Mode I crack propagation under high cyclic loading in 316L stainless steel

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Abstract

Mode I fatigue crack propagation under high mechanical-thermal stress in large-scale yielding condition is observed in an experimental setup (named PACIFIC) developed by EDF. However, the crack growth within the ductile zones cannot be quantified numerically: the traditional methods based on the fracture mechanics such as the Paris propagation law are not appropriated, and no other ductile fatigue propagation law is yet approved. The objective of this study is to come up with a robust fatigue propagation law in large-scale plastic condition.

An incremental model was developed for mode I fatigue crack growth under complex load spectra in consideration of non-linearity of material in the crack tip region. The displacement field in the crack tip region is partitioned into an elastic field and a plastic field which presents the plastic deformation within the crack tip region. Each field is approached by the product of a spatial reference field and its intensity factor. This approach allows dealing with the crack growth at large-scale. Our study aims at extending this model to account for the large-scale plasticity. For this purpose, besides the elastic and plastic field, the velocity field of the crack tip region is approached by a third field which is assumed to introduce the large-scale plasticity and is in the form of a product of a spatial reference field and its intensity factor rate. Non-linear FE analyses were used to show that such an approximation is reasonably precise. The analyses conducted in this study indicate that the large yielding plasticity has no direct contribution to the crack propagation, thus the crack growth rate is merely proportional to the small-scale plasticity rate as in small-scale yielding condition. In addition, the 316L displays a very significant cyclic hardening effect and a memory effect. A constitutive law for this material in large strain range was identified that accounts for this memory effect, and the simulations are in good agreement with the experiments.

Keywords: Fatigue crack propagation; Large-scale plasticity; 316L stainless steel

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1. Introduction

The 316L stainless steel is currently widely used as structural material for some components of pressurized water reactor (PWR). In PWR, as a result of temperature gradients which occur during plant operating transients, some of these components are subjected to high repeated non-cyclic thermal loadings, which could introduce a subcritical mode I fatigue crack propagation in large-scale yielding conditions if we assume a crack initiation on a hypothetical defect (Vallet et al., 2012). Thus in order to guarantee the functioning life of a PWR power station, it is essential to be able to quantify the crack propagation rate.

Fatigue crack propagation has been largely studied for years. However no validated model exists in large-scale yielding conditions since the general methods based on linear elastic fracture mechanics are no longer appropriate. An incremental model was developed for mode I fatigue crack growth under variable amplitude loadings. This model accounts for the history effects through the modelling of the non-linear behaviour of the crack tip plastic zones in small-scale yielding conditions (Pommier et al., 2007). Decreuse and Fremy (2010 &2012) have enriched and validated the model by experimental results of crack propagation in non-proportional mixed mode and for materials with a combined non-linear isotropic and kinematic behaviour. This paper is devoted to extending this model from small-scale yielding conditions to large-scale yielding conditions.

To achieve this, a non-linear finite element method was applied in the development of model by introducing a third decomposition of velocity field of the crack tip region so as to take account of the contribution of large-scale plasticity to the kinematics of the crack tip zone. In addition, a constitutive law for 316L stainless steel for a total stain range from 0.1% to 2.0% was identified.

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>FE finite element method</td>
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<tr>
<td>LEFM linear elastic fracture mechanics</td>
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<tr>
<td>316L SS 316L stainless steel</td>
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<tr>
<td>CTOD crack tip opening displacement</td>
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<tr>
<td>$K_{I}^{\infty}$ mode I nominal stress intensity factor</td>
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<tr>
<td>$\tilde{K}_{I}$ mode I pseudo-elastic intensity factor</td>
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<tr>
<td>$p_{I}$ mode I small-scale plastic intensity factor</td>
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<td>$g_{I}$ mode I large-scale plastic intensity factor</td>
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<tr>
<td>$u_{I}^f$ mode I elastic reference displacement field</td>
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<td>$u_{I}^p$ mode I large-scale plastic reference displacement field</td>
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<td>$C_{1R}, C_{2R}, C_{3R}$ related errors associated with elastic, small-scale elastic-plastic or large-scale elastic-plastic approximation of velocity field of crack tip region respectively</td>
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2. Material constitutive behavior

316L SS used in the present study has the following chemical composition (in wt.%): C: 0.026, Mn: 1.81, Si: 0.42, S: 0.004, P: 0.033, Ni: 12.00, Cr: 16.96, Mo: 3.43, Cu: 0.03, N: 0.05.

Experiments were conducted to characterize and identify the cyclic elastic-plastic constitutive law for 316L SS. Stain-controlled push-pull tests were carried out on cylindrical specimens of extensometer length of 10mm and a diameter of 8mm. A constant strain rate of $10^{-3}$ s$^{-1}$ was employed for increasing strain amplitude from $\pm 0.1\%$ to $\pm 2.0\%$ at room temperature. Decreasing numbers of cycle for each strain level was chosen from 200 cycles for strain amplitude 0.1% to 10 cycles for strain amplitude 2.0% according to the compromise between the cyclic stabilization and the damage accumulation of material.

The 316L SS displays a significant hardening effect and a memory effect according to experimental results. Hence, the cyclic elastic-plastic behaviour of this material was modelled with the von Mises yield criterion and the combined one non-linear isotropic and two non-linear kinematic hardening in consideration of memory effect. The
material constitutive law parameter identification was carried out using code-aster FE software. A good agreement is found between experiments and simulations.

3. Development of multi-scale approach

The main idea of this paper is to extend the condensed incremental crack tip plasticity model from small-scale yielding conditions to large-scale yielding conditions. This model was developed initially for mode I fatigue crack propagation in account of the plasticity induced history effect in crack tip region. It aims at establishing a model reasonably precise compared with elastic-plastic FE computation but condensed into a set of partial derivative equations so as to avoid numerous elastic-plastic FE computations in the future. (Pommier et al., 2005)

For this purpose, the kinetics of the crack tip region is represented by a set of condensed variables. In linear elastic fracture mechanics (LEFM), the displacement field of the crack tip region is approached by the product of spatial reference field \( u_f^{\circ} \) and nominal stress intensity factor \( K_f^{\circ} \). In small-scale cyclic plastic conditions, an intensity factor \( K_f^t \) was firstly defined for elastic spatial reference field \( u_f^t \), and then an additional spatial reference field \( u_f^c \) to represent the small-scale plasticity was introduced as well as its intensity factor \( \rho_i \).

So in order to generalize this approach to large-scale plasticity, besides the intensity factors of the elastic part \( K_f^t \) and of the small-scale yielding plastic part \( \rho_i \), a third intensity factor \( \gamma_i \) was introduced to account for the contribution of large-scale plasticity to the kinematics of the crack tip region. Each of them is associated to a spatial reference field given once for all: \( u_f^t (r, \theta) \) for the elastic part, \( u_f^c (r, \theta) \) for the small-scale yielding part and \( u_f^\gamma (r, \theta) \) for the large-scale yielding part.

The three reference fields were extracted through FE computations in order to study the influence of large-scale plasticity on crack propagation and to develop the model to be able to account for large-scale plasticity.

3.1. Hypothesis

Some hypotheses were made. First of all, Fig 1 presents a point P in a local coordinate system \((e_x,e_y,e_z)\) attached to the crack tip \( T \). The velocity field in the global coordinate system \( O(X,Y,Z) \) may be written as in equation 1. The first term in the equation relates to crack growth, the second term come from the velocity of crack tip, the two terms are not subjected to this paper. The last term \( v(P)_{R_T} \) denoted by \( v(P) \) afterwards, stems from the elastic-plastic strain within the crack tip region (domain D).

\[
v(P)_{R_0} = v(T)_{R_0} + w \left( \frac{R_0}{r_0} \right)^k TP + v(P)_{R_T}
\]

Then the velocity field \( v(P) \) is partitioned into elastic and plastic components according to the material and loading conditions. Each component is approximated as a product of an intensity factor rate and a spatial reference displacement field. In our study, for a material of elastic-plastic behaviour in large-scale yielding condition, the velocity field is approached by a sum of products of spatial reference fields and intensity factor rates in equation 2. The first term is to represent the elastic part, the second is for small-scale plasticity and the last term is for large-scale plasticity with hypothesis that the kinetics of small-scale plasticity and large-scale plasticity on the velocity field within domain D could be separated.

\[
v(P, t) = \dot{K}(t) * u_f^t (P) + \dot{\rho}(t) * u_f^c (P) + \dot{\gamma}(t) * u_f^\gamma (P)
\]
This approximation is performed using a proper orthogonal decomposition, such as the Karhuen-Loeve transform for instance. The details of the procedure applied to FE computation to determine the reference fields are reported in section 3.2. This approximation implies that for a domain D:

\[
\sum_{\text{PED}}(u_i^f(P)) \cdot (u_j^f(P)) = 0, \quad \sum_{\text{PED}}(u_i^g(P)) \cdot (u_j^g(P)) = 0, \quad \sum_{\text{PED}}(u_i^g(P)) \cdot (u_j^g(P)) = 0
\]  

(3)

Hence, if the velocity field \(v(P)\) is known via FE computation and provided the reference fields have been determined beforehand, the three intensity factor rates could be determined as follows:

\[
\dot{K}(t) = \frac{\sum_{\text{PED}}(u_{EF}(P_{t,n+1})-u_{EF}(P_{t,n}))*u_{f}^g(P)}{\sum_{\text{PED}}u_{f}^g(P)*u_{f}^g(P)}
\]  

(4)

\[
\dot{\rho}(t) = \frac{\sum_{\text{PED}}(u_{EF}(P_{t,n+1})-u_{EF}(P_{t,n}))*u_{f}^g(P)}{\sum_{\text{PED}}u_{f}^g(P)*u_{f}^g(P)}
\]  

(5)

\[
\dot{g}(t) = \frac{\sum_{\text{PED}}(u_{EF}(P_{t,n+1})-u_{EF}(P_{t,n}))*u_{f}^g(P)}{\sum_{\text{PED}}u_{f}^g(P)*u_{f}^g(P)}
\]  

(6)

And the related error associated with this approximation is calculated as follows:

\[
C1R = \frac{\sqrt{\sum_{\text{PED}}(v_{EF}(P_{t})-K(t)*u_{f}^g(P))^2}}{\sqrt{\sum_{\text{PED}}(v_{EF}(P_{t}))^2}}
\]  

(7)

If the velocity field is approached only by elastic field.

\[
C2R = \frac{\sqrt{\sum_{\text{PED}}(v_{EF}(P_{t})-K(t)*u_{f}^g(P)-\dot{\rho}(t)*u_{f}^g(P))^2}}{\sqrt{\sum_{\text{PED}}(v_{EF}(P_{t}))^2}}
\]  

(8)

If the velocity field is approached by elastic and small-scale plasticity field.

\[
C3R = \frac{\sqrt{\sum_{\text{PED}}(v_{EF}(P_{t})-K(t)*u_{f}^g(P)-\dot{\rho}(t)*u_{f}^g(P)-\dot{g}(t)*u_{f}^g(P))^2}}{\sqrt{\sum_{\text{PED}}(v_{EF}(P_{t}))^2}}
\]  

(9)

If the velocity field is approached by elastic, small-scale plasticity and large-scale plasticity.

This approximation in equation 2 is very useful since it allows representing the velocity field in domain D in cyclic elastic-plastic conditions via a set of condensed variables \((\dot{K}(t), \dot{\rho}(t), \dot{g}(t))\). The quality of the approximation is measured by the related errors in equation 7-9.

3.2. Implementation

3.2.1. FE model

In order to apply the procedures presented in section 3.1, a 2D FE model was prepared (Fig 2). The model is an “infinite” sheet of dimension 2m×2m and contains a through thickness central crack with 2\(a\)=20mm in length. ¼ of the model is simulated as a result of symmetry. Linear plane strain elements were employed. A refined zone with structural mesh is defined by a semi-circular with a radius \(R=1\)mm. The displacement of all the nodes located between \(R=1\)mm and \(r=100\)μm from the crack tip is recorded at every time increment during the computation.
3.2.2. Construction of the reference fields

First of all, the elastic spatial reference field $u_i^e(P)$ was constructed through an elastic numerical simulation, using imposed displacement boundary conditions which made $K_i^p = 1 \text{MPa}\sqrt{m}$. Then, monotone elastic-plastic FE calculation were performed to determine $u_i^e$ and $u_i^p$. To simplify the problem, a combined non-linear isotropic and kinematic hardening for 316L SS behaviour without history effect identified by Chaboche et al. (1979), was employed with the parameters as follows: $E=196\text{GPa}$, $v=0.3$, $R_0=180\text{MPa}$, $R_{inf}=223\text{MPa}$, $b=5$, $C_1=50400\text{MPa}$, $J_1=280$, $C_2=2250\text{MPa}$, $\gamma_1=280$, $C_2=2250\text{MPa}$, $\gamma_2=15$. Such a load was employed that plastifies the whole model. The displacement field at each time increment was stored.

Therefore, the small-scale plastic reference field was determined when the plasticity was constrained in elastic bulk. The variation of $\tilde{K}(t)$ was calculated according to equation 4. The remainder of the displacement field for each time increment is then calculated as follows:

$$u^R(P,t_{n+1} \rightarrow t_n) = (u^{EF}(P,t_{n+1}) - u^{EF}(P,t_n)) - \tilde{K}(t)u_i^e(P)$$ (10)

Then the Karhunen-Loeve transform is applied to the remainder displacement field, by extracting the first decomposition term, we have:

$$u^R(P,t_{n+1} \rightarrow t_n) \approx f(t)g(P)$$ (11)

The spatial distribution $g(P)$ is made non-dimensional by a multiplication factor A so that the intensity factor $\rho$ can be directly representative of the crack tip opening displacement (CTOD). The small-scale plastic spatial reference field is defined as follows:

$$u_i^e(P) = Ag(P)$$ (12)

Therefore the small-scale plastic intensity factor $\rho$ for every time increment is obtained as follows:

$$\rho(t) = \frac{\Sigma_{P\in D}(u^{EF}(P,t_{n+1}) - u^{EF}(P,t_n))u_i^e(P)}{\Sigma_{P\in D} u_i^e(P)^2}$$ (13)

The same procedure is employed to construct the spatial reference field of large-scale plasticity while the remainder of the displacement field during each time increment for the whole step is calculated as follows:

$$u^R(P,t_{n+1} \rightarrow t_n) = (u^{EF}(P,t_{n+1}) - u^{EF}(P,t_n)) - \tilde{K}(t)u_i^e(P) - \rho(t)u_i^p(P)$$ (14)

Then the Karhunen-Loeve transform is applied to the remainder displacement field, by extracting the first decomposition term, we have:

$$u^R(P,t_{n+1} \rightarrow t_n) \approx f(t)g(P)$$ (15)

This time, we take the spatial distribution $g(P)$ directly as the spatial reference field of large-scale plasticity $u_i^p$. 

![Fig 2: FE model; Mesh detail of the crack tip region](image)
3.2.3. Extraction of related errors for a monotone cyclic loading

A monotone cyclic loading in mode I was simulated. The intensity factors were extracted according to equations 4-6. The related errors were calculated according to equation 7-9 and reported in Fig 3(a). It is observed that C2R and C3R are much lower than C1R, and C3R lower than C2R. This indicates that an elastic plastic approximation is better than an elastic approximation; Furthermore in large-scale plasticity, addition of the third reference field improves the quality of velocity field’s approximation. On the other hand, for increment t=0, t=50 and t=72, an elastic approximation is as precise as an elastic plastic approximation, which means that the material behaves elastically during the time increments. Moreover, at increments when the material behaves totally plastically, C3R is relatively low, indicating the approximation of the velocity field of the crack tip region with three terms of decomposition is sufficiently precise, this validate our assumption in section 3.1.

3.2.4. Large-scale plastic reference field analysis

Three models with different size of domain D were simulated to construct \( u_t^\rho (P) \) using the procedures described in section 3.2.2. Each \( u_t^\rho (P) \) obtained is then approximated by a product of an angular function and a radial function through Karhunen-Loeve transform, i.e. \( u_t^\rho (P) \approx f(r)g(\theta) \). \( f(r) \) was dimensioned in order to make the comparison (Fig 3(b)).

Two conclusions could be drawn. Firstly, the three curves have almost the same trend, which means the size of the crack tip zone has little influence on the large-scale plastic reference field. Secondly, take the red curve for example, it is observed that the curve tends to zero when it approaches the crack tip. This phenomenon is essential to our study, which means that the plastic flow of structure has little or no direct contribution to the crack propagation as the crack propagate as a result of the plasticity of the crack tip region. Hence, it is indicated that the propagation law in the condensed model remains the same in large-scale yielding conditions than in small-scale yielding conditions, namely the crack growth rate \( \dot{a} \) is merely proportional to the small-scale plasticity rate \( \dot{\rho} \). This will be validated by the cracking experiment in large-scale yielding conditions.

4. Conclusions and prospects

This article deals with crack propagation in large-scale yielding conditions by extending the condensed incremental model proposed by S.Pommier. Apart from the intensity factors of the elastic part \( \dot{K} \) and of the small-scale yielding plastic part \( \dot{\rho} \), a third intensity factor \( \dot{g} \) was introduced to account for the influence of large-scale plasticity to the kinematics of the plastic crack tip zone. Non-linear FE analyses were used to show that such an approximation is reasonably precise. The analyses conducted in this study indicate that the large-scale yielding plasticity has no direct contribution to the crack propagation, thus the crack growth rate \( \dot{a} \) is merely proportional to the small-scale plasticity rate \( \dot{\rho} \) as in small-scale yielding condition. In addition, the 316L displays a very significant
cyclic hardening effect and a memory effect. A constitutive law for this material in large strain range was identified that accounts for this memory effect, and the simulations are in good agreement with the experiments.

In the next future, the propagation law of the model should be validated for different material, material with history effect for instance. The second part of the model which is a non-local elastic-plastic constitutive model for the crack tip region provided the small-scale plasticity rate $\dot{\rho}$ as a function of the nominal loading conditions $K^{oo}$ and the internal variables of the model (locus and size of the elastic domain) has to be modified for large-scale yielding conditions. Furthermore, the crack propagation experiment in large-scale yielding condition will be carried out so as to validate the simulation results.

References


