ELSEVIER
Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia - Social and Behavioral Sciences 39 (2012) 831 – 844
Procedia
Social and Behavioral Sciences

The Seventh International Conference on City Logistics

Maximizing resource effectiveness of highway infrastructure maintenance inspection and scheduling for efficient city logistics operations

Manoj K. Jha[a*], Shaghayegh Shariat[a], Jawad Abdullah[b], Bimal Devkota[c]

[a]*Department of Civil Engineering, Morgan State University, Baltimore, MD 21251, USA*
[b]*Maryland Sate Highway Administration, Baltimore, MD, USA*
[c]*City of Baltimore Department of Transportation, MD, USA*

Abstract

The safe and efficient movement of people and cargo on roadways is dependent on a well functioning highway system which requires effective maintenance policies focused on the maximum use of invested resources. For efficient city logistics operations various components of highway infrastructure, such as pavements, guardrails, and roadside signs must be maintained and kept in acceptable operating condition as elements of an integrated highway network. Due to the shrinking budget for undertaking infrastructure inspection and maintenance activities Resource Effectiveness (RE) becomes paramount. Theoretically, Resource Effectiveness (RE) is intended to get the optimum performance out of a project or investment. Instead of adding more resources (like labor, equipment, etc.), it will concentrate on the prudent utilization of resources or investment. Thus, RE in a broader sense, is concerned with the prudent use of labor, equipment, and material. In this paper we develop an optimization approach for maximizing resource effectiveness of highway infrastructure maintenance investments, subject to budget constraints, based on the concept of the well-known Cobb-Douglas production function. Several examples are presented.

Keywords: Resource effectiveness; highway infrastructure maintenance; Cobb-Douglas production function; highway asset management; optimization

[*] Corresponding author. Tel.: +1-443-8851446; fax: +1- 443-8858218.
E-mail address: manoj.jha@morgan.edu

 doi:10.1016/j.sbspro.2012.03.151

1. Introduction

We are living in challenging economic times which require thoughtful use of available resources in order to maximize their productivity. Often times in organizations resources are wasted since minimal effort is devoted to maximize their effectiveness. For effective city logistic operations a well functioning highway system is desired, which requires timely inspection and maintenance of various components of the highway system, such as pavements, bridges, tunnels, and other assets. In our previous works ([5]&[6]) we have developed optimization models for obtaining optimal inspection and maintenance schedules of highway assets over a planning horizon. A comprehensive maintenance strategy requires an intense resource study to serve as the basis for increased reliability of best practices. This paper formulates a special class of maintenance inspection and scheduling problem in which the resource effectiveness is optimized to achieve a desired service output level.

The concept of resource effectiveness has primarily been studied in the area of economics and dates back to 1951. Debreu (1951) called the wasted resource as "dead loss" and developed a mathematical approach to calculate the coefficient of resource utilization. Intuitively, the issue of resource effectiveness can be expressed as shown in Fig. 1(a) & Fig. 1(b). Fig. 1(a) shows that an optimal output level Q*can be achieved at a resource effectiveness level RE_1. Thus any additional resource, say RE_2 will be wasted since it will not improve the output level to any further extent. The hatched area in Fig. 1(a) shows wasted resource.

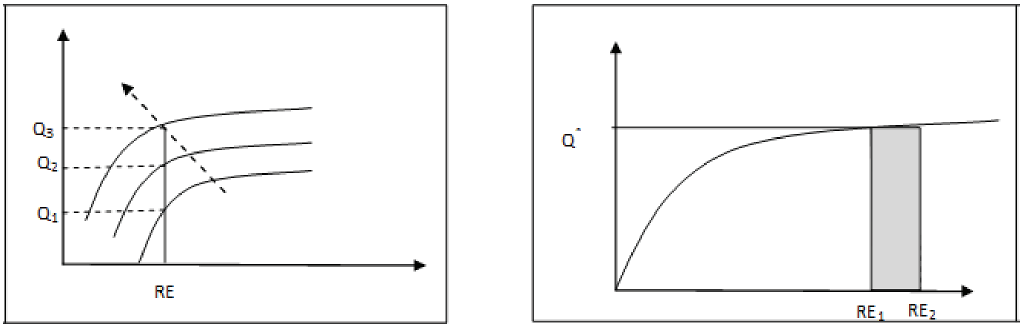

Fig. 1. (a) Graphical illustration of service quality (Q) vs. resource effectiveness (RE); (b) Different levels of service quality (Q) for a given resource effectiveness (RE)

In many real-life situations, especially in agencies and organizations this is what is observed, i.e., resources are wasted quite often given that no further improvement in the service quality is possible. In an era of budgetary and resource limitations modern methods are continually being sought to increase the reliability and availability of available resources.

Fig. 1(b) shows how different output levels (Q_1…,Q_3) can be achieved for the same resource effectiveness level RE. Since output levels are generally a combination of Labor (L), Equipment (E), and Material (M), different L, E, and M combinations may yield different Q's subject to a given resource effectiveness level RE.

Theoretically, Resource Effectiveness (RE) desires to get the optimum performance out of a project or an investment for accomplishing specified objectives. Rather than over-exerting undue maintenance efforts, the ideal circumstance would be to exhaust cost-effective means to predict transport system failures. This consequence would afford a highway agency the ability to effectively maintain its' transportation system networks, while cost-effectively accruing a safety benefit by avoiding unpredicted breakdowns. Instead of adding more resources (like labor, equipment etc.), it will concentrate on the prudent utilization of resources and investments.

Thus, RE in a broader sense, is concerned with the prudent use of labor, equipment, and material. Equipment effectiveness quantifies the time it uses to do the job, while human resource effectiveness is concerned with the time that an individual takes to perform assigned tasks on a project. If an organization wants to maximize its' resource effectiveness, it should prepare a clear definition of all the activities performed by its resources and then identify the most important resources, highlighting the constrained tasks. A higher degree of resource effectiveness should result in a reduced cycle-time, with minimum maintenance requirements and lesser supervision. As a result, the organization can increase its' capacity and achieve greater resource life-cycle. Since there is always some limitations on budget allocations [8] there should be a way to optimize the resource effectiveness based on the specified constraints.

1.1. Resource effectiveness and city logistics problems

City logistics activities entail the movement of people and goods in an efficient manner. For example, vehicle routing problems deal with obtaining optimal vehicle routes subject to specified objectives and constraints [9] & [10]. The objective of resource effectiveness is to derive the optimum performance out of an invested infrastructure instead of contributing additional funds. The guarantee of service quality is important to the users and maintenance agencies with restrictive budgets charged with ensuring that the highway network capacity is suitable and adequate for service. The resource life-cycle suggests a sequence of time-events to occur prior to accomplishing the resource goals or objectives. A segment of the engineering profession considers reliability engineering as seeking best practices and the right tools to govern the entire life-cycle of a resource from designation to removal from service.

In the highway maintenance arena, the operational inspection work is strategically intended to determine the right amount of maintenance work to keep the resources at an optimal level. Achieving this goal requires a logistic centre and an extensive application of information and communication technologies, namely the tracking of vehicles with, individual route guidance, electronic data interchange protocols, to also include tracking routing and scheduling in real-time. Additionally, the coordination of all operations in real-time is desirable in order to promptly react to any unexpected events. Transport planners and managers of logistic centers, support individual City Logistics systems in a given urban municipality that provide scheduling tours and routing of the highway network.

2. Problem formulation

We formulate the resource effectiveness maximization problem by introducing the notion of the production function generally covered in Economics [1]. The nomenclature used in the formulation is shown in the table below. Since the conceptualized output level in Fig. (1) can be thought of as a production function, its' measure can be regarded as the service level in the infrastructure inspection and scheduling problem described in our previous works [5]&[6]. The production functions are generally a measure of a certain combination of Labor, Equipment, and Material, and generally exhibit a constant elasticity of substitution property. In the Infrastructure Inspection and Scheduling (IIS) problem, a production function refers to the output level Q reflecting the measure of effort by a highway crew to carry out the inspection and maintenance operations.

2.1. Production functions

In economics, constant elasticity of substitution (CES) is a property of some production and utility functions. More precisely, it refers to a particular type of collective function which combines two or more types of expenditure, or two or more types of productive inputs into a collection. This collective function shows signs of constant elasticity of substitution. The CES production function is a type of production function that displays constant elasticity of substitution between capital and labor [2].

Nomenclature

Q	Output
F	Factor of productivity
a	Share parameter
X	Production factors (i = 1,2...n)
s	Elasticity of substitution.
y	Total production (the monetary value of all goods produced in a year)
L	Labor input
K	Capital input
A	Total factor productivity
α and β	Output elasticities of labor and capital, respectively. These values are constants determined by available technology.
L*	Optimal labor work hours
E*	Optimal equipment usage hour
M*	Optimal required material in kg
C*	Total Cost
Q*	Optimal Resource Effectiveness Production Function Value
RE*	Optimal Resource Effectiveness Utility Function Value

The general form of the CES production function is [1]:

$$Q = F.\left[\sum_{i=1}^{n} a_i^{1/s} X_i^{\frac{(s-1)}{s}}\right]^{\frac{s}{(s-1)}} \tag{1}$$

Leontief, Linear and Cobb-Douglas production functions are special cases of CES Production function [3]. If s approaches 1, we get the Cobb-Douglas function expressed as:

$$y = \gamma x_1^{\alpha_1} \times .... \times x_n^{\alpha_n} \tag{2}$$

If s approaches infinity we get the linear (perfect substitutes) function expressed as:

$$y = \alpha_1 x_1 + ... + \alpha_n x_n \tag{3}$$

if s approaches 0, we get the Leontief (perfect complements) function expressed as:

$$y = \min\left\{\frac{x_1}{\alpha_1} + ... + \frac{x_n}{\alpha_n}\right\} \tag{4}$$

2.1.1. Cobb-Douglass production function

Knut Wicksell (1851–1926), proposed the Cobb–Douglas functional form of production functions in economics which is used to represent the relationship of an output to inputs. It was then tested by Charles Cobb and Paul Douglas in 1900–1928 against statistical evidence. Cobb–Douglas function used at micro level is expressed as (Cobb and Douglas, 1928):

$$y = AL^{\alpha} K^{\beta} \tag{5}$$

Output elasticity measures the responsiveness of output to a change in levels of either labor or capital used in production.

Cobb–Douglas function used at macro level is expressed as:

$$y = K^{\alpha} L^{1-\alpha} \tag{6}$$

Where K is capital and L is labor. When the model coefficients sum to one, the production function is first-order homogeneous, which implies constant returns to scale, that is, if all inputs are doubled then the output will double.

2.2. Utility functions

Nonetheless, the Cobb–Douglas function has been applied in a lot of other contexts besides production. It can be applied to utility as follows:

$$U(x_1, x_2) = x_1{}^{\alpha} x_2{}^{\beta} \tag{7}$$

Where x_1 and x_2 are the quantities consumed of good 1 and good 2, respectively. In its generalized form, where x_1, x_2, ... ,x_L are the quantities consumed of good 1, good 2, ..., good L, a utility function representing the Cobb–Douglas preferences may be written as:

$$\tilde{u}(x) = \prod_{i=1}^{L} x_i{}^{\lambda_i} \quad \beta \tag{8}$$

With x = (x_1, x_2, ... ,x_L). Setting $\lambda = \lambda_1 + \lambda_2 + ... + \lambda_L$ and because the function $x \rightarrow x^{1/\lambda}$ is strictly monotone for x > 0, it follows that $u(x) = \tilde{u}(x)^{1/\lambda}$ represents the same preferences. Setting $\alpha_i = \lambda_i / \lambda$ it can be shown that:

$$u(x) = \prod_{i=1}^{L} x_i{}^{\alpha_i}, \sum_{i=1}^{L} \alpha_i = 1 \tag{9}$$

The utility function may be maximized by looking at the logarithm of the utility:

$$\ln u(x) = \sum_{i=1}^{L} \alpha_i \ln x_i \tag{10}$$

Thus, the utility maximization problem can be expressed as:

$$\text{Max} \sum_{i=1}^{L} \alpha_i \ln x_i \tag{11}$$

$$\text{s.t.} \sum_{i=1}^{L} p_i x_i = w \tag{12}$$

2.3. Solution procedure

The above formulation is solved by using the Lagrangian Multipliers which provide a strategy for finding the maximum/minimum of a function subject to constraints. The concept of Lagrangian Multipliers can be found in standard references [11] and has been skipped here for brevity.

3. Numerical examples

We show several examples to demonstrate the applicability of the developed formulation. The first two examples are relatively simpler with 2 and 3 variables to optimize the resource effectiveness in a highway maintenance project. In the examples, we have employed the concepts of both the production function maximization and utility function maximization. The third example is an application of the developed methodology in a real IIS problem.

3.1. The 2-variable numerical example

Suppose that in a highway maintenance project the cost of labor is \$40/hr, the cost of equipment is \$36/hr, and the available budget is \$70,000. The output elasticity of labor work is considered as 2/3, output elasticity of equipment is considered as1/2, and the factor of productivity (A) is estimated to be 12.

3.1.1. Cobb-Douglas production function application

Maximize $$Q = AL^{\alpha} E^{\beta} \tag{13}$$

Based on budget constraint $$B(L, E) = lL + eE \leq W \tag{14}$$

$$Q = 12L^{2/3}E^{1/2} \tag{15}$$

$$B = 40L + 36E = \$70{,}000 \tag{16}$$

Using the Lagrange multiplier (lambda) will lead to:

$$F'(L, E) = 12L^{2/3}E^{1/2} - \lambda\,(40L + 36E - 70{,}000) \tag{17}$$

Solving the above equation, we obtain:

$$L^{*} = 999;\ E^{*} = 832.5 \tag{18}$$

$$\text{Total Cost } C^{*}: (40(\$/hr) \times 999\ (hr)\ ) + (36(\$/hr) \times 832.5\ (hr)\ ) = \$69{,}930 \tag{19}$$

$$\text{Optimal } Q^{*} = 34{,}601$$

The results indicate that the optimal output level Q* is achieved by investing L* units of labor and E* units of equipment. Therefore, any excess expenditure towards labor and equipment expenditures will be a waste since the productivity cannot be improved any further.

3.1.2. Cobb-Douglas utility function application

Maximize $$RE = \ln(L^{\partial 1} E^{\partial 2}) \tag{20}$$

Based on budget constraint $$B(L, E) = lL + eE \leq W \tag{21}$$

$$\sum \partial i = 1 \quad \partial 1 = \frac{\alpha}{\alpha + \beta} \quad \partial 2 = \frac{\beta}{\alpha + \beta} \tag{22}$$

$$\partial 1 = \frac{\alpha}{\alpha + \beta} = \frac{2/3}{2/3 + 1/2} = \frac{4}{7} \tag{23}$$

$$\partial 2 = \frac{\alpha}{\alpha + \beta} = \frac{1/2}{2/3 + 1/2} = \frac{3}{7} \tag{24}$$

$$RE = \frac{4}{7}\ln(L) + \frac{3}{7}\ln(E) \tag{25}$$

$$B = 40L + 36E = \$70,000 \tag{26}$$

Using the Lagrangian multiplier will lead to:

$$F'(L, E) = \frac{4}{7}\ln(L) + \frac{3}{7}\ln(E) - \lambda\,(40L + 36E - 70000) \tag{27}$$

Solving the above equation, we obtain:

$$L^* = 1,000\ ,\ E^* = 833 \tag{28}$$

$$Total\ Cost\ C^*: (40(\$/hr) \times 1000\ (hr)) + (36(\$/hr) \times 833\ (hr)) = \$69,988 \tag{29}$$

$$Optimal\ Resource\ Effectiveness\ RE^* = 6.8$$

The results indicate that an optimal resource effectiveness RE* is sufficient to achieve the desired productivity by investing L^* and E^* units of labor and equipment, respectively.

3.2. The 3-variable numerical example

Given:
The cost of labor is \$40/hr;
The cost of equipment is \$36/hr;
The cost of material is \$20/kg;
The available budget is \$180,000;
The output elasticity of labor work is considered as 2/3;
The output elasticity of equipment is considered as 1/2;
The output elasticity of material is considered as 1/3;
The factor of productivity (A) is estimated to be 12.

3.2.1. Cobb-Douglas production function application

Maximize $$Q = AL^{\alpha} E^{\beta} M^{\gamma} \tag{30}$$

Based on budget constraint $B(L,E) = lL + eE + mM \leq W$ *(31)*

$Q = 12L^{2/3}E^{1/2}M^{1/3}$ *(32)*

B = 40L+ 36E +20M= $180,000 *(33)*

Using Lagrangian multiplier will lead to:

$F'(L, E) = 12L^{2/3}E^{1/2}M^{1/3} - \lambda\,(40L + 30E + 20M - 180{,}000)$ *(34)*

Solving the above equation, we obtain:

L= 2,000 (hr), E*=1,666.67 (hr), M*= 2,000 (kg)* *(35)*

Total Cost C: (40($/hr) x 2,000 (hr))+ (36($/hr) x 1,666.67 (hr)) +*

(20($/kg) x 2,000 (kg))= $180,000 *(36)*

Optimal Productivity Q=979,797* *(36a)*

The results indicate that in order to achieve the desired output level of Q^*, the amount of investments needed towards labor, equipment, and material are L^*, E^*, and M^*, respectively.

3.2.2. Cobb-Douglas utility function application

Maximize $RE = \ln(L^{\partial 1}E^{\partial 2}M^{\partial 3})$ *(37)*

Based on budget constraint $B(L,E,M) = lL + eE + mM \leq W$ *(38)*

$$\sum \partial i = 1 \quad \partial 1 = \frac{\alpha}{\alpha+\beta+\delta}, \quad \partial 2 = \frac{\beta}{\alpha+\beta+\delta}, \quad \partial 3 = \frac{\delta}{\alpha+\beta+\delta} \tag{39}$$

$$\partial 1 = \frac{\alpha}{\alpha+\beta} = \frac{2/3}{2/3+1/2+1/3} = \frac{4}{9} \tag{40}$$

$$\partial 2 = \frac{\alpha}{\alpha+\beta} = \frac{1/2}{2/3+1/2+1/3} = \frac{3}{9} \tag{41}$$

$$\partial 3 = \frac{\alpha}{\alpha+\beta} = \frac{1/2}{2/3+1/2+1/3} = \frac{2}{9} \tag{42}$$

$$RE = \frac{4}{9}\ln(L) + \frac{3}{9}\ln(E) + \frac{2}{9}\ln(M) \tag{43}$$

B = 40L+ 30E +20M= $180,000 *(44)*

Using Lagrangian multiplier will lead to:

$$F'(L, E, M) = \frac{4}{9}\ln(L) + \frac{3}{9}\ln(E) + \frac{2}{9}\ln(M) - \lambda\,(40L + 36E + 20M - 180{,}000) \quad (45)$$

Solving the above equation, we obtain:

$$L^* = 2{,}000\ (hr),\ E^* = 1{,}666.67(hr),\ M^* = 2{,}000(kg) \quad (46)$$

$$\text{Total Cost } C^* = (40(\$/hr) \times 2{,}000\ (hr)) + (36(\$/hr) \times 1{,}666.67\ (hr)) +$$

$$(20(\$/kg) \times 2{,}000\ (kg)) = \$180{,}000 \quad (47)$$

$$\text{Optimal } RE^* = 7.54 \quad (47a)$$

The results indicate that an optimal resource effectiveness RE* is sufficient to achieve the desired productivity by investing L^* and E^*, and M^* units of labor, equipment, and material, respectively.

3.3. An example using a real highway network

We make use of the example from Jha et al (2010) for the application of Infrastructure Inspection and Scheduling Problem (IIS) to which the developed resource effectiveness maximization concept is employed. The road network shown in Fig. 2 is a section of the local highway network in the City of Baltimore, Maryland, surrounding the intersection of interstate 695 and Harford road with the arcs numbered in a counterclockwise direction. The labor, equipment, and material input values for conducting maintenance activities along each of the arcs is shown Table 1. In addition, the following input values are given:

The cost of labor is $40/hr;
The cost of equipment is $30/hr;
The cost of material is $20/kg;
The available budget is $80,000;
The output elasticity of labor work is considered to be 2/3;
The output elasticity of equipment is considered to be 1/2;
The output elasticity of material is considered to be 1/3;
The factor of productivity (A) is estimated to be 12.

The forms of utility function and cost are as follows:

$$RE = \frac{4}{9}\ln(L) + \frac{3}{9}\ln(E) + \frac{2}{9}\ln(M) \quad (48)$$

$$B = 40L + 30E + 20M = \$80{,}000 \quad (49)$$

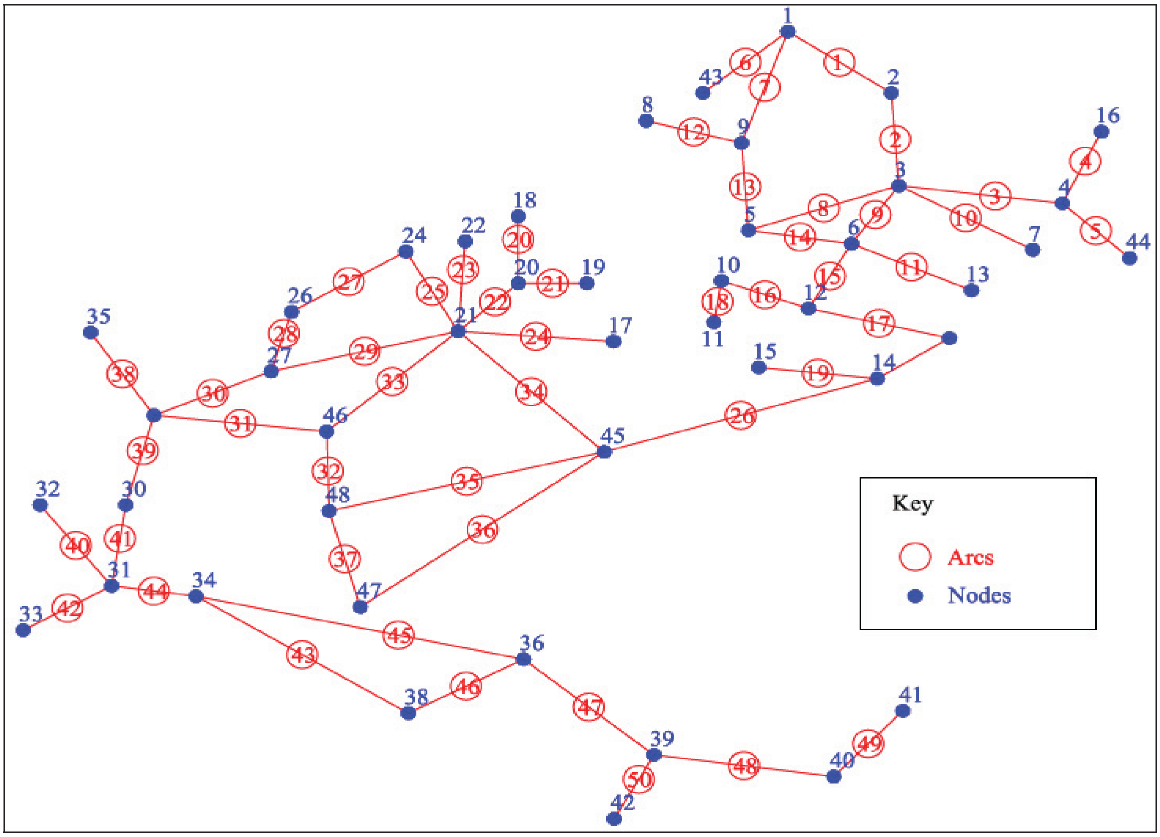

Fig. 2. A schematic of Baltimore, MD (I-695) highway network

For the aforementioned problem, different combinations of routes are obtained as optimal solutions, subject to the budget constraint. Jha et al. (2010) used Floyd's shortest path algorithm to obtain the optimal routes. The following routes were obtained as a set of an optimal solution for a budget constraint of $80,000:

- Route 1: 37, 32, 35, 36, 18, 13, 1, 3, 2, 6;
- Route 2: 8, 4, 12, 5, 15, 10, 11, 9, 7, 19;
- Route 3: 14, 16, 17, 26, 27, 30, 38, 40, 42, 44;
- Route 4: 48, 50, 45, 46, 43, 47, 49, 41, 39, 28;
- Route 5: 25, 22, 23, 24, 20, 33, 31, 29, 34, 21;

Table 1. Labor, equipment, and material input values for the example study

Arc #	Labor hours	Equipment hours	Material quantity (kg.)
1	8	5	375
2	1	3	178
3	4	6	247
4	9	8	332
5	3	10	336
6	9	5	289
7	9	3	310
8	5	6	401
9	5	2	251
10	3	5	250
11	3	6	172
12	9	9	260
13	1	7	460
14	5	6	170
15	5	9	135
16	7	8	131
17	4	1	425
18	6	1	461
19	1	3	453
20	3	2	391
21	10	3	185
22	3	7	334
23	8	1	459
24	1	1	308
25	9	10	451
26	2	5	263
27	3	10	243
28	6	9	231
29	2	4	351
30	8	1	124
31	8	8	454
32	8	7	298
33	3	6	418
34	8	6	306
35	9	9	271
36	5	4	424
37	8	7	429
38	7	5	247
39	7	1	479
40	6	2	335
41	2	5	262
42	6	6	169
43	9	3	363
44	10	6	382
45	3	2	176
46	2	3	217
47	10	2	250
48	5	4	109
49	6	7	473
50	9	4	250

The optimal solution is obtained using the Lagrangian Multiplier procedure as explained in examples 1 and 2 above. The optimal labor, equipment, and material quantities as well as optimal resource effectiveness are shown in Table 2 below. It can be seen that total optimal cost is at or below the appropriated budget:

Table 2. Optimal quantities for the third example

Route #	L*	E*	M*	C*	Q* (Production)	RE* (Utility)
1	51	51	3228	68130	17,417.71	4.85
2	52	61	2900	61910	18,620.06	4.90
3	58	50	2489	53600	17,230.31	4.85
4	59	40	2810	59760	16,231.10	4.81
5	55	48	3657	76780	18,524.73	4.89

The results indicate that for different optimal routes different combinations of L*, E*, and M* are required to produce the desired productivity, which can be achieved at different optimal RE* values as shown in Table 2 above. Further, it can be observed that while the desired productivity for different routes are different, the optimal resource effectiveness values are very close, which means resources are generally not wasted.

4. Results and discussions

This paper presented a numerical optimization approach for resource effectiveness which is defined as the minimum resource level required to achieve desired productivity at the expense of different levels of labor, equipment, and material expenditures, subject to a budget constraint. The approach is similar to that covered in economics related to consumer theory and behaviour. We employed the well known Cobb-Douglass production function and the technique of Lagrangian Multipliers to obtain optimal resource effectiveness on highway infrastructure maintenance inspection and scheduling problems. We showed one example application using a real highway network in the Baltimore area. The proposed optimization approach can be integrated in the computerized maintenance management system of a city or state department of transportation, which in turn, can be used to compare various alternatives and scenarios for maximizing resource effectiveness.

5. Conclusions and future works

In this study we developed a computationally feasible numerical modelling framework for maximizing resource effectiveness of a highway infrastructure maintenance and management. In future works we will develop an integrated modelling framework for infrastructure inspection, scheduling, and resource effectiveness.

Acknowledgements

This work was completed at the Center for Advanced Transportation and Infrastructure Engineering Research (CATIER) at the Morgan State University. The authors thank the reviewers for their valuable comments.

References

[1] Armington PS. A theory of demand for products distinguished by place of production. *IMF Staff Papers* 1969; 16: 159–178.

[2] Arrow KJ, Chenery HB, Minhas BS, Solow RM. Capital-labor substitution and economic efficiency. *Review of Economics and Statistics* 1961; 53: 225-251.

[3] Cobb CW, Douglas PH. A theory of production. *American Economic Review 18 (Supplement)* 1928; 139–165.

[4] Debreu G. The coefficient of resource utilization. *Econometric* 1951; 19(3): 273-292.

[5] Jha MK, Kepaptsoglou K, Karlaftis M, Abdullah J. A genetic algorithms-based decision support system for transportation infrastructure management in urban areas. In: Taniguchi E, Thompson RG, editors. *Recent advances in city logistics: Proceedings of the 4th International Conference on City Logistics*, Elsevier; 2006: 509-523.

[6] Jha MK, Chacha S, Udenta F, Abdullah J. Formulation and solution algorithms for highway infrastructure maintenance optimization with work-shift and overtime limit constraints. *Procedia Social and Behavioral Sciences* 2010; 2: 6323–6331.

[7] Jorgensen DW. *Econometric modeling of producer behavior*. MIT Press, Cambridge; 2000.

[8] Maji A, Jha MK. Modelling highway infrastructural maintenance schedule with budget constraint. *Transportation Research Record 1991, Journal of the Transportation Research Board* 2007; 19-26

[9] Samanta S, Jha MK. A conceptual framework for solving the multiple depot probabilistic vehicle routing problem with time window (MDPVRPTW). In: Taniguchi E, Thompson RG, editors. *Recent advances in city logistics: Proceedings of the 4th International Conference on City Logistics,* p. 495-507.

[10] Samanta S, Jha MK. Multi depot probabilistic vehicle routing problems with a time window: Theory, solution, and application. *International Journal of Operations Research and Information Systems* 2011; 2(2): 40-64.

[11] Vapnyarskii IB. Lagrange multipliers. In:Hazewinkel M, editor. *Encyclopaedia of mathematics,* Springer; 2001.