Numerical simulation of crack growth through particulate clusters in brittle matrix using the XFEM technique

Zhiyong Wang, Li Ma*, Linzhi Wu

Center for Composite Materials, Harbin Institute of Technology, Harbin 150001, PR China

Abstract

The purpose of the present work is a numerical study of the interaction between a crack and second phase particles in a reinforced composite material. Simulations are accomplished using the extended finite element method (XFEM) without tip-enriched functions, a new domain expression of an interaction integral for evaluating stress intensity factors, and the maximum hoop stress criterion for crack-growth direction prediction. Crack deflection mechanisms and the associated energy release rate variations are investigated. It is found that although the energy release rate is affected by the particle at relatively large distances, the crack trajectory is not substantially altered until the crack is very close to the particles. A pre-existing interface crack is also observed by introducing adaptive enriched functions in the XFEM. By this method, expensive meshing strategies can be avoided and considerable flexibility is obtained. The results show that the flaw could attract the main crack and increase the energy release rate.

© 2011 Published by Elsevier Ltd. Open access under CC BY-NC-ND license. Selection and peer-review under responsibility of ICM11

Keywords: crack; particles; interaction; energy release rate; the extended finite element method.

1. Introduction

To improve material performances, a widely used way is introducing strengthening phases into a matrix to form a multi-phase structure, such as fiber-reinforced materials or particle-reinforced materials. It is now well known, for example, that although the mechanical properties of particle-reinforced metal matrix composites are generally better than the properties of the monolithic alloy alone, it is often found that the fracture properties are not improved. Indeed, the fracture toughness may be significantly lower than that of the matrix material. Therefore, understanding the fracture behavior of this kind of composites materials is vitally important to make guidance for practical manufactures and applications. Observations indicate that the crack may remain entirely within the matrix material, but it is more likely for the crack

* Corresponding author. Tel.: +86 451 86402739; fax: 86 451 86402386.
E-mail address: mali@hit.edu.cn; wzy_hit@126.com.
propagation to be associated with fracture of the interfaces between the matrix and the particles, or fracture of the particles themselves. In some materials we may see one mechanism dominating the others, although all mechanisms are usually present[1]. Furthermore, flaws or defects in the material like matrix crack or interface crack can complicate the modeling of a microstructure. The finite element related method (FEM) is perhaps the most widely applied numerical method in this area today because its flexibility and stability in handling material heterogeneity, non-linearity and complex boundary conditions[2]. However, the generation of conforming meshes is still an expensive task for complex microstructure geometries. The boundary element related method (BEM) requires discretization at the boundary of the solution domains only, thus reducing the problem dimensions by one and greatly simplifying the input requirements[3]. However, the premise of the BEM is that one should know the fundamental solution of the differential operator. In addition, in general, the BEM is not as efficient as the FEM in dealing with material heterogeneity, because it cannot have as many sub-domains as elements in the FEM[4]. In the present paper, the extended finite element method (XFEM) without tip-enriched functions, which works within the framework of the standard finite element method, is employed. Both the weak and strong discontinuities including the matrix crack, interface crack and material interface are described by means of the level set method[5]. This method insures that all the discontinuities are still independent of the mesh. Furthermore, the asymptotic crack-tip displacement fields are no longer to be incorporated. Since the analytical expressions of crack-tip fields may be difficult to obtain or may be complex themselves.

By this method, we investigate the interaction between a propagating crack and a single particle or multiple particles in a brittle matrix. The objectives are to study the effects of particle and interface flaw on main crack path and stress intensity factors.

2. The extended finite element method without tip-enriched functions

By enriching the standard approximation with additional functions, the extended finite element method (XFEM) allows for the modeling of arbitrary geometric features independently of the finite element mesh. This advance has provided a robust and accurate computational tool for modeling discontinuities and their evolvement. We refer the reader to a big number of publications concerning the XFEM[6-9]. In the present work, the extended finite method without tip-enriched functions is employed. Fig.1 (a) illustrates the different discontinuous interfaces in a domain \( \Omega \) and Fig.1 (b) shows the finite element mesh and refined mesh around crack tips. The crack surfaces are assumed to be traction-free.

The governing equations and more details can be found in the original papers of the XFEM written by Belytschko and co-workers[6-7]. What we should point out is that, in this paper, only the straightforward enrichment functions for material interfaces and crack surfaces are adopted. In order to improve the numerical precision, the mesh around the crack tips is refined as shown in Fig.1 (b). The degenerate elements are employed around the crack tips just like the idea used in the traditional finite element method. Thus, this method can easily be applied to the problems in which the analytical expressions of crack tip field are difficult to obtain or are complex themselves. The excellent accuracy and convergence of this numerical technique has been demonstrated by Yu[10] by resolving several benchmark problems in fracture mechanics of non-homogeneous materials. Here, we develop the method to make it suitable for crack-growth modeling in particle reinforced composites. First, the location of the refined mesh are directly associated with the location of the crack tips during the simulation. Namely, we can find the tip-element according to the coordinate of the crack tip. Then, the domain occupied by elements around the tip-element and the tip-element (in this paper, 9 elements as shown in Fig.1 (b) are adopted) is refined as mentioned above. Finally, we lay the refined mesh over the original structured mesh. They share the same nodes with each other on the boundary of the refined domain. The initial values of stiffness of the 9
original quadrilateral elements are assigned to zero, and will not be computed any more. These manipulations insure that only the refined mesh works in the refined domain and the structured mesh is not destroyed during the crack-growth modeling. With this method, only small region should be refined and there is no need to ensure the crack surfaces and the material interfaces confirm with the mesh boundaries. Up to now, we can find that this method not only maintains the good idea of the XFEM, but also can be used in more general situations.

Fig. 1. (a) Illustration of different discontinuous interfaces in a domain \( \Omega \); (b) Finite element mesh and refined mesh around crack tips.

Consequently, we adopt the approximation of the displacement \( u(x) \) as:

\[
\mathbf{u}(x) = \sum N_i(x) \mathbf{u}_i + \sum_{i=1}^{2} \sum_{j=1}^{2} N_{ij}(x) \mathbf{b}_{ij} \psi_{ij}(x) + \sum_{j=1}^{2} \sum_{k=1}^{2} N_{K,j}(x) \mathbf{c}_{Kj} \varphi_{K,j}(x). \tag{1}
\]

Here, \( N(x) \) is the standard finite element shape function; \( \psi(x) \) and \( \varphi(x) \) are the shifted enrichment functions for cracks and material interfaces, respectively; \( \mathbf{b} \) and \( \mathbf{c} \) are the additional degrees of freedom for the corresponding enriched nodes. Of course, there are several similar choices for the enriched functions\(^8-9\).

3. The interaction integral

The interaction integral is derived from the \( J \)-integral for two admissible states (actual and auxiliary fields). Through appropriate algebraic manipulations, this method can easily decouple the mixed-mode stress intensity factors. Some related works can be refer to the papers written by Kim\(^{11}\). If we choose the incompatibility formulation of the auxiliary fields, the expression of the interactions integral could be changed. The new expression of the interaction integral does not contain any derivatives of material properties. As a result, the interaction integral does not need the material properties to be differentiable. Since it may be difficult to obtain the derivatives of material properties or there are no derivatives in actual case. Futhermore, the interaction integral is still valid even when the integral domain contains material interfaces. More details can be found in the paper written by Yu\(^{10}\).

4. The maximum hoop stress criterion

There are several criteria for predicting crack-growth direction in non-homogeneous materials. The maximum hoop stress criterion, which is a commonly used criterion, is adopted herein. According to this criterion, the crack will grow in the direction along which the maximum hoop stress occurs. Namely, once \( K > K^* \), the crack grows in the direction by a small increment set in advance and the process repeated. In general, however, smaller increments made little difference to the calculated trajectory in the examples to follow.
5. Numerical examples

In order to demonstrate the accuracy of the numerical technique proposed in this paper, we first present a numerical example of crack-particle interaction which has been investigated by Williams[12]. The geometry features and material properties are chosen to be consistent with Ref. [12]. Table 1 and 2 give our results compared to those of the analytical and numerical results from Ref. [12]. As can be seen from the tables, the results produced by the XFEM without tip-enriched functions are in good agreement with the available results.

Table 1 Normalized stress intensity factors at crack tip A

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.5,1.5)</td>
<td>0.6149</td>
<td>0.614</td>
<td>0.613</td>
<td>0.0586</td>
<td>0.055</td>
<td>0.061</td>
</tr>
<tr>
<td>(0.5,1.75)</td>
<td>0.7584</td>
<td>0.752</td>
<td>0.750</td>
<td>-0.0424</td>
<td>-0.043</td>
<td>-0.041</td>
</tr>
<tr>
<td>(0.5,2)</td>
<td>0.8342</td>
<td>0.835</td>
<td>0.834</td>
<td>-0.066</td>
<td>-0.063</td>
<td>-0.062</td>
</tr>
<tr>
<td>(0.5,3)</td>
<td>0.9560</td>
<td>0.956</td>
<td>0.956</td>
<td>-0.0349</td>
<td>-0.034</td>
<td>-0.035</td>
</tr>
</tbody>
</table>

Table 2 Normalized stress intensity factors at crack tip B

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.5,1.5)</td>
<td>0.8150</td>
<td>0.817</td>
<td>0.817</td>
<td>0.0673</td>
<td>0.067</td>
<td>0.067</td>
</tr>
<tr>
<td>(0.5,1.75)</td>
<td>0.8758</td>
<td>0.878</td>
<td>0.878</td>
<td>-0.0613</td>
<td>-0.062</td>
<td>-0.0621</td>
</tr>
<tr>
<td>(0.5,2)</td>
<td>0.9133</td>
<td>0.916</td>
<td>0.915</td>
<td>-0.0516</td>
<td>-0.052</td>
<td>-0.052</td>
</tr>
<tr>
<td>(0.5,3)</td>
<td>0.9725</td>
<td>0.972</td>
<td>0.973</td>
<td>-0.0231</td>
<td>-0.024</td>
<td>-0.024</td>
</tr>
</tbody>
</table>

We go on by considering an edge crack contained in a square plate interacts with a circular particle. The plate is subjected to uniformly distributed traction loading along both the top and bottom edge. The particle is offset from the crack surface by a distance \( d \). In this example, \( d/r=0.8 \), where \( r \) is the radius of the particle.

![Computational crack trajectories for an edge crack approaching a particle](image1)

![The non-dimensional energy release rate for the case indicate (a)](image2)

Fig. 2. (a) Computed crack trajectories for an edge crack approaching a particle; (b) The non-dimensional energy release rate for the case indicate (a).
The energy release rate is made non-dimensional by dividing by the corresponding value for the non-reinforced case with the same crack geometry. The computed crack trajectories are shown in Fig. 2 (a) and the corresponding energetics are shown in Fig. 2 (b). It can be found that as the crack approaches the particle it experiences a substantial shielding effect, as expected, but that after it passes the particle an amplifying effect occurs. It also can be found that although the energy release rate is affected by the particles at relatively large distances, the crack trajectory is not substantially altered until the crack is very close to the particles. Here, the shielding effect implies the increased effective toughness and could cause crack deceleration. In the opposite case, the amplifying effect implies the decreased effective toughness, so crack acceleration can be anticipated. If there is an interface crack located between the angular position 150 and 210, the main crack could be attracted and the non-dimensional energy release rate could be increased in some extent, as shown in Fig. 2.

Fig.3. Computed crack trajectory for an edge crack approaching a cluster consisting of two particles.

Fig.4. The non-dimensional energy release rate: (a) the interface is perfect; (b) an interface flaw exists

At last, we consider that the edge crack interacts with clusters consisting of two equal particles. The other geometry information is the same as before. If the particles and the matrix are perfectly bonded, the main crack would grow along the straight line due to the symmetrical loads and geometry. In addition, both shielding and amplifying effects are enhanced as the spacing between the particles is reduced, as shown in Fig. 4 (a). If there is an interface crack located between the angular position 240 and 300, as shown in Fig.3, the interface flaw tends to attract the crack, which initially produces a substantial increase in the energy release rate. Thereby, such flaws appear to promote rapid crack propagation and subsequent
fracture of the material. In this case, however, we see that as the crack turns into alignment with the direction of the externally applied load the energy release rate subsequently undergoes a reduction, as shown in Fig. 4 (b).

6. Conclusions

The extended finite element method has been adopted and developed for modeling the interaction between a propagating crack and the second phase particles. This technique of directly evaluating the fracture parameters in particle reinforced composite materials is found to be very convenient and reliable. The results show that the presence of particles may substantially affect the energy release rate. Consequently, the crack growth trajectories are also changed. A pre-existing flaw on the interface of a particle could attract the crack and increase the energy release rate. Such flaws appear to promote rapid crack propagation and subsequent fracture of the material.

Acknowledgements

The present work is supported by the Program for New Century Excellent Talents in University under grant No. NCET-08-0152 and the Program of Excellent Team in Harbin Institute of Technology.

References