Influence of the cylindrical source size on impact probability caused by fragments

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Abstract

The movement equations of fragment with the size of the bursting vessel were developed. The ground distributions of fragments, and the probability of impact between the fragments and the target were investigated in consideration of cylindrical source size using Monte-Carlo simulations. The results showed that the distributions of the fragments from the lower half of the source vessels on the ground were larger than 0, it was simply probable that the fragments would hit the target around the source. The relative deviation of impact probability would be larger than 10% when the target vessel was located in the distance less than 8 times of the cylindrical source diameter. The proportion of impact probability of lower part of the source to the total impact probability decreased with the distance, but that of upper part increased. The proportion of upper and lower parts would be equal if the distance was about 5 times of the source diameter. The source size should be considered with the distance from the source to the target less than 14 times of the source diameter, and its effect on the impact probability was significant.

1. Introduction

In chemical industries, an explosion of some equipment may generate lots of fragments which can be projected over long distances, damage other sites fixed nearby, and cause more severe consequences. This is the domino effect,
a well-known cause of major accidents [1–3]. Moreover, Fragment projection in an explosive accident is one important origin of the domino effect on chemical process vessel [4].

2. Analysis of previous work

Each cycle of the overall domino effect caused by fragments includes three detailed steps: the source term, the fragment trajectory term, and the target term. Some research on the three components described above has been performed in previous work [3,5–28]. In recent work[3,18–21], the corresponding probabilistic distributions of the source terms were developed, the trajectory equations of the fragments were proposed, and the ground distributions of the fragments were evaluated. Afterwards, the probabilistic models of fragment impact were developed in the target term, a calculation of the impact probability was implemented, and its effects on the impact probability were evaluated. Then, a simplified plastic model for assessing the rupture probability with high reliability was proposed, and its influence on penetration depth was investigated. On the basis of these findings [3,18–21], in the work implemented by Sun et al. [27], more specific and accurate probabilistic models of the number of fragments from a horizontal cylindrical vessel explosion were defined by collecting and analyzing data from past accidents leading to fragment projection, and a more reasonable probability density function for the number of fragments from a spherical vessel explosion was recommended. The effects of the algorithms (movement approach, fragment rotation, wind, and number of simulation runs) on the fragment trajectory and target terms (the ground distributions of fragments, the probability of impact between the fragments and the target, and the rupture probability of the impacted target) and the influence of the calculation parameters (the objective volume, the degree of filling of the source vessel, and the kind of explosion) on the target term (the probability of fragment impact and the rupture probability of the target) were explored using Monte Carlo simulations. Besides, in Qian, Xu, & Liu’s task [23], for the factory being used or not meeting the required safety protection distance, the barrier net was put between the accident source and the objective vessel to make up for the lack of actual distance, and the method of heading off the fragments from a vessel explosion was put forward based on the analysis of the fragment trajectory and the probability of fragment impact using Monte Carlo simulations.

However, in the analysis described above [3,18–21,23,27], all the sources were defined as points, and their sizes were neglected (e.g. both the initial height and displacement of fragments were considered as 0). Actually, the part of the source from which the fragments (e.g. the initial height and displacement of fragments) were generated was random. In view of the previous research, the industrial site installed in Shanghai Petrochemical Company Limited (vertical cylindrical vessels) was taken as an example, the fragment trajectory and target terms (the ground distributions of fragments, the probability of impact between the fragments and the target, and the rupture probability of the impacted target) were investigated in consideration of different source sizes (different sizes of vertical cylindrical vessels) using Monte-Carlo simulations, and the obtained results were compared with those without the source sizes. Then, the contribution characteristics of the domino effect risk initiated by fragments respectively from the upper and lower part of the sources were revealed.

3. Fragment trajectory and target terms

3.1. Movement approach of fragment

The equations of fragment trajectory were summarized and are shown in Table 1 [3,15–16,18,22], where $x$, $y$, and $z$ are the center coordinates of the fragment; $k$ is the drag factor, $t$ is the flight time; $g$ is the acceleration due to gravity.

When the size of the source vessel is considered for the sources of the vertical cylindrical vessels (see Fig. 1), the solutions of the equations in Table 1 can be obtained as follows:
Table 1. Equations of fragment trajectory.

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movement equation</td>
<td>( \frac{d^2x}{dt^2} + k\left( \frac{dx}{dt} \right)^2 = 0 )</td>
<td>( \frac{d^2y}{dt^2} + k\left( \frac{dy}{dt} \right)^2 = 0 )</td>
<td>In the ascending part ( \frac{d^2z}{dt^2} + k\left( \frac{dz}{dt} \right)^2 + g = 0 )</td>
</tr>
<tr>
<td>In the descending part</td>
<td>( \frac{d^2z}{dt^2} - k\left( \frac{dz}{dt} \right)^2 + g = 0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the upper half of the source vessel \((0^\circ < \varphi \leq 90^\circ)\)

**x-direction**

\[
v_x = v \cos \varphi \cos \theta
\]

\[
x(t) = \begin{cases} \frac{1}{k} \ln(1 + kv_x) + x_0, v_x \geq 0 \\ -\frac{1}{k} \ln(1 - kv_x) + x_0, v_x < 0 \end{cases}
\]

**z-direction**

\[
v_z = v \cos \varphi \sin \theta
\]

\[
z(t) = \begin{cases} \frac{1}{k} \ln(1 + kv_z) + z_0, v_z \geq 0 \\ -\frac{1}{k} \ln(1 - kv_z) + z_0, v_z < 0 \end{cases}
\]

**y-direction**: including the ascending and descending parts

In the ascending part

\[
v_y = \frac{\tan(\beta - \alpha t)}{\alpha}
\]

\[
y(t) = -\frac{1}{2k} \ln \left[ \frac{\alpha^2 v_y + 1}{\alpha^2 (v \sin \varphi)^2 + 1} \right] + y_0
\]

In the descending part

\[
v_y = \frac{1 - \exp(\chi t - 2\beta)}{\alpha[1 + \exp(\chi t - 2\beta)]}
\]

\[
y(t) = Z_M + \frac{1}{2k} \ln \left[ -\alpha^2 v_y \right] + y_0
\]

For the lower half of the source vessel \((-90^\circ \leq \varphi \leq 0)\)

The equations of the velocity and displacement of the fragment are the same as those for the upper half of the source vessel.
\( y \)-direction: including only the descending part of the fragment

\[
v_y = \frac{1 + \varepsilon \gamma}{\alpha (1 - \varepsilon \gamma)}
\]

\( y(t) = \frac{t}{\alpha} - \frac{2}{\alpha} \ln \left( \frac{1 + \varepsilon \gamma}{1 - \delta} \right) + y_0
\]

where \( \alpha = (k/g)^{0.2} \); \( \beta = \arctan(\alpha \varepsilon \sin \phi) \); \( \chi = 2k/\alpha \), \( x_0, y_0, \) and \( z_0 \) are the initial displacement of the fragment; \( v \) is the initial departure velocity; \( \theta \) and \( \varphi \) are the initial departure angles (horizontal and vertical angles).

For the vertical cylindrical source, the following correlations can be derived for \( \varphi, h, \) and \( r \) from Fig. 1:

\[
h = y_0 = R + R \sin \varphi
\]

\[
r = R \cos \varphi
\]

\[
x_0 = R \cos \varphi \cos \theta
\]

\[
z_0 = R \cos \varphi \sin \theta
\]

where \( h \) is the initial height of fragment; \( R \) is the distance from the source center to wall of the vessel; \( r \) is the initial horizontal displacement.

\[\text{Fig. 1. Description of fragment trajectory for explosion of vertical cylindrical source.}\]

3.2. Fragment impact, penetration, and damage

As discussed in detail in the references [3,18–21], the impact probability between the projectiles and the potential target \( \text{Targ}\) can be calculated using Monte Carlo simulations in the light of Eq. (15):

\[
P_{\text{imp}} = \frac{\sum_{s=1}^{N_{\text{sim}}} \sum_{j=1}^{N} \left( j, s \right)}{N_{\text{sim}}} \quad \text{and} \quad n\left( j, s \right) = \begin{cases} 1 & \text{if} \left( V_{\text{Targ}} \cap V_{\text{fragment}} \neq \emptyset \right) \\ 0 & \text{otherwise} \end{cases}
\]

where \( N_{\text{sim}} \) is the total number of Monte Carlo simulations; \( n \) is a parameter that indicates whether the projectile impacts the target; \( V_{\text{fragment}} \) is the fragment trajectory; and \( V_{\text{Targ}} \) is the target volume with a given location, dimensions, and shape. The numerical algorithm for the Monte Carlo method for fragment impact has been
described in detail by [3,18–21].

Then, an impact between the fragment and a target occurs, it may cause partial or complete damage (penetration or perforation) to the target and then result in the explosion of the target. As discussed in the references [3,18–21], the penetration depth of the fragment can be expressed using Eqs. (16) or (17), and the rupture probability of the impacted target, \( P_{\text{rup}} \), can be expressed as in Eq. (18):

\[
\begin{align*}
\mathbf{h}_p &= -d_p \cos \eta + \frac{\sqrt{\left( d_p \cos \eta \right)^2 + \frac{4}{\pi} \tan \eta \left( \frac{E_c}{f_u \varepsilon_u} \right)^{2/3}}}{2 \tan \eta} \quad \text{for the case } \eta \neq 0 \\
\mathbf{h}_p &= \frac{1}{\pi d_p} \left( \frac{E_c}{f_u \varepsilon_u} \right)^{2/3} \quad \text{for the case } \eta = 0
\end{align*}
\]

\[
P_{\text{rup}} = P(E_c \leq 0) = \frac{\sum_{i=1}^{N_{\text{imp}}} \sum_{j=1}^{N_{\text{simu}}} q(j,i)}{N_{\text{simu}}} \quad \text{and } q(j,i) = \begin{cases} 1 & \text{if } E_c \leq 0, \\ 0 & \text{otherwise} \end{cases}, \quad E_c = (e_t - h_p) - e_{cr}
\]

where \( d_p \) is the fragment diameter; \( \eta \) is the incidence angle of the fragment; \( f_u \) is the ultimate strength of the target constitutive material and \( \varepsilon_u \) its ultimate strain; \( E_c \) is the kinetic energy expended when the penetration process occurs; \( E_c \) is the limit state function; \( N_{\text{imp}} \) is the number of the fragments impacting the target, obtained through Eq. (15) in each simulation; \( N_{\text{simu}} \) is the total number of Monte Carlo simulations indicating that \( \sum_{j=1}^{N} n(j,s) / N \) in Eq. (15) is not zero in each simulation; \( e_t \) is the target thickness; \( h_p \) is the penetration depth; and \( e_{cr} \) is the critical plate thickness. Use of the Monte Carlo method for evaluation of the rupture probability of the impacted target has been described in detail in the references [3,18–21].

The domino effect risk \( P_{\text{domino}} \) can be expressed as: \( P_{\text{domino}} = P_{\text{gen}} \times P_{\text{imp}} \times P_{\text{rup}} \), where \( P_{\text{gen}} \) is the generation probability of fragments.

4. Simulation results and discussions

4.1. Characteristics of the source vessels

As a case study, the industrial site installed in Shanghai Petrochemical Company Limited was considered, including 6 vertical cylindrical vessels (for each type of tank, the volumes are lied between 100–10000 m³). The distributions of the fragments crashing on the ground can be derived according to the movement approach above under the set of hypotheses proposed by the authors [3,18–21] by means of Monte Carlo simulations.

All the features of the source terms discussed in detail in the references [3,13,16,18–19,22–23,27] are used here. The fragment impact and penetration into the target, i.e. the probability of impact \( P_{\text{imp}} \) and rupture probability of the impacted target \( P_{\text{rup}} \) can be calculated considering and not considering the source size as discussed in the references [3,22], and then the domino effect risk caused by fragments \( P_{\text{domino}} \) can be respectively derived.
4.2. Results and analysis

4.2.1. The ground distribution of fragments

With the source size considered, the results of the distributions of the fragments from the lower half of the source vessels on the ground were obtained and are shown in Fig. 2. All the fragments distributions were larger than 0. Otherwise, the source was a point source without the source size considered, the distributions of the fragments from the lower half of the source were 0.

The calculation results showed that the maximum distances respectively from the vertical cylindrical vessels of 100m³ and 10000 m³ reached 500m and 1800 m. Thus, it was simply probable that the fragments from the lower half of the source vessels would hit the target around the source, the source size could not be neglected in the quantitative assessment of the domino effect risk caused by fragments.

Fig. 2. Distributions of the fragments from the lower half of the vertical cylindrical vessels on the ground.

4.2.2. The impact probability of fragments

(1) Investigation on the probability of fragments impact on a given target

A spherical vessel of 50m³ in Shanghai Petrochemical Company Limited was chosen as the target, and the probability of fragments impact was evaluated using Eqs. (1)-(18). The cases were shown in Table 2. For the vertical cylindrical vessels, the volumes of 100, 1000, and 10000 m³ were respectively chosen, and each volume was respectively considered with and without the source size.

For the vertical cylindrical vessels, all were equivalent to spheres, and the diameters were the equivalent diameters, as shown in Table 2.

The impact probability of fragments respectively generated by the vertical cylindrical vessels of volumes 100, 1000, and 10000 m³ on the target were investigated neglecting and not neglecting the source size, and the results
were shown in Fig. 3(a)-(c) \( (D\) is the equivalent diameter of the source tank). The corresponding relative deviations were demonstrated in Table 3–5. The results illustrated that the relative deviation of impact probability would be larger than 10% when the target vessel was located in the distance less than \(8D\) for the sources, but it would be smaller than 10% if the distance was greater than \(8D\).

Table 2. Cases in the simulations.

<table>
<thead>
<tr>
<th>Vessel type</th>
<th>Case No.</th>
<th>Volume/m(^3)</th>
<th>Equivalent diameter/m</th>
<th>Source size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical cylindrical</td>
<td>1</td>
<td>100</td>
<td>5.76</td>
<td>no consideration</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
<td>5.76</td>
<td>consideration</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1000</td>
<td>12.42</td>
<td>no consideration</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1000</td>
<td>12.42</td>
<td>consideration</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>10000</td>
<td>26.72</td>
<td>no consideration</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>10000</td>
<td>26.72</td>
<td>consideration</td>
</tr>
</tbody>
</table>

(a) 100 m\(^3\) vessel

(b) 1000 m\(^3\) vessel

(c) 10000 m\(^3\) vessel

Fig. 3. Description of fragment trajectory.
Comparison between impact probability of fragments respectively from the upper and lower part of the sources

The proportion of impact probability of upper and lower parts of the sources to the total impact probability was investigated with the size of the bursting tank, and the results were revealed in Fig. 4. The results demonstrated that: the proportion of impact probability of lower part of the source to the total impact probability decreased with the distance, but that of upper part increased according to the distance. When the target was located in the distance less than $5D$ for the three types of sources, the proportion of impact probability of lower part would larger; if the distance was about $5D$, the proportion of impact probability of upper and lower parts would be equal; while the target was located in the distance greater than $5D$, the proportion of impact probability of upper part would be larger. When the distance from the source was larger than $14D$, the proportion of impact probability of lower part was 0. Thus, with the distance from the source to the target less than $14D$, the source size should be considered, and its effect on the impact probability was significant.

Table 3. Relative deviation of impact probability of case 1 to case 2 for the three types of vessels.

<table>
<thead>
<tr>
<th>Vessel type</th>
<th>Target distance</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Relative deviation/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>D</td>
<td>0.07464</td>
<td>0.09837</td>
<td>31.8</td>
</tr>
<tr>
<td>Vertical</td>
<td>2D</td>
<td>0.02435</td>
<td>0.03155</td>
<td>29.6</td>
</tr>
<tr>
<td>Vertical</td>
<td>3D</td>
<td>0.01963</td>
<td>0.02334</td>
<td>18.9</td>
</tr>
<tr>
<td>Vertical</td>
<td>4D</td>
<td>0.01710</td>
<td>0.01945</td>
<td>13.8</td>
</tr>
<tr>
<td>Vertical</td>
<td>5D</td>
<td>0.01639</td>
<td>0.01850</td>
<td>12.9</td>
</tr>
<tr>
<td>Vertical</td>
<td>6D</td>
<td>0.01377</td>
<td>0.01536</td>
<td>11.6</td>
</tr>
<tr>
<td>Vertical</td>
<td>7D</td>
<td>0.00843</td>
<td>0.00928</td>
<td>10.1</td>
</tr>
<tr>
<td>Vertical</td>
<td>8D</td>
<td>0.00608</td>
<td>0.00649</td>
<td>6.9</td>
</tr>
<tr>
<td>Vertical</td>
<td>9D</td>
<td>0.00554</td>
<td>0.00571</td>
<td>3.2</td>
</tr>
<tr>
<td>Vertical</td>
<td>10D</td>
<td>0.00452</td>
<td>0.00461</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 4. Relative deviation of impact probability of case 3 to case 4 for the three types of vessels.

<table>
<thead>
<tr>
<th>Vessel type</th>
<th>Target distance</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Relative deviation/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>D</td>
<td>0.01162</td>
<td>0.01402</td>
<td>20.7</td>
</tr>
<tr>
<td>Vertical</td>
<td>2D</td>
<td>0.00318</td>
<td>0.00380</td>
<td>19.8</td>
</tr>
<tr>
<td>Vertical</td>
<td>3D</td>
<td>0.00231</td>
<td>0.00273</td>
<td>18.2</td>
</tr>
<tr>
<td>Vertical</td>
<td>4D</td>
<td>0.00207</td>
<td>0.00243</td>
<td>17.6</td>
</tr>
<tr>
<td>Vertical</td>
<td>5D</td>
<td>0.00199</td>
<td>0.00227</td>
<td>14.4</td>
</tr>
<tr>
<td>Vertical</td>
<td>6D</td>
<td>0.00166</td>
<td>0.00182</td>
<td>10.1</td>
</tr>
<tr>
<td>Vertical</td>
<td>7D</td>
<td>0.00137</td>
<td>0.00150</td>
<td>10.0</td>
</tr>
<tr>
<td>Vertical</td>
<td>8D</td>
<td>0.00119</td>
<td>0.00128</td>
<td>7.8</td>
</tr>
<tr>
<td>Vertical</td>
<td>9D</td>
<td>0.00116</td>
<td>0.00124</td>
<td>6.9</td>
</tr>
<tr>
<td>Vertical</td>
<td>10D</td>
<td>0.00096</td>
<td>0.00100</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 5. Relative deviation of impact probability of case 5 to case 6 for the three types of vessels.

<table>
<thead>
<tr>
<th>Vessel type</th>
<th>Target distance</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Relative deviation/%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>D</td>
<td>0.00292</td>
<td>0.00380</td>
<td>30.2</td>
</tr>
<tr>
<td>Vertical</td>
<td>2D</td>
<td>0.00267</td>
<td>0.00343</td>
<td>28.6</td>
</tr>
<tr>
<td>Vertical</td>
<td>3D</td>
<td>0.00203</td>
<td>0.00243</td>
<td>19.9</td>
</tr>
<tr>
<td>Vertical</td>
<td>4D</td>
<td>0.00186</td>
<td>0.00211</td>
<td>13.9</td>
</tr>
<tr>
<td>Vertical</td>
<td>5D</td>
<td>0.00156</td>
<td>0.00175</td>
<td>12.8</td>
</tr>
<tr>
<td>Vertical</td>
<td>6D</td>
<td>0.00143</td>
<td>0.00159</td>
<td>11.2</td>
</tr>
<tr>
<td>Vertical</td>
<td>7D</td>
<td>0.00138</td>
<td>0.00152</td>
<td>10.5</td>
</tr>
<tr>
<td>Vertical</td>
<td>8D</td>
<td>0.00138</td>
<td>0.00148</td>
<td>7.6</td>
</tr>
<tr>
<td>Vertical</td>
<td>9D</td>
<td>0.00127</td>
<td>0.00135</td>
<td>6.6</td>
</tr>
<tr>
<td>Vertical</td>
<td>10D</td>
<td>0.00066</td>
<td>0.00068</td>
<td>4.3</td>
</tr>
</tbody>
</table>
5. Conclusions

The domino effect risk was investigated in consideration of different source sizes (different sizes of vertical cylindrical vessels) using Monte-Carlo simulations:

- With the source size considered, the results of the distributions of the fragments from the lower half of the source vessels on the ground were larger than 0. The calculation results showed that the maximum distances respectively from the vertical cylindrical vessels of 100 m³ and 10000 m³ reached 500 m and 1800 m. Thus, it was simply probable that the fragments from the lower half of the source vessels would hit the target around the source, the source size could not be neglected in the quantitative assessment of the domino effect risk caused by fragments.

- The results illustrated that the relative deviation of impact probability would be larger than 10% when the target vessel was located in the distance less than $8D$ for the three types of sources, but it would be smaller than 10% while the distance was greater than $8D$.

- The proportion of impact probability of lower part of the source to the total impact probability decreased with the distance, but that of upper part increased according to the distance. When the target was located in the distance less than $5D$ for the three types of sources, the proportion of impact probability of lower part would be larger; if the distance was about $5D$, the proportion of impact probability of upper and lower parts would be equal; while the target was located in the distance greater than $5D$, the proportion of impact probability of upper part would be larger. When the distance from the source was larger than $14D$, the proportion of impact probability of lower part would be 0. Thus, with the distance from the source to the target less than $14D$, the source size should be considered, and its effect on the impact probability was significant.

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