \in F(X)
\]

\[
Z_k = \{z_{k,1}, z_{k,2}, \ldots, z_{k,N(k)}\} \in F(Z)
\]

where \( X_k \) and \( Z_k \) are, respectively the state and measurement sets. \( X \in \mathbb{R}^n \) and \( Z \in \mathbb{R}^m \) are the state and measurement spaces, respectively. \( x_{k,1}, x_{k,2}, \ldots, x_{k,M(k)} \in X \) are the states of \( M(k) \) targets at time \( k \) and \( z_{k,1}, z_{k,2}, \ldots, z_{k,N(k)} \in Z \) are the measurements of \( N(k) \) at time \( k \). \( F(X) \) and \( F(Z) \) denote the spaces of all finite subsets of \( X \) and \( Z \), respectively. Using the FISST notion of integration and density, the multi-target Bayes filter is generally intractable and it is necessary to resort to more tractable approximations. The PHD filter is a first-order moment approximation to the full multi-target Bayes filter in Eq. (3) and Eq. (4), which operates on the single-target state space \( X \). The prediction and update equations of the PHD recursion that respectively model the multi-target transition density, the probability of detection, and the intensity of the clutter, respectively.
The integral of PHD over any given area $S \subset \mathcal{X}$ specifies the expected number of targets $N_k$ present in that area, that is,

$$N_k = \int_S v_{n|x}(x)dx$$

(8)

The general approach to extracting multi-target states is looking for the $n$ (the nearest integer rounding $N_k$) largest local maxima of the PHD and taking their coordinates as the state estimates of the targets.

2.3. FPHD smoother formulations

Similar to the multi-target forward filtering generalization in Eq. (3) and Eq. (4), the backward smoothing can also be represented by RFS. Given the measurements up to time $k$, the smoothed multi-target density is propagated backward, from time $k$ to time $k' < k$, via the multi-target backward smoothing recursion

$$p_{k|k}(X) = p_{k'|k}(X) \int f_{k'|k}(Y|X) \frac{p_{k+1|k}(Y)}{p_{k+1|k'}(Y)} \delta Y$$

(9)

As with the multi-target Bayes filter, the multi-target backward smoothing recursion involves set integration, and a computationally tractable first-order approximation is given by

$$v_{k|x}(x) = v_{k'|x}(x)(1 - P_S(k|x) + P_{k|x}) \times \int \frac{v_{k+1|x}(x)f_{k+1|x|k'}(x)}{v_{k+1|x|k'}(x)} \delta x$$

(10)

where $v_{k|x}$ and $v_{k'|x}$ are the smoothed and filtered PHD at time $k'$, respectively. $v_{k+1|x}^{'}$ and $v_{k'+1|x'}^{'}$ are the smoothed and predicted PHD at time $k' + 1$, respectively.

3. Variable-lag PHD smoother

3.1. Drawback of FPHD smoother

As stated before, the PHD smoother adjusts the forward filtered PHD sequentially. The lag of the smoother decides how many more measurements will be used for backward smoothing. The innovation given by the measurements propagates backward recursively to the smoothed time instants. Afterward, the smoothed cardinality and the target states can be extracted from the smoothed PHD distribution. It is shown that the PHD smoother can indeed improve target state estimates, but does not necessarily work well at cardinality estimates. To express the major defect of the PHD smoother more explicitly, a proposition is summarized as follows.

**Proposition.** If target track dropping occurs at the time instant $k$, then the corresponding smoothed target tracks are also dropped at the time instant $k - L$, where $L$ is the lag of the smoother.

**Proof.** The following discussion will adopt the particle implementation of the smoother. In the forward filter, suppose that the predicted PHD at time $k$ is of the form

$$v_{k|x}(x) = \sum_{j=1}^{L_{k|x}} w_{k|x}^{(0)} \delta_{x_{k|x}^{(0)^j}}(x)$$

Then the posterior PHD at time $k$ can be expressed as

$$v_{k|x}(x) = \sum_{j=1}^{L_{k|x}} w_{k|x}^{(0)} \phi_{k+1|x}^{(1)}(x)$$

where $L_{k|x}$, $w_{k|x}^{(0)}$, $w_{k|x}^{(1)}$, and $w_{k+1|x}^{(2)}$ are respectively the predicted number of particles, the predicted weight of particles, the particle weight corresponding to missed detections and measurement updates.

While in backward smoothing, suppose that the filtered PHD at time $k'$ and the smoothed PHD at time $k' + 1$ from time $k$ are formulated by the weighted samples

$$v_{k|x}(x) = \sum_{j=1}^{L_{k|x}} w_{k|x}^{(0)} \delta_{x_{k|x}^{(0)^j}}(x)$$

(13)

$$v_{k'|x'}(x') = \sum_{j=1}^{L_{k'|x'}} w_{k'|x'}^{(0)} \delta_{x_{k'|x'}^{(0)^j}}(x')$$

(14)

Then the smoothed PHD at time $k'$ from time $k$ is given by the reweighted samples

$$v_{k|x}(x) = \sum_{j=1}^{L_{k|x}} w_{k|x}^{(0)} \delta_{x_{k|x}^{(0)^j}}(x)$$

(15)

where

$$w_{k|x}^{(0)} = w_{k|x}^{(0)} \left(1 - P_S(x^{(0)}|x^{(0)}) + P_S(x^{(0)}|x^{(0)}) \sum_{j=1}^{L_{k+1|x}} \eta_{k+1|x}^{(0)^j} \phi_{k+1|x}^{(1)} \right)$$

(16)

$$\eta_{k+1|x}^{(0)^j} = w_{k+1|x}^{(0)} \left(1 - P_S(x^{(0)}|x^{(0)}) \right) \sum_{j=1}^{L_{k+1|x}} \phi_{k+1|x}^{(1)} \delta_{x_{k+1|x}^{(0)^j}}(x_{k+1|x}^{(0)^j})$$

(17)

$$\phi_{k+1|x}^{(1)} = \sum_{i=1}^{L_{k|x}} w_{k|x}^{(0)} P_S(x^{(0)}|x^{(0)}) f_{k+1|x|k}^{(1)}(x_{k+1|x}^{(0)^j})$$

(18)

As it is shown by the above equations, in SMC-PHD smoothing, the smoothed particle weights at time $k - L$ are evaluated using the backward iterations giving the filter outputs as $\{v_{k|x}, x_{k|x}^{(0)^j}\}_{j=1}^{L_{k|x}}$ for $t = k - L, k - L + 1, ..., k$.

**Assume that at time instant $k$, the PHD filter drops one target, so the state estimate of the target is also lost at this time. Note the particles corresponding to the dropped target as $x_{k-1|x}^{(d)^j} (d$ is the index of the particles for this target) at time $k - 1$, and then according to Eq. (17), the transition density $f_{k|x}^{(d)}(x_{k|x}^{(0)^j}|x_{k-1|x}^{(d)^j})$ decrease sharply because of the lost particles at time $k$, and $\eta_{k+1|x}^{(0)^j}$ becomes very small. At the same time, $\phi_{k+1|x}^{(1)}$, $j = 1, 2, ..., L_0$ are constant for all the particles at time $k - 1$, so the adjusted weights at time $k - 1$ are dependent on the transition density. As a result, the adjusted weights at time $k - 1$ related to the dropped target are very small. At time $k - 2$, the adjusted weights are dependent on the smoothed weights at time $k - 1$, and the weights corresponding to the dropped target at time $k$ are also smaller than the non-dropped targets. Therefore, at time $k - L$, the weights corresponding to the dropped target are so small that the target cannot be extracted by summing the weights of the particles. Then the smoothed cardinality estimate will lose the dropped target at
time $k$. Similarly, the death of targets during the tracking can also cause the same problem, for the process is almost like the targets being dropped by the PHD filter. It is analyzed as above that, if there are some target states missing within the lag during the forward filtering, the backward smoothing cannot correctly adjust the corresponding weights of the particles. The mentioned drawback of the FPHD smoother degrades its performance.

3.2. Novel variable-lag PHD smoother

The explanations in the previous section have showed the reason why the FPHD smoother cannot sufficiently improve the estimates of the number of targets, especially when targets die in the tracking. The main idea of this paper is to devise a variable-lag smoothing approach called variable-lag PHD (VPHD) smoother for adjusting the smoothing lag in the backward smoothing process according to the forward filtering. The proposed method is summarized in the following steps.

- **Step 1 (PHD filtering):** assuming the initial smooth lag is $\ell$, we now smooth the PHD at time $k - \ell$. We can obtain the filtered PHD and cardinality $\{v_0(x), N_i\}_{t=k-\ell}^T$. Note that for output purposes, the estimated cardinality is given by $N_i = \text{round}(N_{t0})$, where round() refers to the nearest integer.

- **Step 2 (Lag adjustment):** comparing the filtered cardinality $N_{k-\ell}^i$ at time $k - \ell$ with $N_i(t = k - \ell + 1, k - \ell + 2, \ldots, k)$, we can get the following three cases, where $\ell'$ represents the adjusted lag.
  
  1. $N_{k-\ell}^i \leq N_i(t = k - \ell + 1, k - \ell + 2, \ldots, k)$. In such a case, we think that some new targets may be born or at least there are no targets dropped by the forward filtering, so the backward smoothing lag remains unchanged, that is, $\ell' = \ell$.
  
  2. $N_{k-\ell} < N_i$ does not hold for all time instants from $t = k - \ell + 1$ to $t = k$. Meanwhile, we think that some targets may be dropped from time $k - \ell$ to $k$ or some targets die and after that some new targets are born. Find the first time $t$ satisfying $N_{k-\ell} < N_i$, and then let $\ell' = \ell - (k - t + 1)$. If $\ell' = 0$, such a case will happen when $t = k - \ell + 1$, and then we get the final lag $\ell' = 1$. If $\ell' < 0$, the case occurs when $N_{k-\ell} > N_{k-\ell+1}$, so we adjust the lag according to Case 3.
  
  3. $N_{k-\ell} > N_i$ if $N_{\text{smooth}, k - \ell + 1} = N_{k-\ell}$, where $N_{\text{smooth}, k - \ell + 1}$ is the smoothed cardinality at time $k - \ell - 1$. In such a situation, it can be shown that some tracks may be dropped by the filter or some targets die after time $k - \ell$. Therefore, the backward smoothing is omitted, and we just use the filtered PHD as the smoothed PHD. Otherwise, we consider that the forward filtering has generated some spurious targets, and then let $\ell' = 3$.

- **Step 3 (PHD smoothing):** let the initial backward smoothed PHD be $\hat{v}_{k-\ell, \ell'}$ and, according to the adjusted smooth lag $\ell'$, we can get the final smoothed PHD at time $k - \ell$.

**Remark.** There may be some other instances in Case 2), such as $N_{k-\ell} > N_i$ and it holds for all times $t$ between $k - \ell + 1$ and $k$ or only a few $t$ times. In such instances, it is difficult to identify missing targets, spurious targets, and the death of targets. Meanwhile, Ref.16 indicates that it is recommended that the time lag of the smoother is three, so here we actually make a tradeoff between the adjustment of smooth lag and the performance of our algorithm. It will be shown in Section 4 that it is worth of doing this.

4. Simulation results

In this section, we present simulation results to demonstrate the performance of the proposed smooth technique (with a lag of 5 time steps) using a linear motion model and a nonlinear nearly-constant turn model. Scenario I considers targets with the linear model and Scenario II considers targets with the nonlinear nearly-constant turn model. Both of the two scenarios verify the effectiveness of the proposed variable-lag forward–backward PHD smoother.

4.1. Scenario I: linear model

This simulation scenario is inspired by the experiment used by Nadarajah et al. in Ref.16. Consider a two-dimensional scenario with a surveillance region of $[-200 200] \times [-200 200]$ m. Three targets appear on the scene one after another, with various births and deaths throughout the 100-time-step scenario. The birth process has a PHD given by the intensity function $\lambda_t(x) = 0.1 N(x; m_{00}, P_{00})$, where $N(x; m_{00}, P_{00})$ represents a normal distribution with mean $m_{00} = [0 3 0 3] \text{T}$ and covariance $P_{00} = \text{diag}([100 100])$. The target state at time $k$, $x_k = [p_{k,0} p_{k,1} p_{k,2}] \text{T}$, consists of position $[p_{k,0} p_{k,1}] \text{T}$ and velocity $[p_{k,2} p_{k,3}] \text{T}$ of the target where $[\cdot] \text{T}$ represents the transpose of a matrix. The survived and detection probabilities are $P_{s} = 0.99$ and $P_{D} = 0.98$, respectively. The state transition of each target is given by

$$x_{k+1} = F_k x_k + v_{k+1},$$

where the target transition matrix $F_k$ is given by

$F_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and the process noise $v_{k+1}$ has a Gaussian distribution with mean zero and covariance matrix $Q_{k+1}$, which is given by

$$Q_{k+1} = \sigma^2 \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix}$$

where $T = 1s$ is the sampling period and $\sigma = 0.2$ is the deviation of the process noise. The ground positions of three tracks over 100 scans are plotted in Fig. 1. The individual $x$ and $y$ components of each track vs. time show the start and finish times of the tracks in Fig. 2 and Fig. 3, respectively. The sensor is located at $[0, -100]^T$, and the measurement model is given by
where $\varepsilon_k \sim N(0, \mathbf{R}_k)$ with $\mathbf{R}_k = \text{diag}(\sigma_{\theta}^2, \sigma_{\sigma}^2)$, and $\sigma_{\theta}$ and $\sigma_{\sigma}$ are the independent zero-mean Gaussian noises with standard deviations 0.05 and 2 for bearing and range, respectively. Clutter is uniformly distributed over the sensor field of view $[-\pi/2, \pi/2] \text{ rad} \times [0, 300] \text{ m}$. The average rate of clutter returns per scan is seven. The SMC method is implemented with 1000 particles per track, while tracks are initialized with 1500 particles. The extraction of state estimates from particle approximation is obtained using the K-mean clustering method.20

The results of a sample run of the FPHD smoother and the proposed VPHD smoother are shown in Figs. 4 and 5 with the true and smoothed $x$ and $y$ positions vs. time. In these two figures, both of the two PHD smoothers can smooth out the majority of states and identify the true tracks. However, the method we proposed has better performance in the estimate of the number of targets, especially when there are target deaths and anomalous smoothing. In this scenario, target 1 dies at time $k = 26$, and target 2 dies at time $k = 61$. As explained in the former section, the FPHD cannot cope with these problems, while the VPHD which takes the target deaths and anomalous smoothing into consideration, can smooth out target 1 between time $k = 21$ and $k = 25$ as well as target 2 between time $k = 56$ and $k = 60$.

In Figs. 6 and 7, the two smoothers compared with their corresponding filtered cardinality estimation are shown. The two filtered cardinality estimates are nearly the same. While in terms of smoothing, our proposed smoother has higher accuracy. In other words, the proposed variable-lag smoothing algorithm can restore both the spurious and the dropped tracks caused by the forward filtering. At the same time, it can almost eliminate the anomalous smoothing resulted from target deaths (at time $k = 26$ and time 61). Hence, VPHD
can significantly improve the performance of cardinality as well as state estimation.

To further demonstrate the improved performance, optimal sub-pattern assignment (OSPA) metric for $p = 1$ and $c = 100 m^2$, which is a counterpart of the root mean square error (RMSE) of a single target problem, is used for our multi-target performance evaluation. Results from 100 Monte Carlo runs are discussed as follows.

Fig. 8 shows the MC average of the estimated OSPA miss-distance for the FPHD and VPHD smoothers. From this figure, the VPHD smoother outperforms the FPHD smoother in terms of total miss-distance (an average improvement of 4.2 m or 14.8%). Especially, the anomalous smoothing (from time $k = 21$ to $k = 25$ and from time $k = 56$ to $k = 60$) in the FPHD smoother is eliminated by our proposed VPHD smoother. Fig. 9 illustrates OSPA localization and cardinality components for both of the smoothers. As can be seen from the figure, both smoothers are roughly on par in terms of location error. However, in terms of cardinality error, the proposed VPHD smoother significantly outperforms the FPHD smoother, especially in the vicinity of the times of target deaths (an average improvement of 4.5 m or 17.2%).

Finally, Fig. 10 shows the RMSE of the smoothed target numbers of both smoothers. The RMSE of cardinality of the VPHD smoother is about 3–5 dB lower than that of the FPHD smoother when target deaths occur. The results also suggest that our proposed multi-target variable-lag smoothing algorithm outperforms the current fixed-lag smoothing algorithm in the aspect of the smoothed number of targets.

4.2. Scenario II: nonlinear model

In this subsection, a typical MTT scenario with a nonlinear nearly-constant turn model inspired by the simulations suggested in the work of Mahler et al., is constructed as follows. A total of five targets appear on the scene with various births and deaths throughout the 100-time-step scenario. Each target follows a nonlinear nearly-constant turn model in which the target state takes the form $x_k = [\tilde{x}_k^T, \omega_k]^T$, where $\tilde{x}_k = [p_{x,k}^T, p_{y,k}^T, \dot{p}_{x,k}, \dot{p}_{y,k}]^T$, where $(p_{x,k}, p_{y,k})$ is the position, $(\dot{p}_{x,k}, \dot{p}_{y,k})$ is the velocity, and $\omega_k$ is the turn rate, respectively. The state dynamics are given by

$$\dot{x}_k = F(x_{k-1})x_{k-1} + G\omega_{k-1} \quad (21)$$

$$\omega_k = \omega_{k-1} + \Delta \omega_{k-1} \quad (22)$$
where
\[ F(\omega) = \begin{bmatrix} \sin \omega \Delta & 0 & -\frac{1 - \cos \omega \Delta}{\omega} \\ \cos \omega \Delta & 0 & -\sin \omega \Delta \\ \frac{1 - \cos \omega \Delta}{\omega} & 1 & \sin \omega \Delta \\ 0 & \sin \omega \Delta & 0 \end{bmatrix}, \quad G = \begin{bmatrix} \Delta^2 & 0 \\ \Delta & 0 \\ \Delta^2 & 0 \\ 0 & \Delta \end{bmatrix} \]
\[ \Delta = 1s, \; \omega_k \sim N(0; \sigma^2_\omega), \; \sigma_v = 3 \text{ m/s}^2, \; \omega_k \sim N(0; \sigma^2_{\omega_k}), \; \text{and} \; \sigma_\theta = 0.1 \pi/180 \text{ rad/s. We assume no spawning, and that the spontaneous birth RFS is Poisson with intensity given by a Gaussian mixture shown below}^{15,17} \]
\[ \gamma_k(x) = \sum_{i=1}^{5} o_{b,i} \mathcal{N}(x, m_b^{(i)}, P_b) \]
where
\[ o_{b,i} = 0.1, \; m_b^{(1)} = [-1500 0 250 0]^T, \; m_b^{(2)} = [-250 0 1000 0]^T, \; m_b^{(3)} = [250 0 750 0]^T, \; m_b^{(4)} = [1000 0 1500 0]^T, \; m_b^{(5)} = [500 0 500 0]^T P_b \]
\[ = \text{diag}([10 10 10 \pi/180]^T). \]

Each target has a probability of detection \( P_{D,k}(x) = 0.98 \) and a survival probability \( P_{S_{B,k}}(x) = 0.99 \). An observation consists of bearing and range measurements
\[ z_k = \begin{bmatrix} \arctan(p_{x,k}/p_{y,k}) \\ \sqrt{p_{x,k}^2 + p_{y,k}^2} \end{bmatrix} + \xi_k \]
where \( \xi_k \sim N(0; R_\xi) \) with \( R_\xi = \text{diag}([\sigma_\xi^2 \sigma_\xi^2]) \), \( \sigma_\xi = 0.3\pi/180 \text{ rad/s} \), and \( \sigma_\theta = 0.7 \text{ m} \). The clutter RFS follows the uniform Poisson model over the surveillance region \([0 \pi] \times [0 2000] \text{ m} \), with \( \lambda_c = 1.1 \times 10^{-3} (\text{rad.m})^{-1} \) (i.e., an average of seven clutter returns in the surveillance region).

The true target trajectories are plotted in Fig. 11 along with the start and stop positions of each track. The SMC method is implemented with 1000 particles per track. The extraction of point estimates from particle approximation is obtained using the K-means clustering method.

The results of a sample run of the FPHD smoother and the proposal VPHD smoother are shown in Figs. 12 and 13 with the true and smoothed \( x \) and \( y \) positions vs. time. Intuitively, both smoothers are able to identify target births and deaths, and maintain target lock for the majority of each track. However, the VPHD smoother is better in terms of state and cardinality estimation, that is, state biases decrease and number of targets estimates are more accurate. Especially, there are smooth anomalies from time \( k = 46 \) to \( k = 50 \), and from time \( k = 56 \) to \( k = 60 \) in the FPHD smoother, when two targets die one after another. Therefore, we can see that the FPHD smoother is sensitive to deaths of targets. Meanwhile, our VPHD smoother is more robust to target deaths which can be verified in Fig. 13.

In Figs. 14 and 15, the target number estimates are plotted. The two smoothers compared with their corresponding filtered cardinality estimations are shown. The two filtered cardinality
estimations are nearly the same. However, in terms of smoothing, our proposed algorithm has advantages, that is, the variable-lag smoothing algorithm can not only restore the spurious or dropped tracks caused by the forward filtering, but also eliminate the anomalous smoothing results from target deaths, which can significantly improve the performance of cardinality as well as state estimation. Note that our proposed smoother may generate anomalous cardinality estimates occasionally, but these false estimates die out very quickly.

To further demonstrate the improved performance, we also use the OSPA as the performance metric, and results from 100 Monte Carlo runs are discussed as follows.

Fig. 16 shows the MC average of the estimated OSPA miss-distance for the FPHD and VPHD smoothers. The VPHD smoother outperforms the FPHD smoother in terms of total miss-distance (an average improvement of 5.7 m or 25.6%).

Especially, the anomalous smoothing in the FPHD smoother is eliminated by our proposed VPHD smoother. Fig. 17 illustrates OSPA localization and cardinality components for both of the smoothers. We can see from the figure that the VPHD smoother slightly outperforms the FPHD smoother in terms of localization error (an average improvement of 0.4 m or 5%). However, in terms of cardinality error, the proposed VPHD smoother significantly outperforms the FPHD smoother, especially in the vicinity of the times of target deaths (an average improvement of 7.2 m or 49%).

Finally, Fig. 18 shows the RMSE of the smoothed target numbers of both smoothing algorithms. The results also suggest that our proposed multi-target variable-lag smoothing algorithm outperforms the current fixed-lag smoothing algorithm in the aspect of the smoothed number of targets.

5. Conclusions

(1) By using the prior knowledge of forward filtering in current PHD smoother, we have proposed a variable-lag PHD smoothing recursion.

(2) The approach involves forward filtering using the standard PHD filter recursion, and adjustment of the lag of smoothing using the cardinality estimations of the forward filtering and the backward smoothing.

(3) Simulations are performed with the proposed method on linear and nonlinear multi-target tracking scenarios. Simulation results confirm that our proposed method can significantly improve both of the state and cardinality estimates, and this approach can be used in current PHD-based multi-target tracking systems conveniently.

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References


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