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Shape invariance method for quintom model in the bent brane background

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Abstract

In the present Letter, we study the braneworld scenarios in the presence of quintom dark energy coupled by gravity. The first-order formalism for the bent brane (for both de Sitter and anti-de Sitter geometry), leads us to discuss the shape invariance method in the bent brane systems. So, by using the fluctuations of metric and quintom fields we obtain the Schrödinger equation. Then we factorize the corresponding Hamiltonian in terms of multiplication of the first-order differential operators. These first-order operators lead us to obtain the energy spectrum with the help of shape invariance method.

1. Introduction

It is known that all analytically solvable potentials in quantum mechanics have the property of shape invariance [1]. In fact shape invariance is an integrability condition, however, one should emphasize that shape invariance is not the most general integrability condition as not all exactly solvable potentials seem to be shape invariance to [2,3]. An interesting feature of supersymmetric quantum mechanics is that for a shape invariant system [4,5] the entire spectrum can be determined algebraically without ever referring to underlying differential equations.

In the present Letter we would like to use this method for an interesting problem in cosmology. Here we consider the quintom model of dark energy [6] in the bent brane background. One of the most important problems of cosmology, is the problem of so-called dark energy. The type Ia supernova observations suggests that the universe is dominated by dark energy with negative pressure which provides the dynamical mechanism of the accelerating expansion of the universe [7–9]. The strength of this acceleration is presently matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these models are based on dynamics of a scalar [10–13] or multi-scalar fields [6]. Primary scalar field candidate for dark energy was quintessence scenario [10,11], a fluid with the parameter of the equation of state lying in the range, $-1 < w < \frac{-1}{3}$. The analysis of the properties of dark energy from recent observations mildly favor models with w crossing -1 in the near past. Meanwhile for the phantom model [12] of dark energy which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian, one always has $w \leqslant -1$. Neither the quintessence nor the phantom alone can fulfill the transition from $w > -1$ to $w < -1$ and vice versa. But one can show [6] that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, dubbed quintom, can produce a better fit to the data than more familiar models with $w \geqslant -1$.

This Letter is organised as follows. In Section 2 we consider the quintom model of dark energy in the background of a five-dimensional space–time with warped geometry. Then we consider the fluctuations of the metric and quintom scalar fields. In Section 3 we review the supersymmetry algebra with the central charge and shape invariance method. In Section 4 we obtain the

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factorized Hamiltonian for the bent brane, which leads us to investigate the shape invariance method with considering the central extended algebra. Finally by taking advantage of shape invariance method we obtain energy spectrum in our interesting geometry.

2. The quintom model in the bent brane background

The quintom model of dark energy [6] is of new models proposed to explain the new astrophysical data, due to transition from $w > -1$ to $w < -1$, i.e., transition from quintessence dominated universe to phantom dominated universe. Containing the normal scalar field ϕ and negative kinetic scalar field χ, the action which describes the quintom model is expressed as the following form

$$S = \int d^4x\, dy \sqrt{|g|}\left(-\frac{1}{4}R + \frac{1}{2}\partial_a\phi\partial^a\phi - \frac{1}{2}\partial_a\chi\partial^a\chi - V(\phi,\chi)\right), \tag{1}$$

where we have not considered the Lagrangian density of matter field, and we take $4\pi G = 1$. The line element of the five-dimensional space–time can be written

$$ds_5^2 = g_{ab}\, dx^a\, dx^b = e^{2A}\, ds_4^2 - dy^2, \tag{2}$$

where $a, b = 0, 1, 2, 3, 4$, and e^{2A} is the warp factor. dS_4^2 represent the four-dimensional metric:

$$ds_4^2 = dt^2 - e^{2\sqrt{\Lambda}t}\left(dx_1^2 + dx_2^2 + dx_3^2\right), \tag{3}$$

where Λ is a four-dimensional cosmological constant. We note that constant Λ is positive for de Sitter (dS_4) space–time, negative for anti-de Sitter (AdS_4) space–time and zero for Minkowski (M_4) space–time.

At first we assume that $\Lambda = 0$, and the functions A, ϕ, χ are $A(y)$, $\phi(y)$, $\chi(y)$.

From the Einstein and Euler–Lagrange equations we obtain,

$$A'' = -\frac{2}{3}\left(\phi'^2 - \chi'^2\right), \qquad A'^2 = \frac{1}{6}\left(\phi'^2 - \chi'^2\right) - \frac{1}{3}V(\phi,\chi), \qquad V_\phi = \phi'' + 4A'\phi', \qquad V_\chi = -\chi'' - 4A'\chi', \tag{4}$$

where a prime denotes a derivative with respect to y, and

$$V_\phi = \frac{dV}{d\phi}, \qquad V_\chi = \frac{dV}{d\chi}. \tag{5}$$

In order to obtain the first-order equation, we use [14]

$$A' = -\frac{1}{3}W, \qquad \phi' = \frac{1}{2}W_\phi, \qquad \chi' = -\frac{1}{2}W_\chi. \tag{6}$$

From (4) and (6) the explicit form of the potential is

$$V(\phi,\chi) = \frac{1}{8}\left({W_\phi}^2 - {W_\chi}^2\right) - \frac{1}{3}W^2. \tag{7}$$

Next we consider the general case with $\Lambda \neq 0$ and we obtain

$$A'' + \Lambda e^{-2A} = -\frac{2}{3}\left(\phi'^2 - \chi'^2\right), \qquad A'^2 - \Lambda e^{-2A} = \frac{1}{6}\left(\phi'^2 - \chi'^2\right) - \frac{1}{3}V(\phi,\chi). \tag{8}$$

The cosmological constant leads us to define the function which corresponds to the scalar fields ϕ and χ. It means that this function is completely coupled and generally responsible for the cosmological constant. Thus we gain

$$A' = -\frac{1}{3}W - \frac{1}{3}\Lambda\gamma Z, \qquad \phi' = \frac{1}{2}W_\phi + \frac{1}{2}\Lambda(\alpha+\gamma)Z_\phi, \qquad \chi' = -\frac{1}{2}W_\chi - \frac{1}{2}\Lambda(\gamma+\beta)Z_\chi, \tag{9}$$

where $Z = Z(\phi,\chi)$ is a new and arbitrary function of the scalar fields and α, β and γ are constants. The potential $V(\phi,\chi)$ is obtained by (8) and (9):

$$\begin{aligned} V(\phi,\chi) = {} & \frac{1}{8}\left(W_\phi + \Lambda(\alpha+\gamma)Z_\phi\right)\left(W_\phi + \Lambda(\gamma-3\alpha)Z_\phi\right) - \frac{1}{3}(W + \Lambda\gamma Z)^2 \\ & - \frac{1}{8}\left(W_\chi + \Lambda(\beta+\gamma)Z_\chi\right)\left(W_\chi + \Lambda(\gamma-3\beta)Z_\chi\right). \end{aligned} \tag{10}$$

Also we assume

$$W_{\phi\chi} = 0. \tag{11}$$

By inserting this potential in the equations of motion, one can obtain the following constraint:

$$\alpha W_{\phi\phi}Z_\phi + \alpha W_\phi Z_{\phi\phi} + 2\Lambda\alpha(\alpha+\gamma)Z_\phi Z_{\phi\phi} - \frac{4}{3}\alpha Z_\phi(W + \Lambda\gamma Z) = 0, \tag{12}$$

$$\beta W_{\chi\chi} Z_\chi + \beta W_\chi Z_{\chi\chi} + 2\Lambda\beta(\beta+\gamma) Z_\chi Z_{\chi\chi} + \frac{4}{3}\beta Z_\chi (W + \Lambda\gamma Z) = 0. \tag{13}$$

For simplicity, we consider $Z(\phi,\chi) = W(\phi,\chi)$, because it is difficult to solve these constraints in the general case:

$$\frac{3}{2} d W_{\phi\phi} - W = 0, \tag{14}$$

$$-\frac{3}{2} d' W_{\chi\chi} + W = 0, \tag{15}$$

where $d = \frac{1+\Lambda(\gamma+\alpha)}{1+\Lambda\gamma}$ and $d' = -\frac{(1+\Lambda(\gamma+\beta))}{1+\Lambda\gamma}$. Then we obtain:

$$d W_{\phi\phi} - d' W_{\chi\chi} = 0. \tag{16}$$

This constraint guide us to consider a superpotential with the following form:

$$W(\phi,\chi) = 3a \sinh(b\phi + c\chi), \tag{17}$$

where $b = \sqrt{\frac{2}{3d}}$, $c = \sqrt{\frac{2}{3d'}}$, and a is a constant. By substituting superpotential (17) into Eqs. (10), (9) we get the following potential, fields, and $A(y)$, respectively:

$$V(\phi,\chi) = -\frac{3}{4} a^2 (1+\Lambda\gamma)\big(2 + \Lambda(2\gamma + 3\alpha + 3\beta)\big) \cosh^2(b\phi + c\chi) + 3a^2(1+\Lambda\gamma)^2 \tag{18}$$

and

$$\phi(y) = \pm\sqrt{\frac{3d}{8}} \ln\big[\tan\big(a(1+\Lambda\gamma)y\big)\big], \tag{19}$$

$$\chi(y) = \pm\sqrt{\frac{3d'}{8}} \ln\big[\tan\big(a(1+\Lambda\gamma)y\big)\big], \tag{20}$$

$$A(y) = \frac{1}{2} \ln\left[\frac{1}{2} \sin\big(2a(1+\Lambda\gamma)y\big)\right]. \tag{21}$$

The shape invariance method helps us to investigate the stability condition. At first we consider fluctuations of the metric and scalar fields.

The perturbed metric is

$$ds^2 = e^{2A}(g_{\mu\nu} + \epsilon h_{\mu\nu})\, dx^\mu\, dx^\nu - dy^2. \tag{22}$$

We use the coordinate z which is defined by

$$dz = e^{-A(y)}\, dy. \tag{23}$$

So, one can obtain the coordinate z as a follow,

$$z = \frac{1}{a(1+\Lambda\gamma)} \ln\big[\tan\big(a(1+\Lambda\gamma)y\big)\big], \tag{24}$$

also y is as

$$y = \frac{1}{a(1+\Lambda\gamma)} \arctan\big(e^{a(1+\Lambda\gamma)z}\big). \tag{25}$$

Therefore, $A(z)$ is given by

$$A(z) = \frac{1}{2} \ln\left[\frac{1}{2} \operatorname{sech}(\eta z)\right], \tag{26}$$

where, $a = 1$ and $\eta = (1+\Lambda\gamma)$.

3. Shape invariance method

If the energy of the ground state is zero, we can factorize the Hamiltonian as,

$$H_1(g) = B^\dagger(g) B(g), \tag{27}$$

where g is (are) the real parameter(s), which give(s) us the potential, and $B(g)$ is a first order differential operator. The ground state of H_1 is annihilated by $B(g)$; the partner Hamiltonian of H_1 will be obtained with reversing the order of B and $B^\dagger$,

$$H_2(g) = B(g)B^\dagger(g), \tag{28}$$

and the spectrum of H_1 and H_2 are degenerate. The only difference is that H_1 has a zero-energy state and in general H_2 does not,

$$H_2 B = B H_1. \tag{29}$$

If we had for $n \geqslant 0$,

$$H_1 \Psi_n^{(1)} = E_n^{(1)} \Psi_n^{(1)} \tag{30}$$

this would imply that

$$H_2\big(B\Psi_n^{(1)}\big) = E_n^{(1)}\big(B\Psi_n^{(1)}\big). \tag{31}$$

So, the relation between the eigenvalues and eigenfunctions of the two Hamiltonians H_1 and H_2 is,

$$E_n^{(2)} = E_{n+1}^{(1)}, \qquad E_0^{(1)} = 0, \qquad \Psi_n^{(2)} \propto A\Psi_{n+1}^{(1)}, \tag{32}$$

where the ground state wavefunction for H_1 (or H_2) can be obtained as follows:

$$\begin{aligned}
&B\Psi_0^{(1)}(x) = 0 \Rightarrow \Psi_0^{(1)}(x) = N\exp\left(-\int^x W(y)\,d(y)\right),\\
&B^\dagger\Psi_0^{(2)}(x) = 0 \Rightarrow \Psi_0^{(2)}(x) = N\exp\left(+\int^x W(y)\,d(y)\right).
\end{aligned} \tag{33}$$

Supersymmetry provides a natural context for understanding the relationship between the states of H_1 and those of H_2, where H_1 and H_2 are partners. If one combines these two operators to

$$H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}, \tag{34}$$

then this matrix can be obtained from the anticommutator $H = \{Q, Q\dagger\}$, where Q and $Q\dagger$ are supercharges, given by

$$Q = \begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & B^\dagger \\ 0 & 0 \end{pmatrix}. \tag{35}$$

In this algebra we have

$$[H, Q] = \big[H, Q^\dagger\big] = 0, \qquad \{Q, Q\} = \big\{Q^\dagger, Q^\dagger\big\} = 0. \tag{36}$$

The shape invariance is a property that arises when there is an additional relationship between the partner Hamiltonian H_1 and H_2. Suppose that these Hamiltonian are linked by the condition

$$B(g_1)B^\dagger(g_1) = B^\dagger(g_2)B(g_2) + c(g_2), \tag{37}$$

where the real parameters g_1 and g_2 are related by a mapping $f: g_1 \to g_2$, and $c(g)$ is a c-number that depends on the parameter(s) of the Hamiltonian. So, in general we can write

$$H_k = B^\dagger(g_k)B(g_k) + c(g_k) + \cdots + c(g_2), \tag{38}$$

where

$$g_{j+1} = f(g_j), \qquad B^\dagger(g_k)H_{k+1} = H_k B^\dagger(g_k). \tag{39}$$

The ground state of each of these sectors satisfies a first-order equation, namely

$$B(g_k)\Psi_1(x; g_k) = 0.$$

Now, we study supersymmetry with a central charge. Supersymmetric quantum mechanics [4] can be formulated as a one-dimensional supersymmetric quantum field theory. A bosonic field is, then, a real-valued function of time, and a fermionic field is a Grassman-valued function of time. The $d = 1$, $N = 1$ superalgebra with a central charge is specified by the following relations:

$$\big\{Q, Q^\dagger\big\} = H, \qquad [H, Q] = \big[H, Q^\dagger\big] = 0, \qquad \{Q, Q\} = \big\{Q^\dagger, Q^\dagger\big\} = C, \tag{40}$$

where Q and C are supercharge and central charge, respectively. The above algebra implies $[Q, C] = [Q^\dagger, C] = 0$. To realize the algebra (40), we represent the supercharges as matrices:

$$Q = \begin{pmatrix} \lambda & 0 \\ B & -\lambda \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} \lambda & B^\dagger \\ 0 & -\lambda \end{pmatrix}, \tag{41}$$

where λ is the real part of a c-number. This approach is meant first to present an implementation of this algebra in a two-sector model, and then to generalize this construction to a four-sector model.

Then the corresponding Hamiltonian and central charge are determined by the superalgebra to be, respectively,

$$H = \begin{pmatrix} B^\dagger B + 2\lambda^2 & 0 \\ 0 & BB^\dagger + 2\lambda^2 \end{pmatrix}, \qquad C = \begin{pmatrix} 2\lambda^2 & 0 \\ 0 & 2\lambda^2 \end{pmatrix}, \quad C \geqslant 0. \tag{42}$$

To construct a model with four sectors, one can concentrate on a two-sector model. It has supercharges

$$Q = \begin{pmatrix} -\lambda_1 & 0 & 0 & 0 \\ B_1 & \lambda_1 & 0 & 0 \\ 0 & 0 & -\lambda_3 & 0 \\ 0 & 0 & B_3 & \lambda_3 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} -\lambda_1 & {B_1}^\dagger & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & -\lambda_3 & {B_3}^\dagger \\ 0 & 0 & 0 & \lambda_3 \end{pmatrix}. \tag{43}$$

By using (40) and (43) for the four sectors we obtain

$$H = \begin{pmatrix} B_1^\dagger B_1 + 2\lambda_1^2 & 0 & 0 & 0 \\ 0 & B_1 B_1^\dagger + 2\lambda_1^2 & 0 & 0 \\ 0 & 0 & B_3^\dagger B_3 + 2\lambda_3^2 & 0 \\ 0 & 0 & 0 & B_3 B_3^\dagger + 2\lambda_3^2 \end{pmatrix} \tag{44}$$

and

$$C = \begin{pmatrix} 2\lambda_1^2 & 0 & 0 & 0 \\ 0 & 2\lambda_1^2 & 0 & 0 \\ 0 & 0 & 2\lambda_3^2 & 0 \\ 0 & 0 & 0 & 2\lambda_3^2 \end{pmatrix}. \tag{45}$$

As we see the sectors one and two are degenerate, with energies bounded from below by $2\lambda_1^2$, and sectors three and four are degenerate, with energies bounded from below by $2\lambda_3^2$. The only exceptions are that sectors one and three each have states that saturate their respective energy bounds while the even sectors do not, and this suggest an enhanced algebraic structure. Therefore, we have to define the shift operator S for the four sectors in order to relate sector two to sector three with the following form:

$$S \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ B_1 & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & B_3 & 0 \end{pmatrix}. \tag{46}$$

We can choose D such that shape invariance condition and $[H, S] = 0$ are satisfied. For this, we suppose that there is a unitary transformation which is represented by an operator Ω such that $B_3 = \Omega^\dagger B_1 \Omega$. And also, we use a unitary operator U such that $U^2 = \Omega$, and the conserved shift operator takes the form

$$S \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ B_1 & 0 & 0 & 0 \\ 0 & U^\dagger B_1 U & 0 & 0 \\ 0 & 0 & U^{\dagger 2} B_1 U^2 & 0 \end{pmatrix}. \tag{47}$$

From conservation of S, one can obtain the shape invariance relation,

$$B_1 B_1^\dagger - U^\dagger B_1^\dagger B_1 U = k, \qquad 2\lambda_3^2 = 2\lambda_1^2 + k + U^\dagger k U. \tag{48}$$

In particular we have

$$H = S^\dagger S + F, \tag{49}$$

where

$$F = \begin{pmatrix} 2\lambda_1^2 & 0 & 0 & 0 \\ 0 & 2\lambda_1^2 + k & 0 & 0 \\ 0 & 0 & 2\lambda_1^2 + k + U^\dagger k U & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix}. \tag{50}$$

In the first three sectors, the energies are constrained by a Bogmol'nyi bound, $H_k \geqslant (F)_{kk}$, because each of the first sector has to be degenerate with the Bogmol'nyi-saturating ground state of one of the first three sectors. The constants in F represent not only the Bogmol'nyi bounds of the various sectors, but also the first three energy eigenvalues of the original Hamiltonian.

In the next section, we shall apply the above information to a bent brane with quintom dark energy.

4. The stability of system with shape invariance method

The corresponding Schrödinger equation for our interesting problem is

$$-\frac{d^2\psi(z)}{dz^2} + V(z)\psi(z) = n^2\psi(z), \tag{51}$$

where

$$V(z) = -\frac{9}{4}\Lambda + \frac{3}{2}A''(z) + \frac{9}{4}A'^2(z). \tag{52}$$

Now we can factorize the corresponding Schrödinger equation in the following form

$$\left[\frac{d}{dz} + \frac{3}{2}A'(z)\right]\left[-\frac{d}{dz} + \frac{3}{2}A'(z)\right]\psi(z) = \left(n^2 + \frac{9}{4}\Lambda\right)\psi(z). \tag{53}$$

The corresponding potential in term of z is as,

$$V(z) = s^2\eta^2 - s(s+1)\eta^2\,\mathrm{sech}^2(\eta z), \tag{54}$$

where $s = \frac{3}{4}$.

By using the new variable $x = \lambda z$ the Schrödinger equation can be written as,

$$-\frac{d^2\psi(x)}{dx^2} + \left[s^2 - s(s+1)\,\mathrm{sech}^2(x)\right]\psi(x) = \left(\frac{n^2}{\eta^2} + \frac{9\Lambda}{4\eta^2}\right)\psi(x), \tag{55}$$

and we have

$$H = -\frac{d^2}{dx^2} - s(s+1)\sec h^2(x) + s^2. \tag{56}$$

Now we are going to factorize H in terms of lowering and raising operators, respectively,

$$B = -\frac{d}{dx} - s\tanh(x), \qquad B^\dagger = \frac{d}{dx} - s\tanh(x), \tag{57}$$

and one can obtain the paired Hamiltonians

$$H_1 = B^\dagger B = -\frac{d^2}{dx^2} - s(s+1)\sec h^2(x) + s^2, \qquad H_2 = BB^\dagger = -\frac{d^2}{dx^2} - s(s-1)\sec h^2(x) + s^2, \tag{58}$$

where

$$H_2(s) = H_1(s-1) + c(s). \tag{59}$$

This relation shows us there is a shape invariance condition with $c(s) = 2s - 1$.

In the case of a central charge, we choose the unitary operator U as follows:

$$U = \exp\left(\frac{\partial}{\partial s}\right), \qquad U^\dagger = \exp\left(-\frac{\partial}{\partial s}\right), \tag{60}$$

where

$$U^\dagger f(s)U \to f(s-1).$$

From Eqs. (47)–(49) we have,

$$S^\dagger S = \begin{pmatrix} B_1^\dagger B_1 & 0 & 0 & 0 \\ 0 & U^\dagger B_1^\dagger B_1 U & 0 & 0 \\ 0 & 0 & \Omega^\dagger B_1^\dagger B_1 \Omega & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix}, \tag{61}$$

with

$$B_1^\dagger B_1 = -\frac{d^2}{dx^2} - s(s+1)\sec h^2(x) + (s-1)^2, \qquad U^\dagger B_1^\dagger B_1 U = -\frac{d^2}{dx^2} - (g-1)(g)\sec h^2(x) + (s-2)^2,$$
$$\Omega^\dagger B_1^\dagger B_1 \Omega = -\frac{d^2}{dx^2} - (s-2)(s-1)\sec h^2(x) + (s-3)^2. \tag{62}$$

Also, by using Eqs. (49), (50), one can obtain F as follows:

$$F = \begin{pmatrix} -\frac{9\Lambda}{4\eta^2} & 0 & 0 & 0 \\ 0 & -\frac{9\Lambda}{4\eta^2} + 2s - 1 & 0 & 0 \\ 0 & 0 & -\frac{9\Lambda}{4q^2} + 4s - 4 & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix}. \tag{63}$$

Therefore, the energy spectrum of H_1 is

$$E_0^{(1)} = \left(-\frac{9\Lambda}{4}\right), \qquad E_1^{(1)} = \left(-\frac{9\Lambda}{4} + \frac{1}{2}\eta^2\right), \qquad E_2^{(1)} = \left(-\frac{9\Lambda}{4} - \eta^2\right), \qquad E_3^{(1)} = H_4. \tag{64}$$

Here we can discuss three cases as $\Lambda = 0$, $\Lambda < 0$ and $\Lambda > 0$ which are corresponding to flat, AdS and dS space, respectively. The energy spectrum of H_1 for flat, AdS and dS spaces are as following respectively. In the another term for $\Lambda = 0$ we have

$$E_0^{(1)} = 0, \qquad E_1^{(1)} = \frac{1}{2}\eta^2 > 0, \qquad E_2^{(1)} = -\eta^2 < 0, \qquad E_3^{(1)} = H_4 \tag{65}$$

for $\Lambda < 0$ we have

$$E_0^{(1)} = \left(-\frac{9\Lambda}{4}\right) > 0, \qquad E_1^{(1)} = \left(-\frac{9\Lambda}{4} + \frac{1}{2}\eta^2\right) > 0,$$
$$E_2^{(1)} = \left(-\frac{9\Lambda}{4} - (1+\Lambda\gamma)^2\right) \leqslant 0 \quad \text{or} \quad \geqslant 0, \qquad E_3^{(1)} = H_4 > 0 \tag{66}$$

and finally for $\Lambda > 0$ we have

$$E_0^{(1)} = \left(-\frac{9\Lambda}{4}\right) < 0, \qquad E_1^{(1)} = \left(-\frac{9\Lambda}{4} + \frac{1}{2}(1+\Lambda\gamma)^2\right) \leqslant 0 \quad \text{or} \quad \geqslant 0,$$
$$E_2^{(1)} = \left(-\frac{9\Lambda}{4} - \eta^2\right) < 0, \qquad E_3^{(1)} = H_4 > 0. \tag{67}$$

We note that in the all above cases we have some tachyonic states with negative energy. From (66) one can see, if $|\frac{9\Lambda}{4}| > (1+\Lambda\gamma)^2$ then for the AdS_4 case, all states have positive eigenvalues. In this case the transition from AdS_4 to M_4 and dS_4 geometry is not stable.

5. Conclusion

In the present Letter we have described the algebra which gives a natural framework for understanding the origins of shape invariance in our interesting problem. The study of shape invariance solutions can be done by the factorization method. Our aim was to solve and discuss the stability of a bent brane in the presence of quintom dark energy and in different geometries with a non-zero cosmological constant. We have done the perturbation to the metric and fields and achieved the corresponding Schrödinger equation which was the second-order equation. Then we have factorized the equation to the first-order equations which are raising and lowering operators and have generated the algebra. From first order equations we easily discussed the energy spectrum and also the stability of the system in the transition to different geometries.

References

[1] L. Gendenstein, JETP Lett. 38 (1983) 356.
[2] F. Cooper, J.N. Ginocchio, A. Khare, Phys. Rev. D 36 (1987) 2458.
[3] J.W. Dabrowska, A. Khare, U.P. Sukhatme, J. Phys. A 21 (1988) L195.
[4] F. Cooper, A. Khar, U. Sukhatme, Phys. Rep. 251 (1995) 268.
[5] L.E. Gendenshtein, I.V. Krive, Sov. Phys. Usp. 28 (1985) 645.

[6] Z.K. Guo, et al., astro-ph/0410654;
G.-B. Zhao, J.-Q. Xia, B. Feng, X. Zhang, astro-ph/0603621;
J.-Q. Xia, G.-B. Zhao, B. Feng, X. Zhang, astro-ph/0603393;
J.-Q. Xia, B. Feng, X. Zhang, Mod. Phys. Lett. A 20 (2005) 2409;
B. Feng, M. Li, Y.-S. Piao, X. Zhang, Phys. Lett. B 634 (2006) 101;
M.R. Setare, Phys. Lett. B 641 (2006) 130.
[7] S. Perlmutter, et al., Astrophys. J. 517 (1999) 565.
[8] P.M. Garnavich, et al., Astrophys. J. 493 (1998) L53.
[9] A.G. Riess, et al., Astron. J. 116 (1998) 1009.
[10] B. Ratra, P.J.E. Peebles, Phys. Rev. D 37 (1988) 3406;
C. Wetterich, Nucl. Phys. B 302 (1988) 302.
[11] I. Zlatev, L. Wang, P.J. Steinhardt, Phys. Rev. Lett. 82 (1999) 896;
P.J. Steinhardt, L. Wang, I. Zlatev, Phys. Rev. D 59 (1999) 123504.
[12] R.R. Caldwell, Phys. Lett. B 545 (2002) 23;
M. Sami, A. Toporensky, gr-qc/0312009;
M. Szydlowski, W. Czaja, A. Krawiec, astro-ph/0401293;
M. Bouhmadi-Lopez, J.J. Madrid, astro-ph/0404540;
Y.H. Wei, Y. Tian, gr-qc/0405038;
V.K. Onemli, R.P. Woodard, gr-qc/0406098;
S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B 632 (2006) 597;
M.R. Setare, Eur. Phys. J. C 50 (2007) 991.
[13] A. Sen, JHEP 0207 (2002) 065, hep-th/0203265;
T. Padmanabhan, Phys. Rev. D 66 (2002) 021301, hep-th/0204150;
M.R. Setare, Phys. Lett. B 653 (2007) 116.
[14] V.I. Afonso, D. Bazeia, hep-th/0601069.