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Equivalence relationship between the general combined-oriented CCR model and the weighted minimax MOLP formulation

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Abstract In many applications, finding target unit is required, particularly when decision maker (DM) wants to search along the effi\cient frontier to locate the most preferred solution. Wong et al. [Wong, Y.H., Luque, M., Yang, J.B., 2009. Using interactive multiple objective methods to solve DEA problem with value judgments. Computers and Operations Research 36, 623–636] established an equivalence model between output-oriented dual DEA models and multiple objective linear programming (MOLP). In a similar vein, the aim of this paper is to establish an equivalence model between the general combined-oriented CCR model and multiple objective linear programming and also using Zionts–Wallenius's method to integrate combined-oriented CCR performance assessment and target setting such that the DM's preference can be taken into account in an interactive fashion. Our proposed model gives the most preferred solution for DM with trade off analysis on both input and output values of DMUs. The applicability of the proposed equivalence model is illustrated, using a real data set as a case study, which consists of 20 bank branches.

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1. Introduction

Data envelopment analysis (DEA) is a mathematical programming technique for identifying efficient frontiers for peer decision making units (DMUs) with multiple inputs and multiple outputs. The units are assumed to operate under similar conditions. Based on information about existing data on the performance of the units and some preliminary assumptions, DEA forms an empirical efficient surface (frontier). If a DMU lies on the surface, it is referred to as an efficient unit, otherwise inefficient. DEA also provides efficiency scores and reference set for inefficient DMU. The efficiency scores are used in practical applications as performance indicators of the DMUs. The reference set for inefficient units consists of efficient units and determines a virtual unit on the efficient surface. The virtual unit can be regarded as a target unit for the inefficient unit. The target unit is found in DEA by projecting an inefficient DMU radially to the efficient surface that usually does not include a decision maker's (DM) preference structure or value judgments. Emrouznejad et al. (2010a,b) proposed a semi-oriented radial measure (SORM) which yield a measure of efficiency and target unit and can handle variables that take positive values for some and negative values for other DMUs. However, to incorporate DM's preference information in DEA, various techniques have been proposed such as the goal and target setting models of Athanassopoulis (1998) and Thompson et al. (1990) and weight restrictions models including imposing bounds on individual weights (Dyson and Thanassoulis, 1988), assurance region (Wierzbicki, 1980), restricting composite inputs and outputs, weight ratios and proportions (Zhu, 1996), and the cone ratio concept by adjusting the observed input–output levels or weights to capture value judgment to belong to a given closed cone (Charnes and Cooper, 1990; Charnes et al., 1994). However, all the above-mentioned techniques would require prior articulated preference knowledge from the DM, which in most cases can be subjective and difficult to obtain.

On the other hand, relationships between DEA and MOLP have been studied from several viewpoints by many authors. For instance, Golany (1988) first proposed an interactive model combining both of these approaches where the DM will allocate a set of level of inputs as resources and be able to select the most preferred set of level of outputs from alternative points on the efficient frontier. Belton (1992) and Belton and Vickers (1993) measured efficiency as a weighted sum of input and output. Thanassoulis and Allen (1998) showed the equivalence between the CCR model and some linear value function model for multiple outputs and multiple inputs. Shin and Ravindran (1991) combined the use of DEA and interactive multiple goal programming where preference information is incorporated interactively with the DM by adjusting the upper and lower feasible boundaries of the input and output levels. Then Joro et al. (1998) proved structural correspondences between DEA models and multiple objective linear programming using an achievement scalarizing function proposed by Wong and Beasley (1990). Further, the concept of value efficiency analysis (VEA) that effectively incorporate preference information in DEA introduced in Joro et al. (2003) and Korhonen et al. (2002). Zionts and Wallenius (1976) proposed a model that calculates efficiency scores incorporating the DM preference information. Halme et al. (1999) evaluated an efficiency of DMU in terms of pseudo-concave value function, by considering a tangent cone of the feasible set as the most preferred solution of the decision maker. Hosseinzadeh Lotfi et al. (2010a,b) obtained an equivalence relationship between min-ordering formulation in MOLP and DEA based on output-oriented CCR dual model. Also, Yang et al. (2009) established the equivalence relationship between the output-oriented DEA dual models and minimax reference point approach of MOLP, showing how a DEA problem can be solved interactively without any prior judgments by transforming it into an MOLP formulation. They applied interactive techniques in MOLP to solve DEA problems and further located the MPS along the efficient frontier for each DMU. They showed that the MPS generated using interactive MOLP methods provides a rich insight into the performance assessment and the efficiency analysis of each DMU with realistic and technically feasible target values that incorporate DM's value judgments. In a similar way, Wong et al. (2009) established equivalence relationship between the output-oriented DEA dual models and the minimax formulations that led to the construction of the three equivalence models namely the super-ideal point model, the ideal point model and the shortest distance model. In a similar vein, the aim of this paper is to establish an equivalence model between the general combined-oriented CCR model and multiple objective linear programming and also using Zionts–Wallenius's method (1976) to integrate the general combined-oriented CCR performance assessment and target setting such that the DM's preference can be taken into account in an interactive fashion.

This paper is organized as follows: Section 2 reviewed some existing approaches on relationships between data envelopment analysis and multiple objective linear programming (MOLP) as well as a brief description of the DEA technique and MOLP structure. Section 3 gives the equivalence relation between the general combined-oriented CCR model and MOLP as well as an interactive multiobjective programming method namely Zionts–Wallenius method. An application on the performance measurement of a bank in IRAN is examined in Section 4. Finally, we conclude in Section 5.

2. Literature review

In this section, we review some existing approaches on relationships between data envelopment analysis and multiple objective linear programming. Before that, we give a brief description of the DEA technique and MOLP structure.

DEA, suggested by Charnes et al. (1978), is a nonparametric frontier estimation methodology based on linear programming to measure the relative efficiency of a decision making unit and provide DMUs with relative performance assessment on multiple inputs and outputs. It also provides reference units known as composite or virtual units which lie on the efficient frontier and are used as target units for inefficient DMUs to benchmark against.

We assume that there are n DMUs to be evaluated, indexed by $j = 1, 2, \dots, n$ and each DMU is assumed to produce s different outputs from m different inputs. Let the observed input and output vectors of DMU_j be $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$, respectively, that all components of vectors X_j and Y_j for all DMUs are non-negative and each DMU has at least one strictly positive input and output. The following linear programming model is the output-oriented CCR model for efficiency analysis:

$$\begin{aligned} \max \quad & \beta_p \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ip}, \quad i = 1, 2, \dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant \beta_p y_{rp}, \quad r = 1, 2, \dots, s \\ & \lambda_j \geqslant 0, \quad j = 1, 2, \dots, n \end{aligned} \tag{1}$$

In this model, the inverse of β_p is the efficiency score of DMU_p. If $\beta_p > 1$, DMU_p is not efficient and the parameter β_p indicates the extent by which DMU_p has to increase its outputs to become efficient. For an inefficient DMU_p, we define its reference set as $E_p = \{\lambda_j^* | \lambda_j^* > 0,\ j = 1, 2, \ldots, n\}$ where $\lambda^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_n^*)$ is the optimal solution of (1). In this case, the point $\left(\sum_{j \in E_p} \lambda_j^* x_j \sum_{j \in E_p} \lambda_j^* y_i\right)$ on the efficient frontier is used to evaluate the performance of DMU_p and can be regarded as a target unit for the inefficient unit DMU_p. This target unit usually does not include a decision maker's preference structure or value judgments. So, it needs to use an interactive method in MOLP.

A multiobjective linear programming problem is to optimize a vector of linear functions in the presence of linear constraints and can be formulated as follows:

$$\begin{aligned} &\max\ f(x) = [f_x(x), \ldots, f_r(x), \ldots, f_x(\lambda)] \\ &s.t. \quad x \in X \end{aligned} \tag{2}$$

Involving $s\ (\geqslant 2)$ conflicting objective functions $f_r : X \to R$ that we want to maximize simultaneously. The decision variables $x = (x_1, x_2, \ldots, x_n)^T$ belong to the nonempty feasible region $X = \{x : Ax = b, x \geqslant 0, x \in R^m\}$, where A is a $m \times n$ matrix. Objective vectors in objective space R^m consist of objective values $f(x) = (f_1(x), f_2(x), \ldots, f_s(x))^T$ and the image of the feasible region is called a feasible objective region $Z = f(X)$.

In MOLP problem, there does not necessarily exist solution that optimizes all objective functions, like in single-objective linear programming, and then the task of MOLP is to find nondominated solutions and to help select a most preferred one. In fact, a solution represented by a point in decision variable space, is a nondominated solution if it is not possible to move the point within the feasible region to improve an objective function value without deteriorating at least one of the other objectives. In multiple criteria terminology, a nondominated solution is also called an efficient solution. Thus, to prevent possible confusion with DEA's efficiency concept, we will use the term "nondominated".

Hosseinzadeh Lotfi et al. (2010a) assumed that the feasible space of MOLP (2) be as follows:

$$X = \left\{ \lambda_j \middle| \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ip}\ (i = 1, 2, \ldots, m),\ \lambda_j \geqslant 0\ (j = 1, 2, \ldots, n) \right\}$$

Then, they wrote the MOLP formulation (2) in a weighted min-ordering approach as:

$$\begin{aligned} &\max \min_{1 \leqslant r \leqslant s} \{f_r(\lambda)\} \\ &s.t. \quad \lambda \in X \end{aligned} \tag{3}$$

The min-ordering formulation (3) can then be written as follows by introducing an auxiliary variable θ:

$$\begin{aligned} &\max\ \theta \\ &s.t. \quad \theta \leqslant f_r(\lambda), \quad r = 1, 2, \ldots, s \\ &\qquad\ \lambda \in X \end{aligned} \tag{4}$$

By the assumption $y_{rp} > 0$ ($r = 1, 2, \ldots, s$), they established the relationship between the output-oriented CCR model (1) and the min-ordering formulation (4). Finally, they proved that efficiency score of the inefficient DMU_p and its target unit can be generated by solving the following formulation and hence used an interactive MOLP method to solve the DEA problem:

$$\begin{aligned} &\max \left[\frac{1}{y_{1j}} \sum_{j=1}^{n} \lambda_j y_{1j}, \ldots, \frac{1}{y_{rj}} \sum_{j=1}^{n} \lambda_j y_{rj}, \ldots, \frac{1}{y_{sj}} \sum_{j=1}^{n} \lambda_j y_{sj} \right] \\ &s.t. \quad x \in X \end{aligned} \tag{5}$$

In a similar manner, Yang et al. (2009) (see also Wong et al., 2009) stated the MOLP formulation (2) in a weighted minimax approach as follows with f^* as the ideal point:

$$\begin{aligned} &\min \max_{1 \leqslant r \leqslant s} \{w_r(f_r^* - f_r(\lambda))\} \\ &s.t. \quad \lambda \in X \end{aligned} \tag{6}$$

The weighted minimax MOLP formulation can then be written as follows by introducing an auxiliary variable θ:

$$\begin{aligned} &\min\ \theta \\ &s.t. \quad \theta \geqslant w_r(f_r^* - f_r(\lambda)), \quad r = 1, 2, \ldots, s \\ &\qquad\ \lambda \in X \end{aligned} \tag{7}$$

By the assumption $y_{rp} > 0$ ($r = 1, 2, \ldots, s$), they established the relationship between the output-oriented CCR model (1) and the weighted minimax formulation (7). Finally, they proved that efficiency score of the inefficient DMU_p and its target unit can be generated by solving the following formulation and hence used some interesting interactive methods in MOLP to solve the DEA problem:

$$\begin{aligned} &\max \left[\sum_{j=1}^{n} \lambda_j y_{1j}, \ldots, \sum_{j=1}^{n} \lambda_j y_{rj}, \ldots, \sum_{j=1}^{n} \lambda_j y_{sj} \right] \\ &s.t. \quad x \in X \end{aligned} \tag{8}$$

These models only consider the output-oriented dual DEA model, which is a radial model that focuses more on output increase. In this paper, we extend the model of Yang et al. (2009) such that it can be considered both the decrease in total input consumption and the increase in total output production based on a radial method.

3. An improved equivalence model

In this section, we extend the approach proposed by Yang et al. (2009) to establish an equivalence model between the general combined-oriented CCR model and the weighted minimax MOLP formulation. In fact, the proposed equivalence model by Yang et al. (2009) is a special case of our proposed equivalence model. Also, Yang et al. (2009) assumed that all of components of output vectors for all DMUs are positive. This assumption does not hold in many applications, while such assumption is not necessary in our model and so our proposed equivalence model is more practical.

3.1. The general combined-oriented CCR model

Based on different empirical axioms and corresponding to different characteristics of the production possibility set and production frontiers, different DEA models, are developed and applied in practice. In this paper, without loss of generality, we will consider a DEA model by using a general directional vector $d = (d_x, d_y)$ (for a discussion on directional distance functions, see (Chambers et al., 1998). Halme et al. (1999) called the model a general combined model. Following Charnes et al. (1978), we assume that the technology set is estimated by:

$$T_c = \{(X, Y) | X \geqslant \lambda X,\ Y \leqslant \lambda Y,\ \lambda \geqslant 0\}$$

To obtain the general combined model, we define the combined efficiency measure as $\max\{\theta_p | (X_p - \theta_p d_x, Y_p + \theta_p d_y) \in T_c\}$, that lead to the following problem:

$$\begin{aligned} & \max\ \theta_p \\ & s.t. \quad (X_p - \theta_p d_x, Y_p + \theta_p d_y) \in T_c \end{aligned} \tag{9}$$

By considering T_c, we have the following linear programming, known as the general combined-oriented CCR model:

$$\begin{aligned} & \max\ \theta_p \\ & s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{ip} - \theta_p d_{ix}, \quad i = 1, 2, \ldots, m \\ & \qquad \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant y_{rp} + \theta_p d_{ry}, \quad r = 1, 2, \ldots, s \\ & \qquad \lambda_j \geqslant 0, \quad j = 1, 2, \ldots, n \end{aligned} \tag{10}$$

where the directional vector $d = (d_x, d_y) = (d_{1x}, \ldots, d_{mx}, d_{1y}, \ldots, d_{sy})$ shows the direction that DMU_p can move to lie on efficient frontier. In fact, DMU_p lies on efficient frontier by at most decreasing in its inputs and at most increasing in its outputs.

In the combined-oriented CCR model (10), λ_j represents the proportion to which DMU_j is used to construct the composite unit for DMU_p ($j = 1, 2, \ldots, n$). Also, the composite unit produces inputs that are at most equal $(X_p - \theta_p d_x)$ and consumes outputs at least equal to $(Y_p + \theta_p d_y)$ with $0 < \theta_p \leqslant 1$, where the $1 - \theta_p$ is the efficiency score of DMU_p. If $0 < \theta_p < 1$, DMU_p is inefficient and the parameter $\theta_p d_x$ indicates the extent by which DMU_p has to decrease its inputs and the parameter $\theta_p d_y$ indicates the extent by which DMU_p has to increase outputs to become efficient.

3.2. *Conducting combined-oriented CCR model performance assessment using an MOLP method*

In the combined-oriented CCR model (10), an efficiency score is generated for a DMU by maximizing outputs and minimizing inputs simultaneously. Either way, this can be regarded as a kind of multiple objective optimization problem. In this subsection, the theoretical considerations of combining MOLP and the combined-oriented CCR model are presented.

Suppose an optimization problem has $m + s$ objectives reflecting the different purposes and desires of the DM. Such a problem can be represented in a general form as follows:

$$\begin{aligned} & \max f(\lambda) = [g_1(\lambda), \ldots, g_i(\lambda), \ldots, g_m(\lambda), h_1(\lambda), \ldots, h_r(\lambda) \ldots, h_s(\lambda)] \\ & s.t. \quad \lambda \in \Lambda \end{aligned} \tag{11}$$

where Λ is the feasible decision space, $g_i(\lambda)$ $(i = 1, 2, \ldots, m)$ and $h_r(\lambda)$ $(r = 1, 2, \ldots, s)$ are continuously differentiable objective functions.

Definition 3.1. A feasible solution $\lambda^* \in \Lambda$ is called efficient solution or nondominated solution to (11), if there is no other $\lambda \in \Lambda$ such that $f(\lambda) \geqslant f(\lambda^*)$and $f(\lambda) \neq f(\lambda^*)$.

Definition 3.2. The point $f^* = (g_1^*, \ldots, g_i^*, \ldots, g_m^*, h_1^*, \ldots, h_r^*, \ldots, h_s^*)$ given by $g_1^* = \max_{\lambda \in \Lambda} g_i(\lambda)$ $(i = 1, 2, \ldots, m)$ and $h_r^* = \max_{\lambda \in \Lambda} h_r(\lambda)$ $(r = 1, 2, \ldots, s)$ is called the ideal point of (11).

We recall that in a MOLP problem, it is generally impossible to find a solution that optimizes all objectives simultaneously. Therefore, in order to reach to a special nondominated extreme point, the MOLP formulation (11) can be written in minimax approach (Stewart, 1996; Wong et al., 2009) as follows:

$$\begin{aligned} & \min \max_{1 \leqslant i \leqslant m, 1 \leqslant r \leqslant s} \{w_i(g_i^* - g_i(\lambda)), w_r(h_r^* - h_r(\lambda))\} \\ & s.t. \quad \lambda \in \Lambda \end{aligned} \tag{12}$$

In the above minimax formulation, for a given weight vector the DM is assumed to be satisfied with an efficient solution $\lambda \in \Lambda$ at which $f(\lambda)$ has the shortest weighted distance from $f^* = (g^*, h^*)$ measured in ∞-norm in the objective space. The minimax formulation (12) can then be written as follows by introducing an auxiliary variable θ:

$$\begin{aligned} & \min\ \theta \\ & s.t. \quad \theta \geqslant w_i(g_i^* - g_i(\lambda)), \quad i = 1, 2, \ldots, m \\ & \qquad \theta \geqslant w_r(h_r^* - h_r(\lambda), \quad r = 1, 2, \ldots, s \\ & \qquad \lambda \in \Lambda \end{aligned} \tag{13}$$

Let

$$G_i(\lambda) = x_{ip} - \sum_{j=1}^{n} \lambda_j x_{ij}, \quad h_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} - y_{rp} \tag{14}$$

Therefore, the combined-oriented CCR model, as shown in formulation (10), can be equivalently rewritten as follows:

$$\begin{aligned} & \max\ \theta \\ & s.t. \quad \theta_p d_{ix} - g_i(\lambda) \leqslant 0, \quad i = 1, 2, \ldots, m \\ & \qquad \theta_p d_{ry} - h_r(\lambda) \leqslant 0, \quad i = 1, 2, \ldots, s \\ & \qquad \lambda \in \Lambda_p \end{aligned} \tag{15}$$

where $\Lambda_p = \{\lambda = (\lambda_1, \ldots, \lambda_n) | \lambda_j \geqslant 0,\ j = 1, 2, \ldots, n\}$.

The reason for establishing the equivalence condition between the combined-oriented CCR model (15) and the minimax formulation (13) is to use the interactive methods in MOLP to locate the most preference solution (MPS) on the efficient frontier for target setting and resource allocation.

Suppose $\bar{g}_{ip}(\lambda) = g_i(\lambda^i)$, in which λ^i is the optimal solution of the following problem:

$$\max_{\lambda \in \Lambda} g_i(\lambda) = x_{ip} - \sum_{j=1}^{n} \lambda_j x_{ij} \tag{16}$$

Also, suppose $\bar{h}_{rp}(\lambda) = h_r(\lambda^r)$ in which λ^r is the optimal solution of the following problem:

$$\max_{\lambda \in \Lambda} h_r(\lambda) = \sum_{j=1}^{n} \lambda_j y_{rj} - y_{rp} \tag{17}$$

Theorem 3.1. *Suppose $d_{ix} > 0$ $(i = 1, 2, \ldots, m)$ and $d_{ry} > 0$ $(r = 1, 2, \ldots, s)$. The combined-oriented CCR model (15) can be equivalently transformed to the minimax formulation (13) using formulations (14), (16) and (17) and the following equations*:

$$w_i = \frac{1}{d_{ix}} \quad (i = 1, 2, \ldots, m), \quad w_r = \frac{1}{d_{ry}} \quad (r = 1, 2, \ldots, s) \tag{18}$$

$$\begin{aligned} & g_i^* = \frac{u^{max}}{w_i} = u^{max} d_{ix} \quad (i = 1, 2, \ldots, m), \\ & h_r^* = \frac{u^{max}}{w_r} = u^{max} d_{ry} \quad (r = 1, 2, \ldots, s) \end{aligned} \tag{19}$$

$$\theta = u^{max} - \theta_p, \quad \Lambda = \Lambda_p \tag{20}$$

$$U^{max} = \max_{1\leqslant i\leqslant m, 1\leqslant r\leqslant s}\{w_i\bar{g}_{ip}, w_r\bar{h}_{rp}\} = \max_{1\leqslant i\leqslant m, 1\leqslant r\leqslant s}\left\{\frac{\bar{g}_{ip}}{d_{ix}}, \frac{\bar{h}_{rp}}{d_{ry}}\right\} \tag{21}$$

Proof. Using (18), the combined-oriented CCR model (15) can be rewritten as follows:

$$\begin{aligned} &\max\ \theta \\ &s.t.\quad \theta_p\frac{1}{w_i} - g_i(\lambda) \leqslant 0, \quad i = 1,2,\ldots,m \\ &\qquad\ \ \theta_p\frac{1}{w_r} - h_r(\lambda) \leqslant 0, \quad i = 1,2,\ldots,s \\ &\qquad\ \ \lambda \in \Lambda_p \end{aligned} \tag{22}$$

The first m constraints in (22) can be equivalently transformed as follows:

$$\begin{aligned} \theta_p\frac{1}{w_i} - g_i(\lambda) \leqslant 0 &\iff -g_i(\lambda)w_i \leqslant -\theta_p \\ &\iff u^{max} - g_i(\lambda)w_i \leqslant u^{max} - \theta_p \\ &\iff w_i\left(\frac{u^{max}}{w_i} - g_i(\lambda)\right) \leqslant \theta \\ &\iff w_i(g_i^* - g_i(\lambda) \leqslant \theta \end{aligned} \tag{23}$$

Similar way the second s constraints in (22) can be equivalently transformed as follows:

$$w_r(h_r^* - h_r(\lambda)) \leqslant \theta \tag{24}$$

Moreover, the objective function of (22) becomes:

$$\max(\theta_p) = -\min(-\theta_p) = -\min(u^{max} - \theta_p) = -\min(\theta) \tag{25}$$

Also, for any $\lambda \in \Lambda$, we have:

$$\theta = u^{max} - \theta_p \geqslant w_i\bar{g}_{ip} - \theta_p \geqslant 0, \quad i = 1,2,\ldots,m \tag{26}$$

$$\theta = u^{max} - \theta_p \geqslant w_r\bar{h}_{rp} - \theta_p \geqslant 0, \quad r = 1,2,\ldots,s \tag{27}$$

$$g_i^* = \frac{u^{max}}{w_i} = u^{max}d_{ix} \geqslant \frac{w_i\bar{g}_{ip}}{w_i} = \bar{g}_{ip} = \max_{\lambda\in\Lambda} g_i(\lambda), \quad i = 1,2,\ldots,m \tag{28}$$

$$h_i^* = \frac{u^{max}}{w_r} = u^{max}d_{ry} \geqslant \frac{w_i\bar{h}_{rp}}{w_r} = \bar{h}_{rp} = \max_{\lambda\in\Lambda} h_r(\lambda), \quad r = 1,2,\ldots,s \tag{29}$$

The equivalence model between the combined-oriented CCR model (15) and the minimax formulation of (13) is established, since (23)–(25) hold. □

From Theorem 3.1 the general combined-oriented CCR model can be equivalently rewritten as a minimax formulation of (13) as follows:

$$\begin{aligned} &\min\ \theta \\ &s.t.\quad \theta \geqslant w_i(g_i^* - g_i(\lambda)), \quad i = 1,2,\ldots,m \\ &\qquad\ \ \theta \geqslant w_r(h_r^* - h_r(\lambda)), \quad r = 1,2,\ldots,s \\ &\qquad\ \ \lambda_j \geqslant 0, \quad j = 1,2,\ldots,n \end{aligned} \tag{30}$$

In fact, the above theorem shows that the general combined-oriented CCR model is actually constructed to locate a specific efficient solution, termed as DEA efficient solution on the efficient frontier of the following generic MOLP formulation for the observed DMU_p:

$$\begin{aligned} &\max\left[x_{1p} - \sum_{j=1}^{n}\lambda_j x_{1j}, \ldots, x_{mp} - \sum_{j=1}^{n}\lambda_j x_{mj}, \sum_{j=1}^{n}\lambda_j y_{1j} - y_{1p}, \ldots, \sum_{j=1}^{n}\lambda_j y_{sj} - y_{sp}\right] \\ &s.t.\quad \lambda \in \Lambda \end{aligned} \tag{31}$$

The generic MOLP problem (31) defines the production possibility set for the observed DMU_p in which there may be more preferred efficient solutions than the DEA efficient solution.

We note that formulation (30) is equivalent to formulation (15) if w_i $(i = 1,2,\ldots,m)$ and w_r $(r = 1,2,\ldots,s)$ are calculated using (18), g_i^* $(i = 1,2,\ldots,m)$ in formulation (30) is calculated using $g_i^* = u^{max}d_{ix}$ and h_r^* $(r = 1,2,\ldots,s)$ is calculated using $h_r^* = u^{max}d_{ry}$. Likewise, formulation (31) is equivalent to formulation (15). So, the efficiency score of DMU_p can be generated by solving formulation (31). Therefore, an interactive MOLP method can be used to solve the DEA problem.

It needs to point out that in the model proposed by Yang et al. (2009) the MPS obtained with trade off only on output values of DMUs. But in our proposed model the MPS obtained with trade off on both input and output values of DMUs.

3.3. Method of Zionts–Wallenius

It is an issue how decision makers decide one from the set of nondominated solutions as the final solution. Consequently, interactive MOLP methods have been developed to this end. In fact, Interactive multiple objective programming methods constitute techniques that allow the DM to search for different solutions along the efficient frontier, so that the DM can reach his most preferred solution (MPS). At each stage, the current solution is adapted to the structure of preferences of the DM. It can be said that an interactive method is designed to drive the DM towards his MPS, in the sense that it is acceptable by the DM.

In this paper, we use the Zionts–Wallenius's method to integrate combined-oriented CCR performance assessment and target setting such that the DMs preference can be taken into account in an interactive fashion. This method is applicable to the problem in (31) where the objective functions are concave and the feasible space is a convex set. The overall utility function is assumed to be unknown explicitly to the DM, but is implicitly a linear function and more generally a concave function of the objective functions. The method makes use of such an implicit function on an interactive basis. The first step of the method is to choose an arbitrary set of positive multipliers or weights and generate a composite objective function or utility function using these multipliers. The composite objective function is then optimized to produce a nondominated solution to the problem. From the set of nonbasic variables, a subset of efficient variables is selected (an efficient variable is one which, when introduced into the basis, cannot increase one objective without decreasing at least one other objective). For each efficient variable a set of tradeoffs is defined by which some objectives are increased and others reduced. A number of such tradeoffs are presented to the DM, who is requested to state whether the tradeoffs are desirable, undesirable or neither. From his/her answers a new set of consistent multipliers is constructed and the associated nondominated solution is found. The process is then repeated, and a new set of tradeoffs is presented to the DM at the current solution, convergence to an overall optimal solution with respect to the DM's implicit utility function is assured.

4. An example with real world data

We now apply this approach to some commercial bank branches in Iran. There are 20 branches in this district. Each branch uses three inputs to produce five outputs. The three inputs are namely payable interest, personnel and non-performing loans, while the five outputs are namely the total sum of four main deposits, other deposits, loans granted, received interest and fee. Table 1 shows the kind of these inputs and outputs. Also the data set for 20 branches of that bank is given in Table 2 (taken from Hosseinzadeh Lotfi et al. (2010a)).

The result of the general combined-oriented CCR model is shown in Table 3. In fact, the general combined-oriented CCR model is run to find the respective efficiency scores, reference set of inefficient branches and the proportion to which efficient branch is used to construct the composite unit for inefficient branches. As shown in Table 3, branches 1, 4, 6, 7, 8, 9, 10, 11, 17 and 19 are efficient branches and branches 2, 3, 5, 12, 13, 14, 15, 16, 18 and 20 are inefficient branches of the bank. For example, branch 12 has an efficient score 0.59 implying that it is operating as an inefficient branch and also its composite unit on the efficient frontier can be represented as a linear combination of 0.29 of branch 4, 0.22 of branch 6, 0.06 of branch 8, 0.04 of branch 10 and 0.06 of branch 11. In fact, the composite unit of branch 12 is given as follows:

(I1, I2, I3) = (4427.9529, 13.6061, 9578.78).
(O1, O2, O3, O4, O5) = (640,527.71, 114,656.46, 496,061.96, 52,111.5666, 511.8362).

This means the first input (payable interest) should be reduced to 4427.9529, the second input (personnel) should be reduced to 13.6061 and the third input (non-performing loans) should be reduced from 9945 to 9578.78 for branch 12 to become efficient. Also, the outputs O1, O2, O3, O4 and O5 should be increased to 640,527.71, 114,656.46, 496,061.96, 52,111.5666 and 511.8362, respectively. But, the DM has not accepted the DEA composite input and output values as the MPS for branch 12. So, it is needed to search MPS along the frontier for branch 12 using interactive MOLP method. The first iteration of the interactive Z–W method gives an unit as a linear combination of 0.27 of branch 10, 0.49 of branch 11 and 0.32 of branch 17 as follows:

(I1, I2, I3) = (7277.3079, 22.8414, 16,004.46).
(O1, O2, O3, O4, O5) = (701,444.14, 137,991.73, 4,316,364.28, 198,096.0937, 1430.594).

The DM is still not satisfied with the solution obtained by the first iteration. In iteration 2, the solution is as a linear combination of 0.04 of branch 10 and 0.67 of branch 11 with the following input and output values:

(I1, I2, I3) = (2338.8402, 14.275, 897.2).
(O1, O2, O3, O4, O5) = (497,597.82, 77,077.73, 1,150,203.67, 38,252.487, 719.7983).

Now, the DM is completely satisfied with the above input and output values. This means the MPS has been found and hence the interactive process terminate.

Table 1 Inputs and outputs.

	Inputs	Outputs
1	Payable interest	The total some of four main deposits
2	Personnel	Other deposits
3	Non-performing loans	Loans granted
4		Received interest
5		Fee

Table 2 Input-data and output-data for 20 branches of bank.

DMU	I1	I2	I3	O1	O2	O3	O4	O5
1	5007.37	36.29	87,243	2,696,995	263,643	1,675,519	108,634.76	965.97
2	2926.81	18.8	9945	340,377	95,978	377,309	32,396.65	304.67
3	8732.7	25.74	47,575	1,027,546	37,911	1,233,548	96,842.33	2285.03
4	945.93	20.81	19,292	1,145,235	229,646	468,520	32,362.8	207.98
5	8487.07	14.16	3428	390,902	4929	129,751	12,662.71	63.32
6	13,759.35	19.46	13,929	988,115	74,133	507,502	53,591.3	480.16
7	587.69	27.29	27,827	144,906	180,530	288,513	40,507.97	176.58
8	4646.39	24.52	9070	408,163	405,396	1,044,221	56,260.09	4654.71
9	1554.29	20.47	412,036	335,070	337,971	1,584,722	176,436.81	560.26
10	17,528.31	14.84	8638	700,842	14,378	2,290,745	662,725.21	58.89
11	2444.34	20.42	500	641,680	114,183	1,579,961	17,527.58	1070.81
12	7303.27	22.87	16,148	453,170	27,196	245,726	35,757.83	375.07
13	9852.15	18.47	17,163	553,167	21,298	425,886	45,652.24	438.43
14	4540.75	22.83	17,919	309,670	20,168	124,188	8143.79	936.62
15	3039.58	39.32	51,582	286,149	149,183	787,959	106,798.63	1203.79
16	6585.81	25.57	20,975	321,435	66,169	360,880	89,971.47	200.36
17	4209.18	27.59	41,960	618,105	244,250	9,136,507	33,036.79	2781.24
18	1015.52	13.63	18,641	248,125	3063	26,687	9525.6	240.04
19	5800.38	27.12	19,500	640,890	490,508	2,946,797	66,097.16	961.56
20	1445.65	28.96	31,700	119,948	14,943	297,674	21,991.53	282.73

Table 3 Efficiency scores and observed DMUs composite unit.

DMU	$1-\theta$	1	4	6	7	8	9	10	11	17	19
1	1	1									
2	0.65		0.25			0.14		0.04	0.14		
3	0.93	0.32				0.45		0.07		0.02	
4	1		1								
5	0.85							0.32	0.36		
6	1			1							
7	1				1						
8	1					1					
9	1						1				
10	1							1			
11	1								1		
12	0.59		0.29	0.22		0.06		0.04	0.06		
13	0.74	0.05	0.07	0.41		0.06		0.06			
14	0.54	0.04	0.15	0.06		0.27					
15	0.97				1.13	0.22	0.04	0.07			
16	0.54		0.27		0.15	0.04		0.17			
17	1									1	
18	0.56		0.29			0.06	0.01				
19	1										1
20	0.57		0.05		0.49	0.05	0.01	0.01		0.02	

Also, the efficiency score of branch 20 is 0.57 implying that it is operating as an inefficient branch too. The general combined-oriented composite unit for inefficient branch 20 is as a linear combination of 0.05 of branch 4, 0.49 of branch 7, 0.05 of branch 8, 0.01 of branch 9, 0.01 of branch 10 and 0.02 of branch 17 with the following input and output values:

(I1, I2, I3) = (842.5937, 16.5435, 20099.27).
(O1, O2, O3, O4, O5) = (171,395.06, 128,620.29, 438,493.23, 33,332.4058, 391.475).

This solution does not satisfy DM and then the interactive Z–W method is used to search MPS. Initially, this method generates a target unit as "0.17 branch 4 + 0.04 branch 9 + 0.29 branch 17" with I1 = 1443.6429; I2 = 12.3576, I3 = 31,929.48, O1 = 238,462.65, O2 = 123,391.16, O3 = 279,262.31, O4 = 22,139.8175 and O5 = 864.3266 that DM does not accept this unit as the MPS for branch 20. The second iteration gives an unit as "0.01 branch 4 + 0.48 branch 7 + 0.07 branch 17" with the input and output values as follows:

(I1, I2, I3) = (586.1931, 15.2386, 16433.45).
(O1, O2, O3, O4, O5) = (124,274.58, 106,035.4, 782,726.93, 22,080.0289, 281.525).

Now, DM completely accepted this solution as the MPS for branch 20 and hence the interactive method terminate.

As we show, the general combined-oriented CCR efficiency results generated do not consider the value judgments of the DM. Hence, interactive MOLP methods search the MPS along the efficient frontier for inefficient branches.

5. Conclusion

In this paper, we obtained an equivalence relation between the general combined-oriented CCR model and the weighted minimax MOLP model and showed how a DEA problem can be solved interactively by transforming it into MOLP formulation. This approach results in a decrease in total input consumption and a permissible increase in total output production instead of only an increase in outputs. Also, in our model it was not necessary that all of components of output vectors for all DMUs be positive. The proposed equivalence model provided the basis to apply interactive methods in MOLP to solve DEA problems and further locate the MPS along the efficient frontier for each inefficient DMU. We used Zionts–Wallenius method to reflecting the DM preferences in the process of assessing efficiency. An application example illustrated how the equivalence model and the interactive procedure can be implemented to support integrated efficiency analysis and target setting. We emphasize that for the sake of illustrating the performance of the our approach, it has been developed using the Z–W method, but there is no restriction at all to use any interactive MOLP method. In fact, comparisons of the results among the several interactive MOLP methods can be made on which method best may fit the data set and the DM's preferences. This will be an interesting research work in the future.

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