Alexandria Engineering Journal (2016) 55, 287–297

HOSTED BY
ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

MHD oscillatory channel flow, heat and mass transfer in a physiological fluid in presence of chemical reaction

J.C. Misra [a,*], S.D. Adhikary [b]

[a] Centre for Healthcare Science and Technology, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
[b] Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur, India

Received 23 July 2015; revised 19 September 2015; accepted 28 October 2015
Available online 17 November 2015

KEYWORDS
Radiative heat transfer;
Chemical reaction;
Oscillatory flow;
Viscoelastic fluid

Abstract In the present paper, the problem of oscillatory MHD flow of blood in a porous arteriole in presence of chemical reaction and an external magnetic field has been investigated. Heat and mass transfer during arterial blood flow are also studied. A mathematical model is developed and analyzed by using appropriate mathematical techniques. Expressions for the velocity profile, volumetric flow rate, wall shear stress and rates of heat and mass transfer have been obtained. Variations of the said quantities with different parameters are computed by using MATHEMATICA software. The quantitative estimates are presented through graphs and table.

1. Introduction

Rates of many physiological functions, including the flow through blood vessels are affected by drugs. The rates of different biochemical reactions that are responsible for the contraction muscles, secretion of different materials such as insulin, mucus and stomach acid by the glands and the transmission of massages by the nerves can be accelerated/decelerated by the action of drugs.

The rate at which the kidney cells perform the regulation of the volume of water/salts in the body is affected by drugs. The rate at which blood flows through arteries can also be enhanced/slowed down by the application of drugs. It may, however, be noted that the damaged structures/functions can only be repaired by drugs, but their restoration is not possible. This is the observation of the clinicians, when they treat patients suffering from various types of degenerative/tissue-destroying diseases, including multiple atherosclerosis (narrowing of arterial lumen due to deposition of different fatty substances, cholesterol, etc.), arthritis, Alzheimer disease, Parkinson disease, heart failure. However, there are some drugs (e.g. antibiotics) that help the body in the damage repair process, when the damage takes place due to some infection. Several drugs (antacids, for example) produce effects, where the function of a cell remains unchanged and a receptor dose not have any cognition. Most of the antacids are bases that interact with stomach acid to neutralize it. Thus stomach acid is reduced simply through chemical reactions.

* Corresponding author.
E-mail address: misrajc@gmail.com (J.C. Misra).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2015.10.005
1110-0168 © 2015 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

Nomenclature

B_0	intensity of the external magnetic field	S_c	Schmidt number
C	concentration	t^*	time
c_p	specific heat at constant pressure	T	fluid temperature
g	gravitational acceleration	u^*	axial velocity
G_c	modified Grashof number	(x^*, y^*, z^*)	space coordinates
G_r	Grashof number	β_0	viscoelastic coefficient
k	permeability factor	β	coefficient of volume expansion due to temperature
K_c	chemical reaction parameter	β_c	coefficient of volume expansion due to concentration
M	Hartmann number	λ	slip parameter
N	thermal radiation parameter	μ	dynamic viscosity
N_u	Nusselt number	v	kinematic viscosity, $\left(\frac{\mu}{\rho}\right)$
p^*	pressure	σ	conductivity of the medium
P_r	Prandtl number	ω^*	angular frequency
q_r	radiative heat flux	ρ	fluid density
R_e	Reynolds number		
Sh	Sherwood number		

The strength (potency in medical terms) of a drug is the quantity that is required to be applied in order to have a visible effect (e.g. reduction of blood pressure, relief of pain); efficacy of a drug refers to its capacity to produce an effect (reduction in blood pressure, for example), while a drug's effectiveness is determined by how well a drug works in the medical treatment of a patient. It may so happen that a drug having higher efficacy (say, in reducing blood pressure) may be less effective. The reason behind this is that the drug may have too many side effects and patient will be highly reluctant to take it such a drug is not at all suitable in real-world use.

Although oxygen binds with blood hemoglobin and does not react, the discussion on drug dynamics, made above emphasizes the need for paying importance to the presence of chemical reactions during various physiological functions. Many researchers, including the present authors have studied various aspects of blood flow in normal/diseased arteries. Some of them will be briefly reviewed in the sequel in this section. While several studies on blood flow in arteries having a single stenosis have already been carried out by different researchers, the literature on multiple atherosclerosis is somewhat scanty. Misra et al. [1] studied blood flow through an arterial segment having multiple stenosis. They presented a schematic diagram of the physical problem and developed an appropriate model, on the basis of which it has been shown that the wall shear stress and pressure drop are maximum at the throat of each stenosis and minimum at the ends of each stenosis, while the volumetric flow rate of blood is maximum at the throat of each stenosis. Their study further reveals an increase in the size of the brain, heat and other organs of the body, leading to stroke, heart attack and other cardiovascular diseases.

The effects of viscosity variation, variable hematocrit and velocity-slip at the arterial wall on blood flow through a porous segment of an artery subject to the action of an external magnetic field were investigated by Sinha and Misra [2]. The study reveals a very important phenomenon that when the erythrocytes concentration is on the rise, the systolic pressure increases, and diastolic pressure diminished. In a separate communication, the same authors (Sinha and Misra [3]) reported an externally applied magnetic field bears the potential to reduce the velocity gradient of a magnetohydrodynamic stagnation point channel flow of a fluid at the surface of the channel, surface heat transfer and the induced magnetic field. In the realm of flow through porous media, Misra and his research group [1–5,21–27] also made valuable contributions by exploring several important phenomena, some of them have the promise of important applications in the study of physiological fluid dynamics. In one of their recent studies dealing with MHD flow through a porous medium with stretching wall, Misra and Sinha [4] made an important observation that both Hall current and thermal radiation play significant roles in controlling the temperature of blood. A fundamental problem concerning biomagnetic fluid flow through a porous medium was solved by Misra et al. [5], by employing the principles of ferrodynamics and biomagnetic fluid dynamics (BFD). This particular investigation was motivated toward estimating blood flow in arteries during electromagnetic hyperthermia (a modern therapeutic procedure used for cancer treatment). The study shows that the presence of a magnetic dipole affects the characteristics of arterial blood flow significantly during electromagnetic hyperthermia.

In a recent study, Adesanya et al. [6] found that hyperviscosity of blood considerably affects the flow and pressure pulse wave propagation. The effect of viscous dissipation on the pulsating flow of a fluid was analyzed by Adesanya et al. [7] by using Adomain decomposition method, while the problem of heat transfer to MHD couple stress pulsatile flow between two parallel porous plates was investigated by Adesanya and Makinde [8]. By using advection–diffusion equations applicable for porous media, Ai and Vafai [9] presented a mathematical model for the transport of macromolecules, like low density lipoproteins (LDL) in the blood stream and in the arterial walls. Wade and Karino [10] reported a computational study on LDL transfer from flowing blood to the wall tissues of arteries.

A porous medium is a material volume that consists of a solid matrix with an interconnected void, while porosity refers to the ratio between the volume of the void space and the total volume of the medium. A porous medium is characterized not only by its porosity but also by its permeability, that is, the measure of flow conductivity in the porous medium. In studies related to the heat transfer in biological tissues, the theory of porous media is preferred by researchers, since the number of assumptions necessary for the development of a model is less than that in the case of other bioheat models. The role of porous media in different studies of the mechanics of different biological organs and tissues is enormous. Khaled and Vafai [11] in a review paper explained the role of porous media in studies related to the flow and heat transfer in biological tissues. The flow characteristics of a Casson fluid in a tube filled with a homogeneous porous medium were examined by Dash et al. [12]. This study was motivated toward modeling blood flow in a pathological state of the artery, when different fatty substances and cholesterol are deposited in the lumen of the coronary artery/artery-clogging blood clots are formed.

Slip-flow is defined as the difference in the velocities between liquids and solids (or liquids and gases) in the vertical flow of two-phase mixtures through a tube, owing to the slip between the two phases. On the basis of an experimental study, Beavers and Joseph [13] proposed a theory according to which the effect of the boundary layer can be replaced by a slip velocity is proportional to the exterior velocity gradient. Beavers and Joseph slip condition has been applied in the study of a large number of different problems dealing with various geometrical configurations. Subsequently Beavers et al. [14] carried out experiments to study the laminar flow in a channel bounded by two parallel plates, one of whose bounding walls is a porous medium. They made an observation that owing to the presence of the porous wall, the mass flow increases. Further experiments were conducted by Beavers et al. [15] with an aim to validate the slip boundary condition in the case of gas flows and to ascertain whether the fluid has a considerable influence on the value of the slip parameter.

In order to understand some physical phenomena, such as transpiration cooling and gaseous diffusion, as well as the physiological flow of blood in presence of an external magnetic field studies of oscillatory MHD flow of viscoelastic fluids in a channel have occupied a central place in fluid dynamics. Many fluids, for example, polymeric solution, polymer melts, paints and oils as well as most physiological fluids, e.g. blood exhibit prominent non-Newtonian behavior. Since non-Newtonian fluids are of different types, various researchers have developed different models to study the behavior of various types of non-Newtonian fluids. Rivlin Ericksen model [16] of second-order fluids is one of several such fluid models that has drawn interest of fluid mechanics researchers.

Fluctuating flow of a viscoelastic fluid in a porous channel was studied by Bhatnagar [17] by using a perturbation scheme. Oscillatory flow of a viscous fluid in a channel was studied by Makinde and Mhone [18]. Ram [19] investigated the effect of Hall current and wall temperature oscillation in a plate where the fluid is rotating. Bastman [20] considered low Reynolds number flow of blood in arteries of slowly varying cross-section, treating blood as a second-order fluid.

The unsteady flow of blood through vessels was studied by Misra and Sahu [21] and Misra et al. [22] by treating blood as a second-order viscoelastic fluid. Electrically conducting viscoelastic fluid flow and heat transfer in a parallel plate channel with stretching walls under the action of an external magnetic field were investigated by Misra and Shit [23]. Subsequently Misra and Shit [24] conducted another study on the flow of a biomagnetic viscoelastic fluid in presence of a magnetic field generated by a magnetic dipole. In this study the non-Newtonian character of blood was represented by Walter's liquid B model. Peristaltic motion of blood in the micro-circulatory system was studied by Misra and Maiti [25], by considering the arterioles to be of varying cross-section. Sinha and Misra [26] developed a numerical model to perform a study on mixed convection hydromagnetic flow over an inclined nonlinear porous stretching sheet. Misra and Sinha [27] also investigated the unsteady flow and temperature fields in a diseased capillary, its lumen being porous and wall permeable. Thermal radiation, velocity slip and thermal slip were incorporated in the study.

However, in none of the aforesaid studies mentioned above, the effect of chemical reaction has been incorporated. Xu et al. [28] developed a theoretical model to study the impact of blood flow on rombus growth, by considering the interaction between different constituents of blood and chemical reaction. Coulson and Hernandez [29] carried out an experimental study that depicts the metabolic activities in the presence of chemical reactions. They made an important observation due to injection of certain drugs, hormones and metabolites, and there occurs considerable increase in plasma concentration of reactants. There are sympathomimetics that bear the potential to increase blood glucose and for days together plasma concentration remains at a higher level. It may be noted that in the body, the rate at which chemical oxidation takes place depends on the amount of oxygen consumption, and the related information is highly useful to explore how various requirements are met.

Studies pertaining to the combined effects of heat and mass transfer on chemical reactions are of considerable importance in physiological flows and also in many industrial processes. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, the transfer of heat and mass occur simultaneously. Heat and mass transfer in magneto-micropolar fluid in a porous plate was studied by Chaudhary and Jain [30]. In their study they considered the radiation term in the heat transfer equation.

Chemical reactions can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be of the first order, if the rate of reaction is directly proportional to the concentration. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes take place in many processes of chemical industries, such as polymer production, the manufacturing of ceramics or glassware, the processing of food and so on. Das et al. [31] considered the effects of a first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer.

Muthucumarswamy and Ganesan [32] as well as Muthucumarswamy [33] studied the first order homogeneous chemical reactions on the flow past an infinite vertical plate. Recently, Kandasamy et al. [34] discussed the heat and mass transfer effect along a wedge with a heat source and concentration in the presence of suction/injection taking into account the chemical reaction of the first order. Chaudhary and Jha [35] studied on the effect of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Chemical physiology of flood flow regulation by red cells was discussed by Singel and Stemler [36].

The aim of the present investigation was to study the effect of chemical reaction, as well as heat and mass transfer on the oscillatory MHD flow of blood, under a single framework, treating blood as a second-grade fluid. We consider a channel flow of blood in the central region of the vessel. The study has been motivated toward examining the flow of blood in an arteriole under some pathological situations, when the fatty plaques of cholesterol and blood clots are formed, so that artery-clogging takes place in some portion of the lumen of the coronary artery. The lumen of the blood vessel also turns to a porous structure due to the presence of multiple atherosclerosis in a diseased state of the artery. Velocity-slip of erythrocytes contained in blood has been taken into account. A magnetic field is considered to be applied externally in a direction transverse to that of blood flow. The radiative heat transfer and first order homogeneous chemical reaction in the mass transfer on the flow field are of particular concern here. Oscillation of the temperature and concentration in the upper side of the artery is considered. Analytical expressions are obtained for the velocity profiles, the temperature, the volumetric flow rate, the wall shear stress and rate of heat transfer on the upper wall of the channel are obtained. These expressions are computed numerically for a specific type of hemodynamical flow. Numerical values of the aforesaid quantities are presented graphically. These graphs depict the variation of the quantities with different values of the parameters involved in the study. The values/range of values considered in carrying out the computational work conform to those of the physiological problem of blood flow in a pathological state. These results reveal the characteristics of blood flow as well as the effects of heat and mass transfer in presence of chemical reaction that takes place during blood flow process due to the action of drugs.

2. Mathematical model

The circulatory system mainly consists of three-dimensional cylindrical vessels. However, in some cases, such as in microvessels of the lungs, motion of blood can be approximately considered as channel flow. With this consideration, as in many other similar theoretical studies (e.g. [5]), the formulation analysis that follows, we use cartesian coordinates. The flow is considered symmetric about the axis of the channel and driven by the stretching of the channel wall, such that the velocity of each wall is proportional to the axial coordinate. In order to study the second-order effects of unsteady MHD flow of blood, let us first consider the flow of a second-order fluid between two parallel plates at $y^* = 0$ and $y^* = h$, where the axis of x^* is taken parallel length of plates and y^*-axis along a direction perpendicular to the plates.

3. Analysis

The model developed here pertains to a pathological state of an arterial segment (as in the case of multiple atherosclerosis), when the lumen has turned porous due to the deposition of different materials such as cholesterol, lipids and fatty substances. The physical sketch of the problem is similar to that in [2,5]. Taking into account the existence of sleep between the velocity of blood and the arterial wall tissues, the relative velocity between blood and the arterial wall is assumed to be proportional to the shear rate at the wall. Blood is considered as a suspension of erythrocytes and other micro-elements in plasma. It is assumed that in the segment under consideration, blood is uniformly dense. A magnetic field of constant intensity B_0 is considered to be applied in the y^*-direction.

The momentum, heat transfer and mass transfer equations are considered in the form (cf. Makinde and Mhone [18])

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\frac{\partial^2 u^*}{\partial y^{*2}} + \beta_0^*\frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} - \frac{\nu}{k^*}u^* - \frac{\sigma B_0^2}{\rho}u^* + g\beta(T - T_0) + g\beta_c(C^* - C_0), \tag{1}$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}, \tag{2}$$

$$\frac{\partial T}{\partial t^*} = \frac{K'}{\rho c_p}\frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y^*} \tag{3}$$

and $$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_c'(C^* - C_0), \tag{4}$$

where the meanings of all the symbols appearing in the equations are listed in the Nomenclature.

In presence of red cell-slip at the boundary wall of the blood vessels reported by Brunn [37] and Nubar [38], the boundary conditions for the problem under consideration are given by

$$u^* = \lambda\frac{\partial u^*}{\partial y^*}, \quad T = T_0 + (T_W - T_0)e^{i\omega^* t^*},$$
$$C^* = C_0 + (C_W - C_0)e^{i\omega^* t^*} \quad at\ y^* = h \tag{5}$$

and

$$u^* = \lambda\frac{\partial u^*}{\partial y^*}, \quad T = T_0, \quad C^* = C_0 \quad at \quad y^* = 0 \tag{6}$$

Using Rosseland approximation [39], the radiative transfer term q_r in Eq. (3) may be expressed as

$$q_r = -\frac{4\sigma^*}{3\alpha_r}\frac{\partial T^4}{\partial y^*}. \tag{7}$$

We assume that the temperature differences within the flow are such that T^4 can be expressed as a linear function of the temperature T. This is accomplished by expanding T^4 in a Taylor series about T_0 (which is assumed to be independent of y^*) and neglecting powers of T higher than the first. Thus we have

$$T^4 = 4T_0^3 T - 3T_0^4. \tag{8}$$

Then the heat transfer equation becomes

$$\frac{\partial T}{\partial t^*} = \frac{K'}{\rho c_p}\frac{\partial^2 T}{\partial y^{*2}} + \frac{16\sigma^* T_0^3}{3\rho c_p \alpha_r}\frac{\partial^2 T}{\partial y^{*2}}. \tag{9}$$

We now introduce the following non-dimensional variables:

$$y = \frac{y^*}{h}, \quad x = \frac{x^*}{h} u = \frac{u^*}{U_0}, \quad R_e = \frac{U_0 h}{\nu}, \quad k = \frac{k^*}{h^2\rho}, \quad t = \frac{t^* U_0}{h},$$
$$p = \frac{h p^*}{\rho\nu U_0}, \quad \theta = \frac{T - T_0}{T_W - T_0}, \quad C = \frac{C^* - C_0}{C_W - C_0},$$
$$G_r = \frac{g\beta(T_W - T_0)h^2}{\nu U_0}, \quad G_c = \frac{g\beta_c(C_W - C_0)h^2}{\nu U_0}, \quad M^2 = \frac{\sigma B_0^2 h^2}{\rho\nu},$$
$$P_r = \frac{\rho c_p}{K}, \quad N = \frac{16\sigma^* T_0^3}{3\alpha_r K}, \quad S_c = \frac{1}{D}, \quad K_c = DK_c'(C_W - C_0),$$
$$\beta_0 = \frac{\beta_0^* U_0}{\nu h}, \quad \omega = \frac{\omega^* h}{U_0}. \tag{10}$$

The governing equations in terms of these dimensionless quantities can then be written as

$$R_e\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \beta_0\frac{\partial^3 u}{\partial y^2 \partial t} - \left\{\frac{1}{k} + M^2\right\}u + G_r\theta + G_c C, \tag{11}$$

$$0 = -\frac{\partial p}{\partial y}, \tag{12}$$

$$P_r\frac{\partial \theta}{\partial t} = (1+N)\frac{\partial^2\theta}{\partial y^2} \tag{13}$$

and

$$S_c\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - K_c C, \tag{14}$$

while the non-dimensional boundary conditions assume the form

$$u = \lambda\frac{\partial u}{\partial y}, \quad \theta = e^{i\omega t}, \quad C = e^{i\omega t} \quad \text{at } y = 1 \tag{15}$$

$$u = \lambda\frac{\partial u}{\partial y}, \quad \theta = 0, \quad C = 0 \quad \text{at } y = 0 \tag{16}$$

From (11) and (12) it follows that $\frac{\partial p}{\partial x}$ is a function of t only. We consider it to be of the form

$$\frac{\partial p}{\partial x} = Be^{i\omega t}. \tag{17}$$

To solve Eqs. (11), (13) and (14) subject to the boundary conditions (15) and (16), we further write the velocity, temperature and concentration as

$$u(y,t) = u_f(y)e^{i\omega t}$$
$$\theta(y,t) = \theta_f(y)e^{i\omega t}$$
$$C(y,t) = C_f(y)e^{i\omega t} \tag{18}$$

Substituting these expressions in (11), (13) and (14) and comparing like terms we have the equations.

$$\{1+\beta_0 i\omega\}\frac{\partial^2 u_f}{\partial y^2} - \left\{R_e i\omega + \frac{1}{k} + M^2\right\}u_f = -B - G_r\theta_f - G_c C_f, \tag{19}$$

$$(1+N)\frac{\partial^2\theta_f}{\partial y^2} - P_r i\omega\theta_f = 0 \tag{20}$$

and $$\frac{\partial^2 C_f}{\partial y^2} - \{S_c i\omega + K_c\}C_f = 0, \tag{21}$$

along with the boundary conditions

$$u_f = \lambda\frac{\partial u_f}{\partial y}, \quad \theta_f = 1, \quad C_f = 1 \quad at\ y = 1 \tag{22}$$

$$u_f = \lambda\frac{\partial u_f}{\partial y}, \quad \theta_f = 0, \quad C_f = 0 \quad at\ y = 0. \tag{23}$$

Solving (19)–(21) subject to the conditions (22) and (23) and using (18), we have

$$\theta(y,t) = \frac{e^{i\omega t}}{e^{m_1} - e^{m_2}}[e^{m_1 y} - e^{m_2 y}] \tag{24}$$

$$C(y,t) = \frac{e^{i\omega t}}{e^{m_3} - e^{m_4}}[e^{m_3 y} - e^{m_4 y}] \tag{25}$$

$$u(y,t) = \left[E_1 e^{m_5 y} + E_2 e^{m_6 y} - \frac{B}{(R_e i\omega + \frac{1}{k} + M^2)} - \frac{G_r}{e^{m_1} - e^{m_2}}\left[\frac{e^{m_1 y}}{A_1} - \frac{e^{m_2 y}}{A_2}\right] - \frac{G_c}{e^{m_3} - e^{m_4}}\left[\frac{e^{m_3 y}}{A_3} - \frac{e^{m_4 y}}{A_4}\right]\right]e^{i\omega t}, \tag{26}$$

which respectively represent the temperature, concentration and velocity fields,where the expressions for the constants $m_i(i = 1, 2, \ldots, 6), A_i(i = 1, 2, 3, 4), E_1, E_2$ are given in Appendix A.

The volumetric flow rate is calculated as

$$Q = \int_0^1 u dy = \left[\frac{E_1}{m_5}(e^{m_5} - 1) + \frac{E_2}{m_6}(e^{m_6} - 1) - \frac{B}{(R_e i\omega + \frac{1}{k} + M^2)} - \frac{G_r}{e^{m_1} - e^{m_2}} \times \left[\frac{(e^{m_1} - 1)/m_1}{A_1} - \frac{(e^{m_2} - 1)/m_2}{A_2}\right] - \frac{G_c}{e^{m_3} - e^{m_4}}\left[\frac{(e^{m_3} - 1)/m_3}{A_3} - \frac{(e^{m_4} - 1)/m_4}{A_4}\right]\right]e^{i\omega t}, \tag{27}$$

while the wall shear stress at the wall of the upper plate (representing the upper wall of the blood vessel) is found as

$$\tau_w = \left[\frac{\partial u}{\partial y} + \beta_0\frac{\partial^2 u}{\partial y \partial t}\right]_{y=1} = \left[E_1 m_5 e^{m_5} + E_2 m_6 e^{m_6} - \frac{G_r}{e^{m_1} - e^{m_2}}\left[\frac{m_1 e^{m_1}}{A_1} - \frac{m_2 e^{m_2}}{A_2}\right] - \frac{G_c}{e^{m_3} - e^{m_4}}\left[\frac{m_3 e^{m_3}}{A_3} - \frac{m_4 e^{m_4}}{A_4}\right]\right] \times (1 + \beta_0 i\omega)e^{i\omega t}. \tag{28}$$

The rates of heat and mass transfer across the upper plate (upper wall) are calculated as

$$N_u = -\left.\frac{\partial\theta}{\partial y}\right]_{y=1} = -\frac{e^{i\omega t}}{e^{m_1} - e^{m_2}}[m_1 e^{m_1} - m_2 e^{m_2}] \tag{29}$$

and

$$Sh = -\left.\frac{\partial C}{\partial y}\right]_{y=1} = -\frac{e^{i\omega t}}{e^{m_3} - e^{m_4}}[m_3 e^{m_3} - m_4 e^{m_4}]. \tag{30}$$

4. Results: Application to dynamics of blood flow in presence of chemical reaction

The motivation behind the study has been to analyze the effects of heat and mass transfer in presence of chemical reaction in the flow of blood, by accounting for the velocity slip and the second-grade effects. For the purpose of numerical computation, the following values of the different parameters involved in the analytical study have been used.

$$B = 1.0, \quad w = 1.0, \quad G_r = 2.0, \quad G_c = 2.0, \quad 0 \leqslant P_r \leqslant 3.0, \quad 0 \leqslant R_e \leqslant 6.0, \quad 0.0 \leqslant S_c \leqslant 1.5, 0.0 \leqslant M \leqslant 5.0, \quad 0.1 \leqslant k \leqslant 2.0, \quad 0.0 \leqslant N \leqslant 3.0, \quad 0.0 \leqslant \beta_0 \leqslant 1.0, \quad 0.0 \leqslant \lambda \leqslant 0.1, 0.0 \leqslant K_c \leqslant 2.0, \quad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

These values/ranges of values of the parameters are mostly representative of blood flow, when a chemical reaction sets in.

By using these values, the analytical expressions derived in the previous section have been computed by employing a suitable software, viz. MATHEMATICA.

Variation in the distributions of velocity, temperature, concentration, volumetric flow rate and wall shear stress has been investigated numerically, with respect to different parameters. Role of the same parameters in executing heat and mass transfer in the blood mass has also been investigated. All the computational data have been presented in graphical/tabular form (cf. Figs. 1–10). Figs. 1–4 focus on the variation of velocity distribution at some specific instants of time for different values of viscoelastic parameter (β_0), slip parameter (λ), magnetic parameter (M), thermal radiation parameter (N) and chemical reaction parameter (K_c). Fig. 1 indicates that at a particular instant of time, blood velocity reduces as blood viscoelasticity (β_0) increases. It may be noted that for $\beta_0 = 0.2$, velocity decreases as time progresses. However, this is not the case when the value of β_0 is larger. It is interesting to see that for $\beta_0 = 0.5$, velocity is greatest at $t = \pi/4$ and for $\beta_0 = 1.0$, velocity is least at $t = 0$.

Both the study $P_r = 0.71$, $R_e = 1.0$, $M = 1.0$, $k = 1.0$, $N = 1.0$, $t = 0$ ($\lambda = 0$, $S_c = 0.0$, $\beta_0 = 0.0$, $K_c = 0.0$, $G_c = 0.0$).

Table 1 gives a comparison between the results computed on the basis of our present study with those of Makined and Mhone [18]. For both of these studies, $P_r = 0.71$, $R_e = 1.0$, $M = 1.0$, $k = 1.0$, $N = 1.0$, $t = 0$. Present study reduces to the case of a Newtonian fluid with no-slip in the absence of chemical reaction when $\lambda = 0$, $\beta_0 = 0.0$, $S_c = 0.0$, $K_c = 0.0$, $G_c = 0.0$.

Fig. 2 reveals that there is no indication of flow separation in the absence of slip velocity at the wall, but flow separation does take place whenever there is velocity-slip at the boundary. It is important to note that the extent of flow separation increases with the increase in the slip velocity parameter λ.

The effects of the magnetic and porosity parameters on the velocity distribution are shown in Fig. 3. It is seen that for a given value of the porosity parameter (k), velocity decreases as the intensity of the magnetic field increases. It is further observed that under an identical strength of the magnetic field, the velocity decreases as the porosity parameter (k) decreases, i.e. as the value of the factor $\frac{1}{k}$ appearing in Eq. (11) increases. The graph displayed in Fig. 4 gives the change in velocity with the variation in the heat radiation, when the value of the chemical reaction parameter remains fixed. The other two sets of graphs in Fig. 4 give an idea of the influence of chemical reaction on the velocity distribution under identical condition of heat radiation.

Figs. 5 and 6 reveal that under the purview of the present computational study, at any given distance the temperature/concentration reduces as the thermal radiation/chemical reaction parameter increases. The same figures further reveal that for any particular values of thermal radiation/chemical reaction parameter, both the temperature and the concentration increase as we move further and further from the lower wall to the upper one.

Fig. 7 shows that irrespective of the magnitude of the magnetic field intensity, the volumetric flow rate reduces as the Reynolds number increases and that the variation takes place almost rectilinearly for all the values of the magnetic parameter considered in the present investigation. It is further observed that when the magnetic parameter M increases from 1 to 3, the volumetric flow rate is reduced drastically.

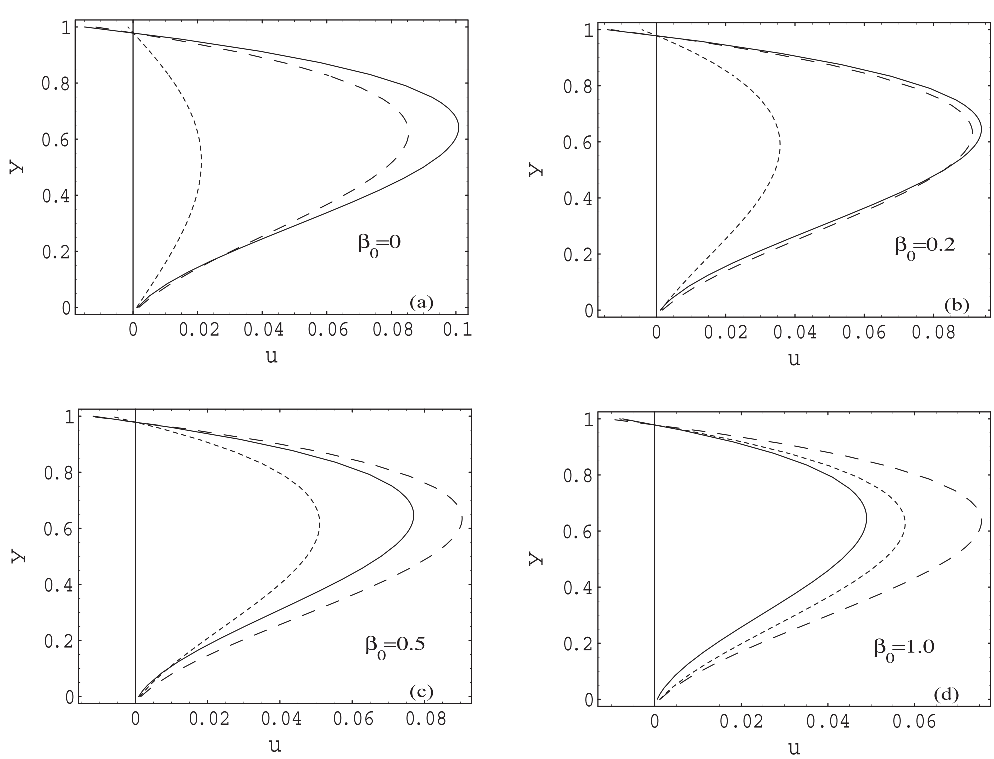

Figure 1 Distribution of velocity in the transverse direction for different values of viscoelastic parameter (β_0), at t: 0 (continuous lines), $\frac{\pi}{4}$ (dashed lines) and $\frac{\pi}{2}$ (dotted lines), in the specific case when $\lambda = 0.02, P_r = 0.71, R_e = 1.0, M = 1.0, k = 1.0, K_c = 0.5, N = 0.5, S_c = 1.0$.

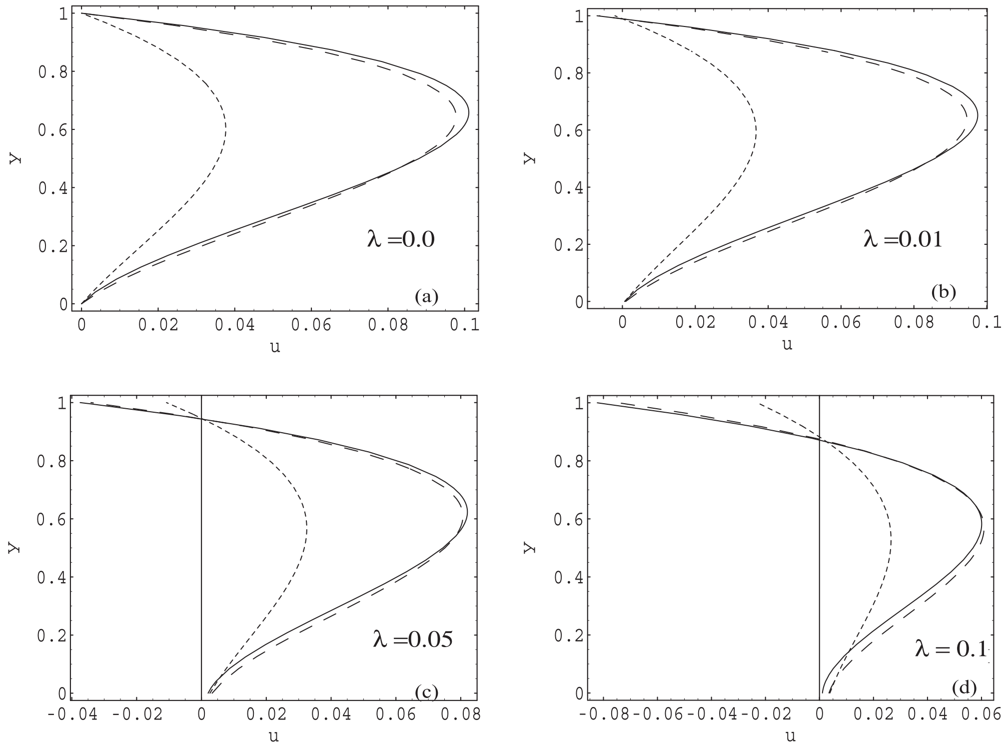

Figure 2 Variation of velocity with transverse distance for different values slip parameter (λ), at three different instants of time, t: 0 (continuous lines), $\frac{\pi}{4}$ (dashed lines) and $\frac{\pi}{2}$ (dotted lines), when $\beta_0 = 0.2, P_r = 0.71, R_e = 1.0, M = 1.0, k = 1.0, K_c = 0.5, N = 0.5, S_c = 1.0$.

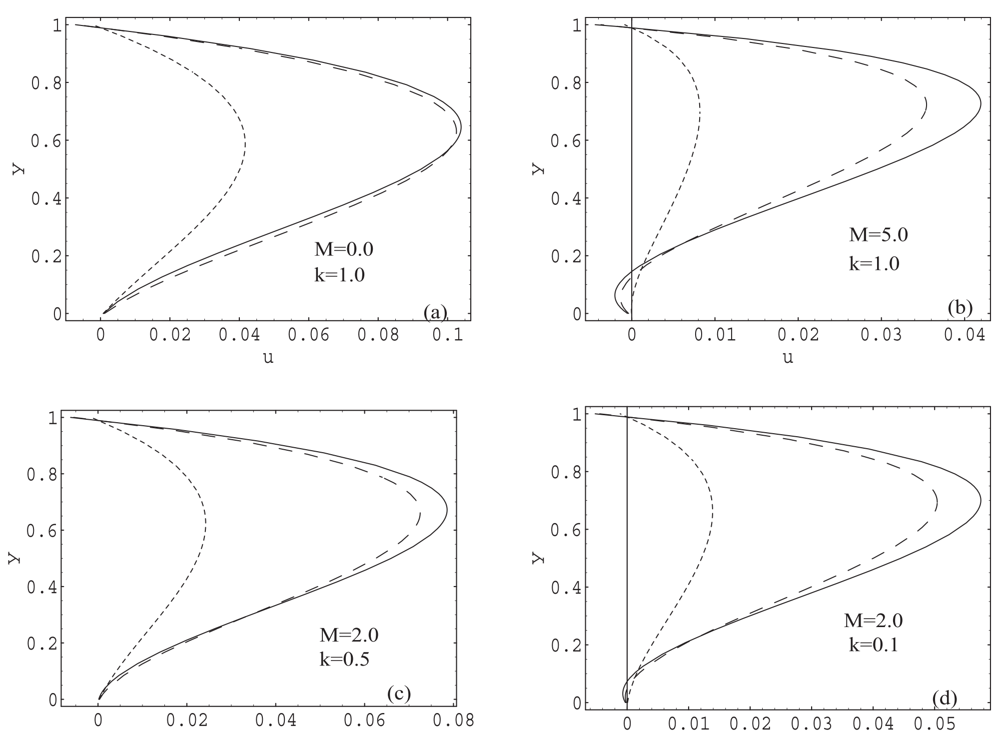

Figure 3 Distribution of velocity for different values of magnetic and porosity parameter (M, k), at different instants of time, t: 0 (continuous lines), $\frac{\pi}{4}$ (dashed lines) and $\frac{\pi}{2}$ (dotted lines) in case $\lambda = 0.02, \beta_0 = 0.2, P_r = 0.71, R_e = 1.0, K_c = 0.5, N = 0.5, S_c = 1.0$.

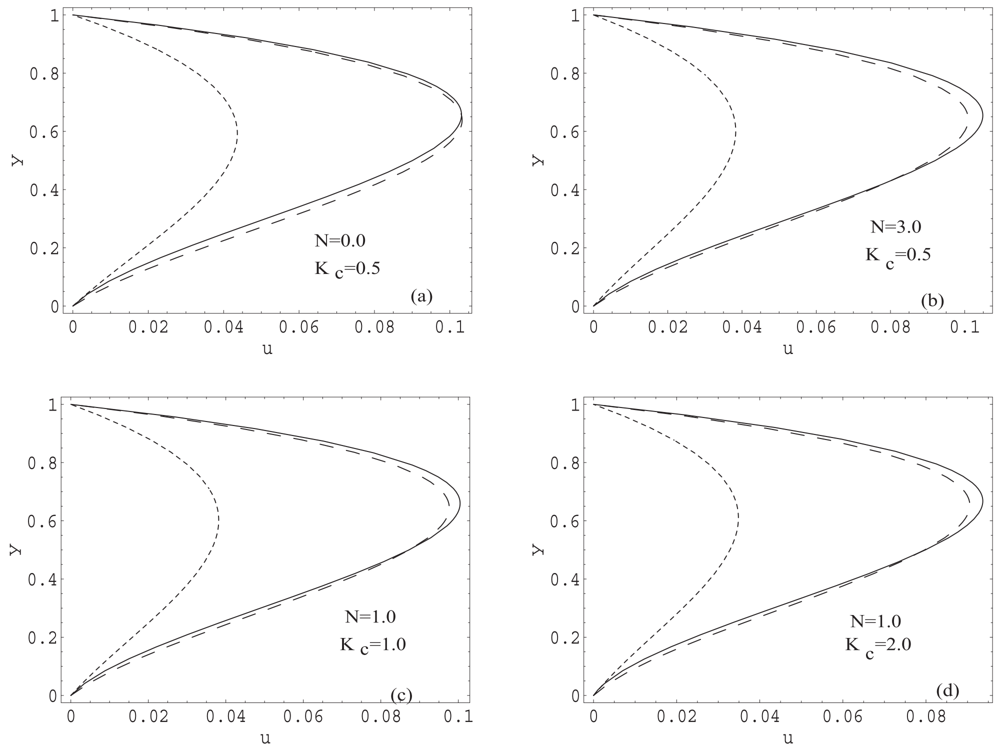

Figure 4 Velocity distribution for different values of heat radiation and chemical reaction parameters (N, K_c), at different time instants, t: 0 (continuous lines), $\frac{\pi}{4}$ (dashed lines) and $\frac{\pi}{2}$ (dotted lines) for the case when $\lambda = 0.0, \beta_0 = 0.2, P_r = 0.71, R_e = 1.0, M = 1.0, k = 1.0, S_c = 1.0$.

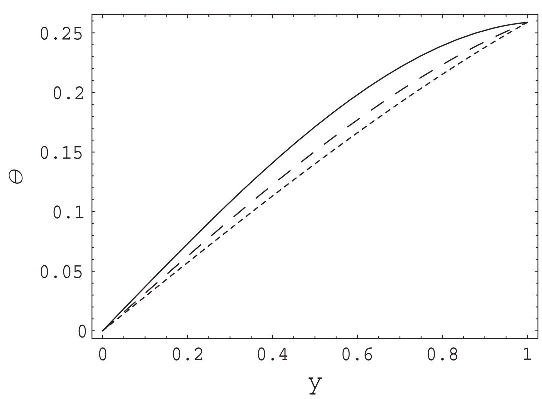

Figure 5 Temperature distribution for different values of thermal radiation parameter (N) when $P_r = 0.71$ at $t = \frac{5\pi}{12}$. The continuous line represents the case when the thermal radiation does not take place, while the dashed and dotted lines stand for the case when the heat radiation parameter (N) equals 1.0 and 3.0 respectively.

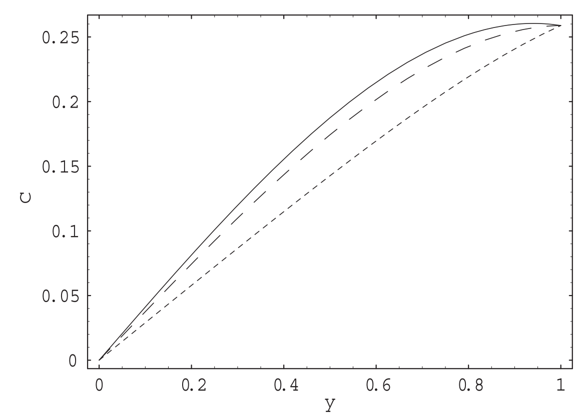

Figure 6 Concentration distribution for different values of chemical reaction parameter (K_c) at the time instant $t = \frac{5\pi}{12}$, in case the value of Schmidt number (S_c) is 1.0 (the continuous line represents the situation when chemical reaction does not take place, while the dashed and dotted lines are representative variations for $K_c = 0.5$ and $K_c = 2.0$ respectively).

The computed values of the shear stress at the upper wall are presented in Fig. 8. This figure shows that in the absence of any magnetic field, the wall shear stress increases with increase in the value of the Reynolds number from 0 to 1.3; around $R_e = 1.3$, and there occurs a sharp reduction in the wall shear stress, which changes its nature from tensile to compressive. A similar nature of the shear stress is observed, even in the presence of a magnetic field of unit strength (M); however the change from tensile to compressive (that occurs in this case around $R_e = 2.5$) is somewhat smooth. It is interesting to note that for $M \geqslant 5$, the shear stress reduces to zero for all values of Reynolds number.

Fig. 9 shows that in the absence of radiation effect, the rate of heat transfer to the upper wall is positive when the Prandtl number exceeds 0.8. When the radiation parameter N is equal to unity, the heat transfer rate is positive, only if $Pr \geqslant 1.6$. It is important to observe that for larger values of N, the heat transfer rate can be positive, only when Prandtl number has a

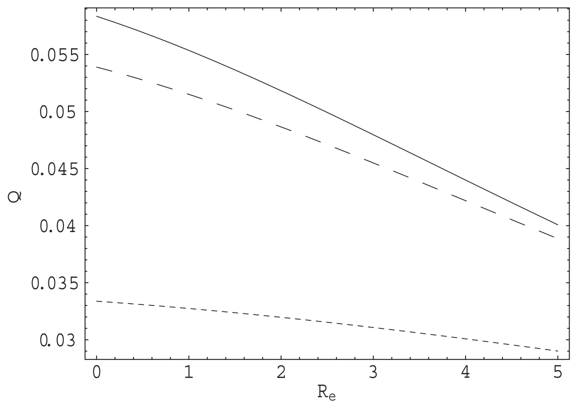

Figure 7 Variation of volumetric flow rate with Reynolds number for different values of magnetic parameter (M) for $P_r = 0.71, \beta_0 = 0.5, k = 1.0, K_c = 1.0, N = 1.0, S_c = 1.0, \lambda = 0.05$, $t = 0$. Here the continuous line gives the variation in the absence of any magnetic field, while the dashed and dotted lines represent the same variation when the magnetic parameter is 1 and 3 respectively.

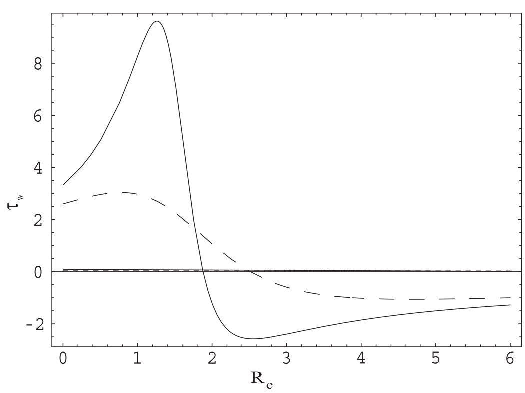

Figure 8 Variation of shear stress at the upper wall with Reynolds number for different values of the magnetic parameter (M) for $P_r = 0.71, \beta_0 = 0.5, k = 1.0, K_c = 1.0, N = 1.0, S_c = 1.0$, $\lambda = 0.02, t = \pi/2$. The continuous, dashed and dotted lines respectively represent the cases when $M = 0$, 1.0 and 5.0.

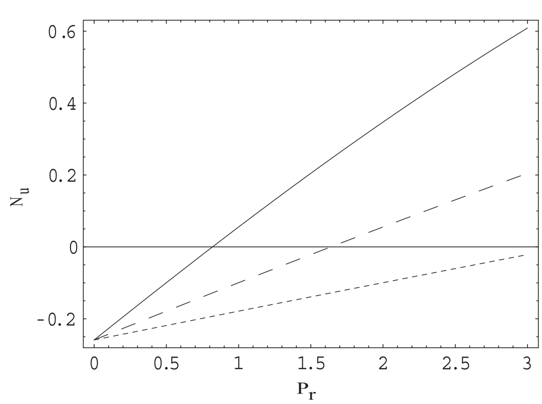

Figure 9 Variation of rate of heat transfer to the upper wall with Prandtl number for different values of radiation parameter N at time $t = 5\pi/12$. The continuous line stand for the case when there is no radiation effect, while the dashed and dotted lines represent the same when $N = 1.0, 3.0$.

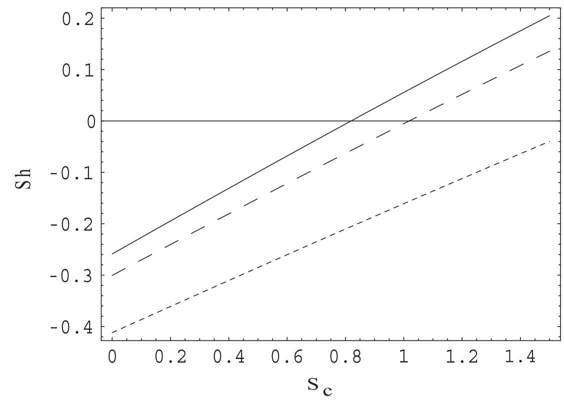

Figure 10 Variation of rate of mass transfer (Sh) to the upper wall with Schmidt number for different values of chemical reaction parameter K_c at $t = 5\pi/12$. Where —— $K_c = 0$, $- - -$ $K_c = 0.5$, ... $K_c = 2.0$.

Table 1 Axial velocity distribution.

y	0	0.2	0.4	0.6	0.8	1.0
Our result	0.0	0.096	0.142	0.150	0.1	0.0
Makinde and Mhone [18]	0.0	0.095	0.14	0.149	0.1	0.0

value greater than 3. For fluids like blood, however, the value of Prandtl number in the normal physiological state is less than 3.

Result for a similar analysis for the variation of mass transfer rate to the upper wall, caused by a chemical reaction in the fluid mass is presented in Fig. 10. This figure reveals that in the case of blood flow, the mass transfer rate is strongly dependent on the value of the chemical reaction parameter K_c. Moreover, for a fixed value of the Schmidt number, the mass transfer rate decreases with increase in K_c.

5. Conclusion

A new model is presented here to study the effects of chemical reaction as well as heat and mass transfer on the magnetohydrodynamic oscillatory flow of blood. Consideration is made of the velocity-slip of erythrocytes. It is important to note that the pulsatility of blood flow owes its origin to the intermittent ejection of blood into the arterial network by the muscular pumping action (systolic and diastolic) of the heart. The analysis is applicable to pertinent problems of physiological fluids and fluid dynamical problems encountered in various industrial processes. However, the computational study has been carried out by using data which conform to those of blood flow in a diseased blood vessel. On the lower wall of the vessel, both the temperature and concentration of blood mass are maintained constant, while the variation of both of them is of oscillatory nature on the upper wall. The study enables us to conclude the following:

(i) Blood viscoelasticity lowers flow velocity significantly.
(ii) The wall shear stress is strongly affected by the Reynolds number.

(iii) At any particular location as the thermal radiation increases, both heat transfer rate and temperature are reduced to an appreciable extent. However, the velocity is not significantly affected by thermal radiation.
(iv) Both mass transfer rate and concentration are reduced due to chemical reaction. Comparatively, the velocity distribution is less affected due to chemical reaction, at least under the purview of the present investigation.
(v) Heat transfer rate increases with increasing Prandtl number.
(vi) Mass transfer rate is enhanced, as the mass diffusivity reduces (i.e. as the Schmidt number increases).

The present study has an important bearing on flow in miniature vessels, like the capillaries, where the diameter of the red cell is larger than that of the capillaries. The results presented here should be of interest to fluid dynamicists and also to physiologists in their study on the flow of blood in the presence of a chemical reaction.

The present study is of particular interest to biophysicists, physiologists and clinicians, because blood flow circulation time is considerably changed after certain classes of drugs are injected into the body. Rate of chemical reaction can also be increased by increasing blood flow. Also, blood flow affects all catabolic reactions and in vertebrates enzyme reaction rate is controlled mainly by adjusting the rate of blood flow by some appropriate means.

Acknowledgments

This work was supported by the Science and Engineering Research Board, Department of Science and Technology (DST), Government of India, New Delhi, under Grant No. SB/S4/MS:864/14. The authors wish to thank the reviewers for their esteemed comments, based upon which the manuscript has been revised to its present form.

Appendix A

$$m_1,\quad m_2 = +\sqrt{\frac{P_r i\omega}{1+N}},\quad -\sqrt{\frac{P_r i\omega}{1+N}} \tag{31}$$

$$m_3,\quad m_4 = +\sqrt{S_c i\omega + K_c},\quad -\sqrt{S_c i\omega + K_c} \tag{32}$$

$$m_5,\quad m_6 = +\sqrt{\frac{R_e i\omega + \frac{1}{k} + M^2}{1+\beta_0 i\omega}},\quad -\sqrt{\frac{R_e i\omega + \frac{1}{k} + M^2}{1+\beta_0 i\omega}} \tag{33}$$

$$A_i = (1+\beta_0 i\omega)m_i^2 - \left(R_e i\omega + \frac{1}{k} + M^2\right),\quad (i = 1,2,3,4) \tag{34}$$

$$\begin{aligned} X_1 = {} & \frac{B}{R_e i\omega + \frac{1}{k} + M^2} + \frac{G_r}{e^{m_1} - e^{m_2}}\left[\frac{1-\lambda m_1}{A_1} - \frac{1-\lambda m_2}{A_2}\right] \\ & + \frac{G_c}{e^{m_3} - e^{m_4}}\left[\frac{1-\lambda m_3}{A_3} - \frac{1-\lambda m_4}{A_4}\right] \end{aligned} \tag{35}$$

$$\begin{aligned} X_2 = {} & \frac{B}{R_e i\omega + \frac{1}{k} + M^2} \\ & + \frac{G_r}{e^{m_1} - e^{m_2}}\left[\frac{(1-\lambda m_1)e^{m_1}}{A_1} - \frac{(1-\lambda m_2)e^{m_2}}{A_2}\right] \\ & + \frac{G_c}{e^{m_3} - e^{m_4}}\left[\frac{(1-\lambda m_3)e^{m_3}}{A_3} - \frac{(1-\lambda m_4)e^{m_4}}{A_4}\right] \end{aligned} \tag{36}$$

$$E_1 = \frac{X_2 - X_1 e^{m_6}}{(e^{m_5} - e^{m_6})(1-\lambda m_5)} \tag{37}$$

$$E_2 = -\frac{X_2 - X_1 e^{m_5}}{(e^{m_5} - e^{m_6})(1-\lambda m_6)} \tag{38}$$

References

[1] J.C. Misra, A. Sinha, G.C. Shit, Theoretical analysis of blood flow through an arterial segment having multiple stenosis, J. Mech. Med. Biol. 8 (2008) 265–279.
[2] A. Sinha, J.C. Misra, MHD flow of blood through a dually stenosed artery: effects of viscosity variation, variable hematocrit and velocity-slip, Can. J. Chem. Eng. 92 (2014) 23–31.
[3] A. Sinha, J.C. Misra, Effect of induced magnetic field on magnetohydrodynamic stragnation point flow and heat transfer on a stretching sheet, ASME J. Heat Transfer 136 (2014) 112701:1–112701:11.
[4] J.C. Misra, A. Sinha, Effect of Hall current and heat radiation on flow of fluid through a porous medium subject to an external magnetic field, Spec. Topices Rev. Porous Media – Int. J. 4 (2) (2013) 147–158.
[5] J.C. Misra, A. Sinha, G.C. Shit, Flow of a biomagnetic viscoelastic fluid: application to estimation of blood flow in arteries during electromagnetic hyperthermia, a therapeutic procedure for cancer treatment, Appl. Math. Mach. 31 (2010) 1405–1420.
[6] S.O. Adesanya, M. Eslami, M. Mirzazadeh, A. Biswas, Shock wave development in couple stress fluid-filled thin elastic tubes, Eur. Phys. J. Plus. 130 (6) (2015) 1–21.
[7] S.O. Adesanya, J.A. Falade, O.D. Makinde, Pulsating flow through vertical porous channel with viscous dissipation effect, U.P.B. Sci. Bull. Ser. D 77 (1) (2015) 25–36.
[8] S.O. Adesanya, O.D. Makinde, Heat transfer to magnetohydrodynamic non-newtonian couple stress pulsatile flow between two parallel porous plates, Z. Naturforsch 67 (a) (2012) 647–656.
[9] L. Ai, K. Vafai, A Coupling model for macromolecular transport in a stenosed arterial wall, Int. J. Heat Mass Transfer 49 (2006) 1568–1591.
[10] S. Wada, T. Karino, Computational study on LDL transfer from flowing blood to arterial wall, in: Clinical Appl. of Comput. Mech. to the Cardiovascular System, Springer, 2000, pp. 157–173.
[11] A.R.A. Khaled, K. Vafai, The role of porous media in modeling flow and heat transfer in biological tissues, Int. J. Heat Mass Transfer 46 (2003) 4989–5003.
[12] R.K. Dash, K.N. Metha, G. Jayaraman, Casson fluid in a pipe filled with a homogeneous porous medium, Int. J. Eng. Sci. 34 (10) (1996) 1145–1156.
[13] G.S. Beavers, D.D. Joseph, Boundary conditions at a naturally permeably wall, J. Fluid Mech. 30 (1967) 197–207.
[14] G.S. Beavers, E.M. Sparrow, R.A. Magnuson, Experiments on coupled parallel flows in a channel and a bounding medium, ASME J. Basic Eng. 92 (1970) 843–848.
[15] G.S. Beavers, E.M. Sparrow, B.A. Masha, Boundary conditions at a porous surface which bounds a fluid flow, A.I.Ch.E.G. 20 (1974) 596–597.
[16] R.S. Rivlin, J.L. Ericksen, Stress-deformation relations for isotropic materials, J. Rat. Mech. Anal. 4 (1955) 323–425.
[17] R.K. Bhatnagar, Fluctuating flow of a viscoelastic fluid in a porous channel, J. Appl. Mech. 46 (1979) 21–25.
[18] O.D. Makinde, P.Y. Mhone, Heat transfer to MHD oscillatory flow in a channel filled with porous medium, Rom. J. Phys. 50 (2005) 931–938.

[19] P.C. Ram, Effect of hall current and wall temperature oscillation on convective flow in a rotating fluid through porous medium, Heat Mass Transfer 25 (1990) 205–208.
[20] A.R. Bastman, Low Reynolds number non-Newtonian flow in slowly varying axi-symmetric tubes, Acta Mech. 44 (1982) 107–119.
[21] J.C. Misra, B.K. Sahu, Unsteady flow of blood through arteries of non-uniform endothelial cross-section, Model. Simul. Control C 15 (1989) 31–64.
[22] J.C. Misra, B. Paul, A.S. Gupta, Hydromagnetic flow of a second-grade fluid in a channel – some applications to the physiological systems, Math. Models Methods Appl. Sci. 8 (8) (1998) 1323–1342.
[23] J.C. Misra, G.C. Shit, Flow and heat transfer of a MHD viscoelastic fluid in a channel with stretching walls: some applications to haemodynamics, Compt. Fluids 37 (2008) 1–11.
[24] J.C. Misra, G.C. Shit, Flow of a biomagnetic viscoelastic fluid in a channel with stretching walls, ASME J. Appl. Mech. 76 (2009) 061006:1–061006:9.
[25] J.C. Misra, S. Maiti, Peristaltic pumping of blood in small vessels of varying cross-section, ASME J. Appl. Mech. 79 (2012), 061003:1-19.
[26] A. Sinha, J.C. Misra, Mixed Convection Hydromagnetic flow with heat generation, thermophoresis and mass transfer over an inclined non-linear porous stretching sheet: a numerical approach, J. Mech. 30 (2014) 491–503.
[27] J.C. Misra, A. Sinha, Effect of thermal radiation on MHD flow of blood and heat transfer in a permeable capillary in stretching motion, Heat Mass Transfer 49 (5) (2013) 617–628.
[28] Z. Xu, N. Chen, S.C. Shadden, J.E. Marsden, M.M. Kamocka, E.D. Rosen, M. Alber, Study of blood flow impact on growth thrombi using a multiscade model, Soft Matter 5 (2009) 769–779.
[29] R.A. Coulson, T. Hernandez, Alligator metabolism studies on chemical reactions in vivo, Comp. Biochem. Physiol. 74 (1983) i–182.
[30] R.C. Chaudhary, P. Jain, Combined heat and mass transfer in magneto-micropolar fluid flow from radiate surface with variable permeability in slip-flow regime, Z. Angew. Math. Mech. (ZAMM) 87 (8–9) (2007) 549–563.
[31] U.N. Das, R.K. Deka, V.M. Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forsch. Ingenieurwesen (Eng. Res.) 60 (1994) 284–287.
[32] R. Muthucumarswamy, P. Ganesan, First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux, Acta Mech. 147 (2001) 45–57.
[33] R. Muthucumarswamy, Effects of a chemical reaction on moving isothermal vertical surface with suction, Acta Mech. 155 (2002) 5–70.
[34] R. Kandasamy, K. Periasamy, K.K.P. Sivagnana, Effects of chemical reaction, heat and mass transfer along wedge with heat source and concentration in the presence of suction or injection, Int. J. Heat Mass Transfer 48 (2005) 388–1394.
[35] R.C. Chaudhary, P. Jain, Effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime, Appl. Math. Mech. 29 (2000) 1179–1194.
[36] D.J. Singel, J.S. Stemler, Chemical physiology of flood flow regulation by red cells: the role of nitric oxide and S-Nitrosohemoglobin, Annu. Rev. Physiol. 67 (2005) 99–145.
[37] P. Brunn, The velocity slip of polar fluids, Rheol. Acta 14 (1975) 1039–1054.
[38] Y. Nubar, Blood flow, slip and viscometry, Biophys. J. 11 (1971) 252–264.
[39] C. Perdikis, A. Raptis, Heat transfer of a micropolar fluid by the presence of radiation, Heat Mass Transfer 31 (1996) 381–382.