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ORIGINAL ARTICLE

MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation (absorption) and slip velocity

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Abstract In this work, the effects of slip velocity on the flow and heat transfer for an electrically conducting micropolar fluid over a permeable stretching surface with variable heat flux in the presence of heat generation (absorption) and a transverse magnetic field are investigated. The governing partial differential equations describing the problem are converted to a system of non-linear ordinary differential equations by using the similarity transformation, which is solved numerically using the Chebyshev spectral method. The effects of the slip parameter on the flow, micro-rotation and temperature profiles as well as on the local skin-friction coefficient, the wall couple stress and the local Nusselt number are presented graphically. The numerical results of the local skin-friction coefficient, the wall couple stress and the local Nusselt number are given in a tabular form and discussed.

1. Introduction

The study of flow and heat transfer past a stretching sheet problems has gained considerable interest because of its extensive engineering applications, such as in the extrusion of plastic sheets, paper production, crystal growing and glass blowing. Crane [1] presented an exact similarity solution in closed analytical form for the laminar boundary flow of an incompressible, steady viscous fluid over a stretching surface with a velocity varying linearly with the distance from a fixed point. Gupta and Gupta [2] extended the Crane's problem to include suction or blowing and studied its influence on the heat and mass transfer in the boundary layer over a stretching surface. Vajravelu and Rollins [3] studied the heat transfer characteristics in an electrically conducting fluid over a stretching sheet with variable wall temperature and internal heat generation or absorption.

The problem of MHD flow and heat transfer over a stretching surface has gained considerable interest because of its applications in industry. For example in the extrusion of a polymer sheet from a die, the sheet is sometimes stretched. During this process, the properties of the final products depend

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considerably on the rate of cooling. By drawing such sheet in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and the final product can be obtained with desired characteristics. Pavlov [4] studied the boundary layer flow of an electrically conducting fluid due to a stretching of a plane elastic surface in the presence of a uniform transverse magnetic field. Chakrabarti and Gupta [5] extended Pavlov's work to study the heat transfer when a uniform suction is applied at the stretching surface.

All the above analysis were restricted to the flow of Newtonian fluids. However, a new stage in the evaluation of fluid dynamic theory whose flow shear behavior cannot be characterized by Newton relationships, was first introduced by Eringen [6,7] as the theory of micropolar and thermomicropolar fluids. This theory can be used to describe the behavior of fluids in many practical applications. These applications include the mathematical model for polymeric fluids, colloidal fluids, real fluids with suspensions, liquid crystal, animal blood and exotic lubricants, as examples, for which the classical Navier–Stokes theory is inadequate. An excellent review of micropolar fluids and their applications was presented by Ariman et al. [8]. Soundalgekar and Takhar [9] investigated the effects of suction and injection on the flow past a continuously moving semi-infinite porous plate in a micropolar fluid at rest. Hady [10] presented analytical solutions for the problem of heat transfer to micropolar fluid from a non-isothermal permeable stretching sheet. Heat transfer over a stretching surface with uniform or variable surface heat flux in micropolar fluids was studied by Ishak et al. [11]. Hassanien and Gorla [12] numerically studied the effects of suction and blowing on the flow and heat transfer of a micropolar fluid over a non-isothermal stretching surface. Hayat et al. [13] analytically studied the problem of a steady two-dimensional mixed convection flow of a micropolar fluid over a non-linear stretching surface by means of HAM. Hayat et al. [14] investigated the two-dimensional magnetohydrodynamic (MHD) stagnation-point flow of an incompressible micropolar fluid over a nonlinear stretching surface. Sajid et al. [15] presented the exact solutions for thin film flows of a micropolar fluid. The boundary layer flow of a micropolar fluid through a porous channel was studied by Sajid et al. [16].

In several physical problems such as fluids undergoing exothermic or endothermic chemical reactions, it is important to study the effects of heat generation and absorption. The presence of heat generation or absorption may alter the temperature distribution in the fluid which in turn affects the particle deposition rate in systems such as nuclear reactors, electronic chips, and semiconductor wafers. The exact modeling of internal heat generation or absorption is difficult but some simple mathematical models may express its average behavior for most physical situations. Following Foraboschi and Federico [17], we shall assume that the volumetric rate of heat generation $Q[W/m^3]$, as:

$$Q = \begin{cases} Q_0(T - T_\infty), & T \geqslant T_\infty \\ 0, & T < T_\infty, \end{cases}$$

where Q_0 is the heat generation or absorption constant. The above relation is valid for the state of some exothermic processes having T_∞ as the onset temperature. The influence of heat generation or absorption on the fluid flow over a stretching surface has been studied by many authors. Cortell [18] studied the flow and heat-transfer in a porous medium over a stretching surface with internal heat generation or absorption. Heat generation/absorption and viscous dissipation effects on MHD flow of a micropolar fluid over a moving permeable surface embedded in a non-Darcian porous medium has been studied by Mahmoud [19]. Damseh et al. [20] investigate the combined heat and mass transfer by natural convection of a micropolar, viscous and heat generating or absorbing fluid flow near a continuously moving vertical permeable infinitely long surface in the presence of a first-order chemical reaction.

In the above mentioned studies, the effect of slip condition has not been taken into consideration. Navier [21] proposed a slip boundary condition where the slip velocity depends linearly on the shear stress. Since then the effects of slip velocity on the boundary layer flow of Newtonian and non-Newtonian fluids have been studied by several authors. Ariel [22] studied the flow of an elastico-viscous fluid past a stretching sheet with partial slip. Hayat et al. [23] analytically studied the solutions of the equations of motion and energy of a second grade fluid for the developed flow over a stretching sheet with slip condition. Hayat et al. [24] studied the effect of the slip condition on flows of an Oldroyd 6-constant fluid. Roux [25] discussed the solvability of the equations of motion for an incompressible fluid of grade two subject to nonlinear partial slip boundary conditions in a bounded simply-connected domain. Liakos [26] studied the steady-state, isothermal flow of a viscoelastic fluid obeying an Oldroyd-type constitutive law with slip boundary condition. Asghar et al. [27] analytically studied the rotating flow of a third grade fluid past a porous plate with partial slip effects. Khan [28] presented the exact analytical solutions for three basic fluid flow problems in a porous medium when the no-slip condition is no longer valid. Asghar et al. [29] obtained the exact analytical solutions for general periodic flows of a second grade fluid in the presence of partial slip and a porous medium. Mahmoud [30] studied the effects of slip velocity on flow and heat transfer of a non-Newtonian power-law fluid on a stretching surface with thermal radiation. Hayat et al. [31] studied the flow and heat transfer of a Newtonian fluid over an unsteady permeable stretching sheet with slip conditions. Mahmoud and Waheed [32] investigated the effect of slip velocity on mixed convection flow of a micropolar fluid over a heated stretching surface in the presence of a uniform magnetic field and heat generation or absorption. The aim of the present work is to investigate the flow and heat transfer of an electrically conducting micropolar fluid over a permeable stretching surface with variable heat flux in the presence of slip velocity at the surface, heat generation (absorption) and a uniform transverse magnetic field.

2. Formulation of the problem

The equations governing the behavior of an incompressible steady micropolar fluid in vectorial form are [6,7]:

Conservation of mass:

$$\nabla \cdot \underline{V} = 0, \tag{1}$$

Conservation of momentum:

$$\rho(\underline{V} \cdot \nabla)\underline{V} = -\nabla p + (\mu + k)\nabla^2 \underline{V} + k\nabla \times \underline{\Omega} + \rho \underline{f}, \tag{2}$$

Conservation of angular momentum:

$$\begin{aligned} \rho j(\underline{V} \cdot \nabla)\underline{\Omega} = (\alpha_0 + \beta_0 + \gamma_0)\nabla(\nabla \cdot \underline{\Omega}) - \gamma_0 \nabla \times (\nabla \times \underline{\Omega}) \\ + k\nabla \times \underline{V} - 2k\underline{\Omega} + \rho \underline{l}, \end{aligned} \tag{3}$$

Energy:

$$\rho c_p(\underline{V}\cdot\nabla)T=\kappa\nabla^2 T+\varphi+Q, \tag{4}$$

where $\underline{f}$ is the body force per unit mass and $\underline{l}$ is the body couple per unit mass. $\underline{V}$ is the translational vector, $\underline{\Omega}$ is the micro-rotation vector and p is the pressure. α_0, β_0, γ_0 and k are the material constants for micropolar fluids. ρ is the fluid density, j is the micro-inertia, μ is the dynamic viscosity, κ is the thermal conductivity. T is the fluid temperature, φ is the dissipation function and c_p is the specific heat at constant pressure.

Consider the two-dimensional flow of an incompressible electrically conducting micropolar fluid past a porous stretching surface which coincides with the plane $y=0$, the flow being in the region $y>0$. The x-axis is taken along the stretching surface in the direction of motion. A uniform magnetic field B_0 is imposed along y-axis, which is normal to the surface. It is assumed that the variable surface heat flux to be $q_w(x)=bx^m$ (where b, m are constants and x measures the distance from the leading edge along the surface of the plate). If the velocity and micro-rotation components are $(u,v,0)$ and $(0,0,N)$, respectively, with neglecting the body couple, the micro-inertia, the viscous dissipation and using the usual boundary layer approximations, the basic equations taking into account the presence of heat generation (absorption) in the energy equation are [9]: Conservation of mass:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{5}$$

Conservation of momentum:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}u, \tag{6}$$

Conservation of angular momentum:

$$G_1\frac{\partial^2 N}{\partial y^2}-\left(2N+\frac{\partial u}{\partial y}\right)=0, \tag{7}$$

Energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho c_p}(T-T_\infty), \tag{8}$$

Subject to the boundary conditions:

$$\begin{aligned}&u=ax+\alpha^*\left[(\mu+k)\frac{\partial u}{\partial y}+kN\right],\quad v=v_w,\\ &N=0,\quad \frac{\partial T}{\partial y}=-\frac{bx^m}{\kappa},\quad \text{at } y=0,\\ &u\to 0,\quad N\to 0,\quad T\to T_\infty,\quad \text{as } y\to\infty,\end{aligned} \tag{9}$$

where u and v are the velocity components in the x and y directions; respectively. N is the micro-rotation or the angular velocity, σ is the electrical conductivity, B_0 is the magnetic field strength, $G_1=\frac{\gamma}{k}$ is the microrotation constant, α^* is the slip coefficient and m is the heat flux exponent. It is noted here that the case of uniform surface heat flux corresponds to $m=0$.

We introduce the following dimensionless variables :

$$\begin{aligned}&\eta=\left[\frac{a}{v}\right]^{1/2}y,\quad \Psi=[av]^{1/2}xf(\eta),\\ &N=ax\left[\frac{a}{v}\right]^{1/2}h(\eta),\\ &T=T_\infty+\frac{q_w(x)}{\kappa}\left[\frac{v}{a}\right]^{1/2}\theta(\eta),\end{aligned} \tag{10}$$

where Ψ is the stream function defined in the usual way as $u=\frac{\partial\Psi}{\partial y}$ and $v=-\frac{\partial\Psi}{\partial x}$ so that the conservation of mass Eq. (5) is automatically satisfied.

Substituting variables (10) into Eqs. (6)–(9), one obtains the following system of non-linear ordinary differential equations:

$$(1+K)f'''+ff''-f'^2+Kh'-Mf'=0, \tag{11}$$

$$Gh''-(2h+f'')=0, \tag{12}$$

$$\frac{1}{Pr}\theta''+f\theta'-mf'\theta+\gamma\theta=0. \tag{13}$$

The transformed boundary conditions are then given by:

$$\begin{aligned}&f=f_w,\quad f'=1+\alpha(1+K)f'',\quad h=0,\quad \theta'=-1\quad \text{at}\quad \eta=0,\\ &f'\to 0,\quad h\to 0,\quad \theta\to 0\quad \text{as}\quad \eta\to\infty,\end{aligned} \tag{14}$$

where primes denote differentiation with respect to η, $M=\frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter, $f_w=-(av)^{\frac{-1}{2}}v_w$ is the suction (>0) or the injection parameter (<0), $K=k/\mu$ is the material parameter, $\alpha=\alpha^*\mu\sqrt{\frac{a}{v}}$ is the slip parameter, $G=\frac{G_1 a}{v}$ is the microrotation parameter, $Pr=\mu c_p/\kappa$ is the Prandtl number and $\gamma=\frac{Q_0}{a\rho c_p}$ is the heat generation (>0) or absorption (<0) parameter.

The physical quantities of interest are the local skin-friction coefficient C_{f_x} , the dimensionless wall couple stress M_x and the local Nusselt number N_{u_x} , which are respectively; defined as:

$$\begin{aligned}&C_{f_x}=\frac{2\tau_w}{\rho(ax)^2},\\ &M_x=\frac{m_w}{\rho avx^3},\\ &Nu_x=\frac{xq_w}{\kappa(T_w-T_\infty)},\end{aligned} \tag{15}$$

where the local wall shear stress τ_w , the wall couple stress m_w and the heat transfer from the plate q_w are defined by:

$$\begin{aligned}&\tau_w=-\left[(\mu+k)\frac{\partial u}{\partial y}+kN\right]_{y=0},\\ &m_w=\gamma_0\left[\frac{\partial N}{\partial y}\right]_{y=0},\\ &q_w=-\left[\kappa\frac{\partial T}{\partial y}\right]_{y=0}.\end{aligned} \tag{16}$$

Using the similarity variables (10), we get

$$\begin{aligned}&\frac{1}{2}C_{f_x}Re_x^{1/2}=-(1+K)f''(0),\\ &M_xRe_x=KGh'(0),\\ &Nu_xRe_x^{-1/2}=\frac{1}{\theta(0)},\end{aligned} \tag{17}$$

where $R_{e_x}=(\frac{ax^2}{v})$ is the local Reynolds number.

In the case of Newtonian fluids ($K=0$) with $\alpha=0$, the momentum Eq. (11) with the boundary conditions (14) has an exact solution in the form:

$$f(\eta)=f_w+\frac{[1-exp(-\beta\eta)]}{\beta},\quad f'(\eta)=exp(-\beta\eta), \tag{18}$$

where β is determined from

$$\beta = \frac{f_w + \sqrt{f_w^2 + 4(1+M)}}{2}.$$

The local skin-friction coefficient given by (17) now becomes:

$$\frac{1}{2} C_{f_x} Re_x^{1/2} = \beta. \tag{19}$$

The exact solution of Eq. (13) satisfying Eq. (14) in terms of Kummer's confluent hyper geometric function $F_1[a,c,x]$ is:

$$\theta(\eta) = (e^{-\beta\eta})^{\frac{a_0-b_0}{2}}\left(c_1 F_1\left[\frac{-2m+a_0-b_0}{2}, 1-b_0, \frac{-e^{-\beta\eta}Pr}{\beta^2}\right] + c_2 (e^{-\beta\eta})^{b_0} F_1\left[-m+\frac{a_0}{2}+\frac{b_0}{2}, 1+b_0, \frac{-e^{-\beta\eta}Pr}{\beta^2}\right]\right),$$

where

$$a_0 = \frac{(1+\beta f_w)Pr}{\beta^2}, \quad b_0 = \frac{\sqrt{(1+\beta f_w)^2 Pr^2 - 4\beta^2 Pr\gamma}}{\beta^2}, \quad c_1 = 0$$

and $c_2 = -1/\left(\frac{-1}{2}\beta(a_0-b_0)F_1\left[-m+\frac{a_0}{2}+\frac{b_0}{2}, \quad 1+b_0, \quad \frac{-Pr}{\beta^2}\right] + F_1\left[1-m+\frac{a_0}{2}+\frac{b_0}{2}, 2+b_0, \frac{-Pr}{\beta^2}\right]\left(\frac{Pr(-m+\frac{a_0}{2}+\frac{b_0}{2})}{\beta(1+b_0)}\right) - b_0\beta F_1\left[-m+\frac{a_0}{2}+\frac{b_0}{2}, 1+b_0, \frac{-Pr}{\beta^2}\right]\right.$

The local Nusselt number given by **(17)** now becomes:

$$Nu_x Re_x^{-1/2} = \frac{1}{c_2 F_1\left[-m+\frac{a_0}{2}+\frac{b_0}{2}, 1+b_0, \frac{-Pr}{\beta^2}\right]}. \tag{20}$$

3. Method of solution

The domain of the governing boundary layer Eqs. (11)–(14) is the unbounded region $[0,\infty)$. However, for all practical reasons, this could be replaced by the interval $0 \leqslant \eta \leqslant \eta_\infty$, where η_∞ is some large number to be specified for computational convenience. Using the algebraic mapping:

$$\chi = 2\frac{\eta}{\eta_\infty} - 1,$$

the unbounded region $[0,\infty)$ is mapped into the finite domain $[-1,1]$.

Eqs. (11)–(14) are transformed into the following Chebyshev spectral equations:

$$(1+K)f'''(\chi) + \left(\frac{\eta_\infty}{2}\right)(f(\chi)f''(\chi) - f'^2(\chi)) + \left(\frac{\eta_\infty}{2}\right)^2 (Kh'(\chi) - Mf'(\chi)) = 0, \tag{21}$$

$$Gh''(\chi) - \left(2\left(\frac{\eta_\infty}{2}\right)^2 h(\chi) + f''(\chi)\right) = 0, \tag{22}$$

$$\frac{1}{Pr}\theta''(\chi) + \left(\frac{\eta_\infty}{2}\right)(f(\chi)\theta'(\chi) - mf'(\chi)\theta(\chi)) + \left(\frac{\eta_\infty}{2}\right)^2 \gamma\theta(\chi) = 0. \tag{23}$$

The transformed boundary conditions are given by:

$$f(-1) = f_w, \quad f'(-1) = \left(\frac{\eta_\infty}{2}\right) + \left(\frac{2}{\eta_\infty}\right)\alpha(1+K)f''(-1), \quad f'(1) = 0,$$

$$h(-1) = 0 \quad h(1) = 0. \tag{24}$$

$$\theta(-1) = -\frac{\eta_\infty}{2}, \quad \theta(1) = 0,$$

where now differentiation in Eqs. (21)–(23) will be with respect to the new variable χ.

Our technique is accomplished by starting with a Chebyshev approximation for the highest order derivatives, f''', h'' and θ'' and generating approximations to the lower order derivatives f'', f', f, h', h, θ' and θ as follows:

Setting $f''' = \phi(\chi)$, $h'' = \psi(\chi)$ and $\theta'' = \zeta(\chi)$, then by integration we obtain:

$$f''(\chi) = \int_{-1}^{\chi} \phi(\chi)d\chi + C_1^f, \tag{25}$$

$$f'(\chi) = \int_{-1}^{\chi}\int_{-1}^{\chi} \phi(\chi)d\chi d\chi + C_1^f(\chi+1) + C_2^f, \tag{26}$$

$$f(\chi) = \int_{-1}^{\chi}\int_{-1}^{\chi}\int_{-1}^{\chi} \phi(\chi)d\chi d\chi d\chi + C_1^f\frac{(\chi+1)^2}{2} + C_2^f(\chi+1) + C_3^f, \tag{27}$$

$$h'(\chi) = \int_{-1}^{\chi} \psi(\chi)d\chi + C_1^h, \tag{28}$$

$$h(\chi) = \int_{-1}^{\chi}\int_{-1}^{\chi} \psi(\chi)d\chi d\chi + C_1^h(\chi+1) + C_2^h, \tag{29}$$

$$\theta'(\chi) = \int_{-1}^{\chi} \zeta(\chi)d\chi + C_1^\theta, \tag{30}$$

$$\theta(\chi) = \int_{-1}^{\chi}\int_{-1}^{\chi} \zeta(\chi)d\chi d\chi + C_1^\theta(\chi+1) + C_2^\theta. \tag{31}$$

From the boundary condition (24), we obtain :

$$C_1^f = -\frac{1}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)}\int_{-1}^{1}\int_{-1}^{\chi}\phi(\chi)d\chi d\chi - \frac{1}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)}\left(\frac{\eta_\infty}{2}\right),$$

$$C_2^f = \left(\frac{\eta_\infty}{2}\right) + \alpha(1+K)\left(\frac{2}{\eta_\infty}\right)C_1^f,$$

$$C_3^f = f_w,$$

$$C_1^h = -\frac{1}{2}\int_{-1}^{1}\int_{-1}^{\chi}\psi(\chi)d\chi d\chi,$$

$$C_2^h = 0,$$

$$C_1^\theta = -\left(\frac{\eta_\infty}{2}\right),$$

$$C_2^\theta = \eta_\infty - \int_{-1}^{1}\int_{-1}^{\chi}\zeta(\chi)d\chi d\chi.$$

Therefore, we can give approximations to Eqs. (25)–(31) as follows:

$$f_i(\chi) = \sum_{j=0}^{N} l_{ij}^f\phi_j + d_i^f, \quad f_i'(\chi) = \sum_{j=0}^{N} l_{ij}^{f1}\phi_j + d_i^{f1},$$

$$f_i''(\chi) = \sum_{j=0}^{N} l_{ij}^{f2}\phi_j + d_i^{f2}, \tag{32}$$

$$h_i(\chi) = \sum_{j=0}^{N} l^h_{ij}\psi_j + d^h_i, \quad h'_i(\chi) = \sum_{j=0}^{N} l^{h1}_{ij}\psi_j + d^{h1}_i, \tag{33}$$

$$\theta_i(\chi) = \sum_{j=0}^{N} l^\theta_{ij}\zeta_j + d^\theta_i, \quad \theta'_i(\chi) = \sum_{j=0}^{N} l^{\theta1}_{ij}\zeta_j + d^{\theta1}_i, \tag{34}$$

for all $i = 0(1)N$, where

$$l^\theta_{ij} = b^2_{ij} - b^2_{Nj}, \quad d^\theta_i = \eta_\infty\left(1 - \frac{(\chi_i+1)}{2}\right),$$

$$l^{\theta1}_{ij} = b_{ij}, \quad d^{\theta1}_i = -\frac{\eta_\infty}{2},$$

$$l^h_{ij} = b^2_{ij} - \frac{(\chi_i+1)}{2}b^2_{Nj}, \quad d^h_i = 0,$$

$$l^{h1}_{ij} = b_{ij} - \frac{1}{2}b^2_{Nj}, \quad d^{h1}_i = 0,$$

$$l^f_{ij} = b^3_{ij} - \frac{1}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)}\left[\frac{(\chi_i+1)^2}{2} + \alpha(1+K)(\chi_i+1)\left(\frac{2}{\eta_\infty}\right)\right]b^2_{Nj},$$

$$d^f_i = f_w + (\chi_i+1)\left(\frac{\eta_\infty}{2}\right) - \frac{\left(\frac{\eta_\infty}{2}\right)}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)} \times \left[\frac{(\chi_i+1)^2}{2} + \alpha(1+K)(\chi_i+1)\left(\frac{2}{\eta_\infty}\right)\right],$$

$$l^{f1}_{ij} = b^2_{ij} - \frac{1}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)}\left[(\chi_i+1) + \alpha(1+K)\left(\frac{2}{\eta_\infty}\right)\right]b^2_{Nj},$$

$$d^{f1}_i = \left(\frac{\eta_\infty}{2}\right) - \frac{\left(\frac{\eta_\infty}{2}\right)}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)}\left[(\chi_i+1) + \alpha(1+K)\left(\frac{2}{\eta_\infty}\right)\right],$$

$$l^{f2}_{ij} = b_{ij} - \frac{1}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)}b^2_{Nj}, \quad d^{f2}_i = -\frac{\left(\frac{\eta_\infty}{2}\right)}{2+\alpha(1+K)\left(\frac{2}{\eta_\infty}\right)},$$

where

$$b^2_{ij} = (\chi_i - \chi_j)b_{ij},$$

and b_{ij} are the elements of the matrix B, as given in Ref. [33]. This system is then solved using Newton's iteration method.

By using Eqs. (32)–(34), one can transform Eqs. (21)–(23) to the following system of nonlinear equations in the highest derivatives:

$$(1+K)\phi_i + \left(\frac{\eta_\infty}{2}\right)\left[\left(\sum_{j=0}^{N} l^f_{ij}\phi_j + d^f_i\right)\left(\sum_{j=0}^{N} l^{f2}_{ij}\phi_j + d^{f2}_i\right) - \left(\sum_{j=0}^{N} l^{f1}_{ij}\phi_j + d^{f1}_i\right)^2\right] + \left(\frac{\eta_\infty}{2}\right)^2\left[K\left(\sum_{j=0}^{N} l^{h1}_{ij}\psi_j + d^{h1}_i\right) - M\left(\sum_{j=0}^{N} l^{f1}_{ij}\phi_j + d^{f1}_i\right)\right] = 0, \tag{35}$$

$$G\psi_i - \left(2\left(\frac{\eta_\infty}{2}\right)^2\left(\sum_{j=0}^{N} l^h_{ij}\psi_j + d^h_i\right) + \left(\sum_{j=0}^{N} l^{f2}_{ij}\phi_j + d^{f2}_i\right)\right) = 0, \tag{36}$$

$$\frac{1}{Pr}\zeta_i + \left(\frac{\eta_\infty}{2}\right)\left[\left(\sum_{j=0}^{N} l^f_{ij}\phi_j + d^f_i\right)\left(\sum_{j=0}^{N} l^{\theta1}_{ij}\zeta_j + d^{\theta1}_i\right) - m\left(\sum_{j=0}^{N} l^{f1}_{ij}\phi_j + d^{f1}_i\right)\right] \left[\left(\sum_{j=0}^{N} l^\theta_{ij}\zeta_j + d^\theta_i\right)\right] + \left(\frac{\eta_\infty}{2}\right)^2\gamma\left(\sum_{j=0}^{N} l^\theta_{ij}\zeta_j + d^\theta_i\right) = 0, \tag{37}$$

4. Results and discussion

In order to asses the accuracy of the present numerical method, we have compared our numerical results taking $\alpha = 0$, $m = 2$, $f_w = 0.2$ and $K = 0$ (for Newtonian fluids case) with those obtained analytically (Eqs. (19) and (20)). It can be seen that the numerical results are in good agreement with those obtained analytically, as shown in Table 1. Table 2 illustrates the comparison between our numerical results for Newtonian fluids case ($K = 0$, $M = 0$ and $f_w = 0$ in Eq. (14)) with those reported by Andersson [34] and Mahmoud [35] for various values of α. The results show a good agreement.

The effects of the slip parameter α on the velocity, micro-rotation and temperature distributions are shown in Figs. 1–3. Fig. 1 depicts the effect of the slip parameter α on the dimensionless velocity. It is noticed that the velocity distribution decreases as α increases for both suction and injection. Also, it is observed from Fig. 2 that the micro-rotation decreases with increasing α in both cases of suction and injection. The temperature distribution increases as α increases as shown in Fig. 3

Figs. 4–6 display the local skin-friction coefficient, the wall couple stress and the local Nusselt number for various values of α and f_w, respectively, keeping all other parameters fixed, respectively. It is noticed that the local skin-friction coefficient increases due to suction and decreases in the case of injection considerably for a fixed value of α. Also, it is observed that for a fixed value of f_w the local skin-friction coefficient decreases as α increases as seen in Fig. 4. It is found from Fig. 5 that the effect of suction is to increase the dimensionless couple stress, and the influence of injection is to decrease the dimensionless couple stress for a fixed value of α. For a fixed f_w the dimensionless couple stress decreases as α increases. From Fig. 6 it is shown that an increase in suction leads to an increase in the local Nusselt number; whereas an increase in the absolute value of injection leads to a decrease in the local Nusselt number considerably for a fixed value of α. Also, it is found that for a fixed f_w the local Nusselt number decreases as α increases. This behavior shows the importance of suction and injection in controlling the velocity, micro-rotation and the temperature in the boundary layer.

The local skin-friction coefficient in the term of $-f''(0)$, the wall couple stress in the term of $h'(0)$ and the local Nusselt

Table 1 Comparison between analytical and numerical values of $\frac{1}{2}C_{f_x}Re_x^{1/2}$ and $Nu_x/Re_x^{1/2}$ for various values of M and γ with $K = 0$, $m = 2$, $\alpha = 0$, $Pr = 0.72$ and $f_w = 0.2$.

M	γ	$\frac{1}{2}C_{f_x}Re_x^{1/2}$		$Nu_x/Re_x^{1/2}$	
		Analytical	Numerical	Analytical	Numerical
0.5	0.1	1.32882	1.32881	1.02938	1.02958
1	0.1	1.51774	1.51773	0.96966	0.969776
1.5	0.1	1.68430	1.68426	0.91730	0.91750
0.5	0.1	1.32882	1.32881	1.02938	1.02958
0.5	0.0	1.32882	1.32881	1.08932	1.08952
0.5	−0.1	1.32882	1.13881	1.13991	1.13991

Table 2 Comparison of $f'(0)$ and $-f''(0)$ for various values of α with $K = 0$, $M = 0$ and $f_w = 0$.

α	$f'(0)$			$-f''(0)$		
	Andersson [34]	Mahmoud [35]	Present work	Andersson [34]	Mahmoud [35]	Present work
0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.9128	0.91279	0.91279	0.8721	0.87208	0.87209
0.2	0.8447	0.84473	0.84472	0.7764	0.77637	0.77639
0.5	0.7044	0.70440	0.70440	0.5912	0.59119	0.59121
1.0	0.5698	0.56984	0.56982	0.4302	0.43016	0.43018
2.0	0.4320	0.43204	0.43199	0.2840	0.28398	0.28400
5.0	0.2758	0.27579	0.27579	0.1448	0.14484	0.14481
10.0	0.1876	0.18758	0.18759	0.0812	0.08124	0.08123
20.0	0.1242	0.12423	0.12420	0.0438	0.04378	0.04381
50.0	0.0702	0.07019	0.07019	0.0186	0.01859	0.01860
100.0	0.0450	0.04501	0.04500	0.0095	0.00955	0.00951

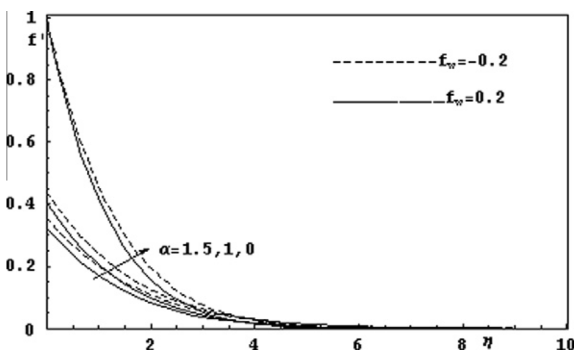

Figure 1 Velocity profiles for various values of α with $G = 2$, $K = 1.2$, $M = 0.5$, $Pr = 0.72$, $m = 2$ and $r = 0.1$.

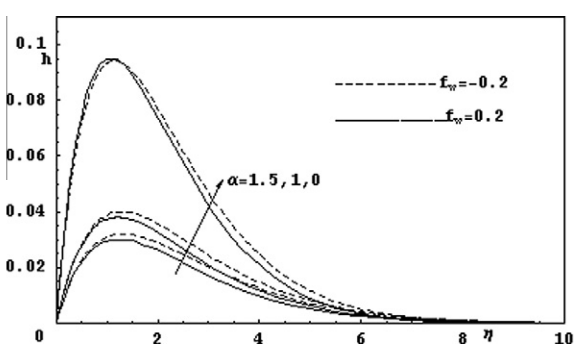

Figure 2 Micro-rotationn profiles for various values of α with $G = 2$, $K = 1.2$, $M = 0.5$, $Pr = 0.72$, $m = 2$ and $r = 0.1$.

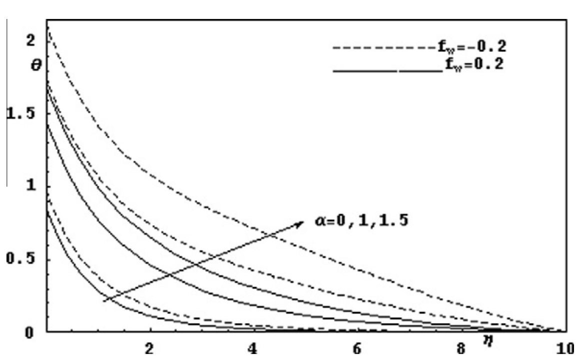

Figure 3 Temperature profile for varioous values of α with $G = 2$, $K = 1.2$, $M = 0.5$, $Pr = 0.72$, $m = 2$ and $r = 0.1$.

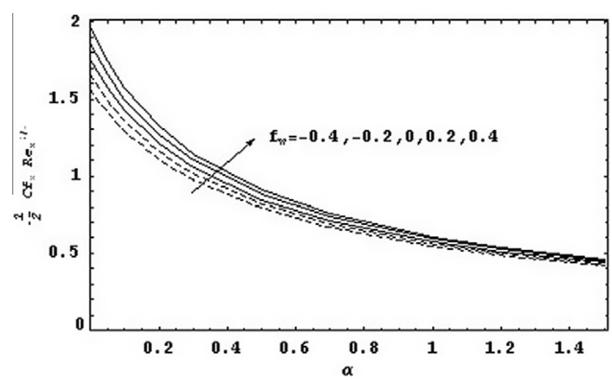

Figure 4 Local skin friction coefficient as a function of α for various values of f_w when $G = 2$, $K = 1.2$, $M = 0.5$, $Pr = 0.72$, $m = 2$ and $r = 0.1$.

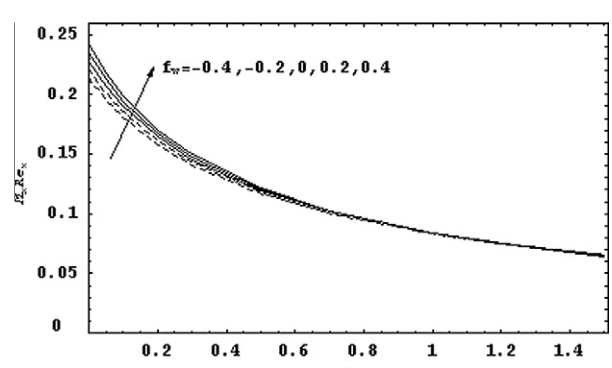

Figure 5 Wall couple stess coefficient as a function of α for various values of f_w when $G = 2$, $K = 1.2$, $M = 0.5$, $Pr = 0.72$, $m = 2$ and $r = 0.1$.

number in the term of $\frac{1}{\theta(0)}$ for various values of M, m and γ are illustrated in Table 3. It is noted from this table that the local skin-friction coefficient and the wall couple stress increase with the increase of M for both suction and injection. The local Nusselt number decreases with the increase of M and the heat generation parameter, while the local Nusselt number increases with the increase of m and the absolute value of the heat absorption parameter for both suction and injection.

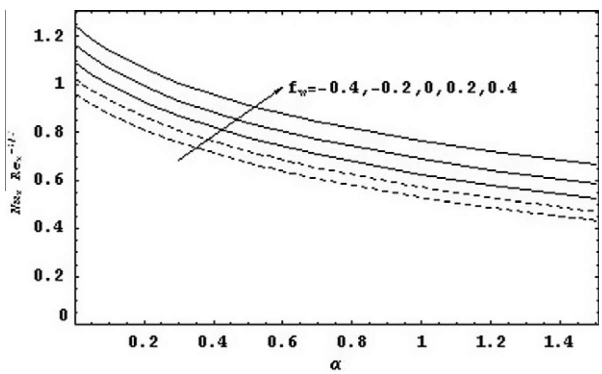

Figure 6 Local Nusselt number cofficient as a function of α for various values of f_w when $G = 2$, $K = 1.2$, $M = 0.5$, $Pr = 0.72$, $m = 2$ and $r = 0.1$.

Table 3 Values of $-f''(0)$, $h'(0)$ and $\frac{1}{\theta(0)}$ with $K = 1.2$, $G = 2$, $\alpha = 0.1$ and $Pr = 0.72$.

M	f_w	γ	m	$-f''(0)$	$h'(0)$	$\frac{1}{\theta(0)}$
0.5	0.2	0.1	2	0.68093	0.194661	1.06836
1	0.2	0.1	2	0.77032	0.204439	1.01252
1.5	0.2	0.1	2	0.84625	0.211354	0.96228
0.5	−0.2	0.1	2	0.61652	0.185523	0.93455
1	−0.2	0.1	2	0.70862	0.197264	0.87108
1.5	−0.2	0.1	2	0.78681	0.205568	0.81090
0.5	0.2	0.2	2	0.68093	0.194661	1.01481
0.5	0.2	0.1	2	0.68093	0.194661	1.06836
0.5	0.2	0	2	0.68093	0.194661	1.11476
0.5	0.2	−0.1	2	0.68093	0.194661	1.15698
0.5	0.2	−0.2	2	0.68093	0.194661	1.19629
0.5	−0.2	0.2	2	0.61652	0.185523	0.87554
0.5	−0.2	0.1	2	0.61652	0.185523	0.93455
0.5	−0.2	0	2	0.61652	0.185523	0.98216
0.5	−0.2	−0.1	2	0.61652	0.185523	1.02466
0.5	−0.2	−0.2	2	0.61652	0.185523	1.06393
0.5	0.2	0.1	0	0.68093	0.194661	0.460672
0.5	0.2	0.1	0.5	0.68093	0.194661	0.641341
0.5	0.2	0.1	1	0.68093	0.194661	0.799354
0.5	0.2	0.1	2	0.68093	0.194661	1.06836
0.5	−0.2	0.1	0	0.61652	0.185523	0.247212
0.5	−0.2	0.1	0.5	0.61652	0.185523	0.463184
0.5	−0.2	0.1	1	0.61652	0.185523	0.642564
0.5	−0.2	0.1	2	0.61652	0.185523	0.934552

5. Conclusions

In this study, we have presented the effects of slip velocity on the flow and heat transfer of an electrically conducting micropolar fluid over a stretching surface with variable heat flux in the presence of heat generation (absorption) and a uniform transverse magnetic field. The results indicate that the numerical values of the local skin-friction coefficient and the wall couple stress increased with the increase of the magnetic parameter and the local Nusselt number decreased with the increase of the magnetic parameter for both cases of suction and injection. The slip parameter has the effect of reducing the value of the local skin-friction coefficient, the wall couple stress and the local Nusselt number for both cases of suction and injection. Also, it was found that injection case leads to reducing the local skin-friction coefficient, the wall couple stress and the local Nusselt number, but the effect of suction gives the opposite. Moreover, the local Nusselt number is decreased as the heat generation parameter is increased. Further, it was found that the local Nusselt number increases with an increase in the absolute value of the heat absorption parameter and the heat flux exponent for both cases of suction and injection.

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