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Nuclear Physics B 795 (2008) 334–360

www.elsevier.com/locate/nuclphysb

Inflationary de Sitter solutions from superstrings [☆]

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Received 26 August 2007; accepted 21 November 2007

Available online 28 November 2007

Abstract

In the framework of superstring compactifications with $N = 1$ supersymmetry spontaneously broken (by either geometrical fluxes, branes or else), we show the existence of *new inflationary solutions*. The time-trajectory of the scale factor of the metric a , the supersymmetry breaking scale $m \equiv m(\Phi)$ and the temperature T are such that am and aT remain constant. These solutions request the presence of special moduli-fields: (i) The universal “no-scale-modulus” Φ , which appears in all $N = 1$ effective supergravity theories and defines the supersymmetry breaking scale $m(\Phi)$. (ii) The modulus Φ_s , which appears in a *very large class of string compactifications* and has a Φ -dependent kinetic term. During the time evolution, $a^4 \rho_s$ remains constant as well (ρ_s being the energy density induced by the motion of Φ_s). The cosmological term $\Lambda(am)$, the curvature term $k(am, aT)$ and the radiation term $c_R = a^4 \rho$ are dynamically generated in a *controllable way* by radiative and temperature corrections; they are effectively constant during the time evolution. Depending on Λ , k and c_R , either a first or second order phase transition can occur in the cosmological scenario. In the first case, an instantonic Euclidean solution exists and connects via tunneling the inflationary evolution to another cosmological branch. The latter starts with a big bang and, in the case the transition does not occur, ends with a big crunch. In the second case, the big bang and the inflationary phase are smoothly connected.

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[☆] Research partially supported by the EU (under the contracts MRTN-CT-2004-005104, MRTN-CT-2004-512194, MRTN-CT-2004-503369, MEXT-CT-2003-509661), INTAS grant 03-51-6346, CNRS PICS 2530, 3059 and 3747, and ANR (CNRS-USAR) contract 05-BLAN-0079-01.

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1. Introduction

In the framework of superstring and M-theory compactifications, there are always moduli fields coupled in a very special way to the gravitational and matter sector of the effective $N = 1$ four-dimensional supergravity. The gravitational and the scalar field part of the effective Lagrangian have the generic form

$$\mathcal{L} = \sqrt{-\det g} \left[\frac{1}{2} R - g^{\mu\nu} K_{i\bar{j}} \partial_\mu \phi_i \partial_\nu \bar{\phi}_{\bar{j}} - V(\phi_i, \bar{\phi}_{\bar{i}}) \right], \quad (1.1)$$

where $K_{i\bar{j}}$ is the metric of the scalar manifold and V is the scalar potential of the $N = 1$ supergravity. (We will always work in gravitational mass units, with $M = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ GeV.) What will be crucial in this work is the non-triviality of the scalar kinetic terms $K_{i\bar{j}}$ in the $N = 1$ effective supergravity theories that will provide us, in some special cases, accelerating cosmological solutions once the radiative and temperature corrections are taken into account.

Superstring vacua with spontaneously broken supersymmetry [1] that are consistent at the classical level with a flat space–time define a very large class of “no-scale” supergravity models [2]. Those with $N = 1$ spontaneous breaking deserve more attention. Some of them are candidates for describing (at low energy) the physics of the standard model and extend it up to $\mathcal{O}(1)$ TeV energy scale. This class of models contains an enormous number of consistent string vacua that can be constructed either via freely acting orbifolds [1,3] or “geometrical fluxes” [4] in heterotic string and type IIA, B orientifolds, or with non-geometrical fluxes [5] (e.g. RR-fluxes or else).

Despite the plethora of this type of vacua, an interesting class of them are those which are described by an effective $N = 1$ “no-scale supergravity theory”. Namely the vacua in which the supersymmetry is spontaneously broken with a vanishing classical potential with undetermined gravitino mass due to at least one flat field direction, the “no-scale” modulus Φ . At the quantum level a non-trivial effective potential is radiatively generated which may or may not stabilize the “no-scale” modulus [2].

What we will explore in this work are the universal scaling properties of the “thermal” effective potential at finite temperature that emerges at the quantum level of the theory. As we will show in Section 4, the quantum and thermal corrections are under control (thanks to supersymmetry and to the classical structure of the “no-scale models”), showing interesting scaling properties.

In Section 2, we set up our notations and conventions in the effective $N = 1$ “no-scale” supergravities of the type IIB orientifolds with D_3 -branes and non-trivial NS–NS and RR three form fluxes H^3 and F^3 . We identify the “no-scale” modulus Φ , namely the scalar superpartner of the Goldstino which has the property to couple to the trace of the energy momentum tensor of a subsector of the theory [6]. More importantly, it defines the field-dependence of the gravitino mass [2]

$$m(\Phi) = e^{\alpha\Phi}. \quad (1.2)$$

Other extra moduli that we will consider here are those with Φ -dependent kinetic terms. These moduli appear naturally in all string compactifications [7]. We are in particular interested in scalars (Φ_s) which are leaving on D_3 -branes and whose kinetic terms scale as the inverse volume of the “no-scale” moduli space.

In Section 3, we display the relevant gravitational, fields and thermal equations of motion in the context of a Friedman–Robertson–Walker (FRW) space–time. We actually generalize the

mini-superspace (MSS) action by including fields with non-trivial kinetic terms and a generic, scale factor dependent, thermal effective potential.

In our analysis we restrict ourselves to the large moduli limit, neglecting non-perturbative terms and world-sheet instanton corrections $\mathcal{O}(e^{-S})$, $\mathcal{O}(e^{-T_a})$. On the other hand we keep the perturbative quantum and thermal corrections.

Although this study looks hopeless and out of any systematic control even at the perturbative level, it turns out to be manageable thanks to the initial no-scale structure appearing at the classical level (see Section 4).

In Section 5, we show the existence of a critical solution to the equations of motion that follows from the scaling properties derived in Section 4. We have to stress here that we extremize the effective action by solving the gravitational and moduli equations of motion and do not consider the stationary solutions emerging from a minimization of the effective potential only. We find in particular that a universal solution exists where all scales evolve in time in a similar way, so that their ratios remain constant: $m(t)/T(t) = \text{const}$, $a(t)m(t) = \text{const}$. Along this trajectory, effective time-independent cosmological term Λ , curvature term k and radiation term are generated in the MSS action, characterizing the cosmological evolution.

Obviously, the validity of the cosmological solutions based on (supergravity) effective field theories is limited. For instance, in the framework of more fundamental theories such as string theory, there are high temperature instabilities occurring at $T \simeq T_H$, where T_H is the Hagedorn temperature of order the mass of the first string excited state. To bypass these limitations, one needs to go beyond the effective field theory approach and consider the full string theory (or brane, M-theory, ...) description. Thus, the effective solutions presented in this work are not valid anymore and *must be corrected* for temperatures above T_H . Moreover, Hagedorn-like instabilities can also appear in general in other corners of the moduli space of the fundamental theory, when space–time supersymmetry is spontaneously broken.

Regarding the temperature scale as the inverse radius of the compact Euclidean time, one could conclude that all the internal radii of a higher dimensional fundamental theory have to be above the Hagedorn radius. This would mean that the early time cosmology should be dictated by a 10-dimensional picture rather than a 4-dimensional one where the internal radii are of order the string scale. There is however a loophole in this statement. Indeed, no tachyonic instability is showing up in the whole space of the moduli which are not involved in the spontaneous breaking of supersymmetry, as recently shown in explicit examples [8]. This leads us to the conjecture that the only Hagedorn-like restrictions on the moduli space depend on the supersymmetry breaking. In our cosmological solutions, not only the temperature T scale is varying, but also the supersymmetry breaking scale m , which turns to be a moduli-dependent quantity. Based on the above statements, we expect that in a more accurate stringy description of our analysis, there should be restrictions on the temperature as well as the supersymmetry breaking scale. This has been recently explicitly shown in the stringy examples considered in [8].

In Section 6, our cosmological solutions are generalized by including moduli with other scaling properties of their kinetic terms.

Finally, Section 7 is devoted to our conclusions and perspective for future work.

2. $N = 1$ no-scale SUGRA from type IIB orientifolds

In the presence of branes and fluxes, several moduli can be stabilized. For instance, in “generalized” Calabi–Yau compactifications, either the $h_{1,1}$ Kähler structure moduli or the $h_{2,1}$ complex structure moduli can be stabilized according to the brane and flux configuration in

type IIA or type IIB orientifolds [4–6,9]. The (partial) stabilization of the moduli can lead us at the classical level to AdS like solutions, domain wall solutions or “flat no-scale like solutions”. Here we will concentrate our attention on the “flat no-scale like solutions”.

In order to be more explicit, let us consider as an example the type IIB orientifolds with D₃-branes and non-trivial NS–NS and RR three form fluxes H^3 and F^3 . This particular configuration induces a well known superpotential $W(S, U_a)$ that can stabilize all complex structure moduli U_a and the coupling constant modulus S [4,5]. The remaining $h_{1,1}$ moduli T_a “still remain flat directions at the classical level”, e.g. neglecting world-sheet instanton corrections $\mathcal{O}(e^{-T_a})$ and the perturbative and non-perturbative quantum corrections [5].

It is also well known by now that in the large T_a limit the Kähler potential is given by the intersection numbers d_{abc} of the special geometry of the Calabi–Yau manifold and orbifold compactifications [10,11]:

$$K = -\log d_{abc}(T_a + \bar{T}_a)(T_b + \bar{T}_b)(T_c + \bar{T}_c). \tag{2.1}$$

Thus, after the S and U_a moduli stabilization, the superpotential W is effectively constant and implies a vanishing potential in all T_a directions. The gravitino mass term is however non-trivial [1,2,4,5,11],

$$m^2 = |W|^2 e^K. \tag{2.2}$$

This classical property of “no-scale models” emerges from the cubic form of K in the moduli T_a and is generic in all type IIB orientifold compactifications with D₃-branes and three form H^3 and F^3 fluxes [4,5]. Keeping for simplicity the direction $T_a = \gamma_a T$ (for some constants γ_a) and freezing all other directions, the Kähler potential is taking the well known $SU(1, 1)$ structure [2],

$$K = -3 \log(T + \bar{T}). \tag{2.3}$$

This gives rise to the kinetic term and gravitino mass term,

$$g^{\mu\nu} 3 \frac{\partial_\mu T \partial_\nu \bar{T}}{(T + \bar{T})^2} \quad \text{and} \quad m^2 = c e^K = \frac{c}{(T + \bar{T})^3}, \tag{2.4}$$

where c is a constant. Freezing $\text{Im } T$ and defining the field Φ by

$$e^{2\alpha\Phi} = m^2 = \frac{c}{(T + \bar{T})^3}, \tag{2.5}$$

one finds a kinetic term

$$g^{\mu\nu} 3 \frac{\partial_\mu T \partial_\nu \bar{T}}{(T + \bar{T})^2} = g_{\mu\nu} \frac{\alpha^2}{3} \partial_\mu \Phi \partial_\nu \Phi. \tag{2.6}$$

The choice $\alpha^2 = 3/2$ normalizes canonically the kinetic term of the modulus Φ .

The other extra moduli that we will consider are those with Φ -dependent kinetic terms. We are in particular interested to the scalars whose kinetic terms scale as the inverse volume of the T -moduli. For one of them, Φ_s , one has

$$K_s \equiv -\frac{\alpha^2}{3} e^{2\alpha\Phi} g^{\mu\nu} \partial_\mu \Phi_s \partial_\nu \Phi_s = -\frac{\alpha^2 c}{3} g^{\mu\nu} \frac{\partial_\mu \Phi_s \partial_\nu \Phi_s}{(T + \bar{T})^3}. \tag{2.7}$$

Moduli with this scaling property appear in a very large class of string compactifications. Some examples are:

- (i) All moduli fields leaving in the parallel space of D_3 -branes [4,5].
- (ii) All moduli coming from the twisted sectors of \mathbb{Z}_3 -orbifold compactifications in heterotic string [7], after non-perturbative stabilization of S by gaugino condensation and flux-corrections [12].

Our analysis will also consider other moduli fields with different scaling properties, namely those with kinetic terms of the form:

$$K_w \equiv -\frac{1}{2}e^{(6-w)\alpha\Phi} g^{\mu\nu} \partial_\mu \phi_w \partial_\nu \phi_w, \tag{2.8}$$

with weight $w = 0, 2$ and 6 .

3. Gravitational, moduli and thermal equations

In a fundamental theory, the number of degrees of freedom is important (and actually infinite in the context of string or M-theory). However, in an effective field theory, an ultraviolet cut-off set by the underlying theory determines the number of states to be considered. We focus on cases where these states include the scalar moduli fields Φ and Φ_s , with non-trivial kinetic terms given by

$$\mathcal{L} = \sqrt{-\det g} \left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu} (\partial_\mu \Phi \partial_\nu \Phi + e^{2\alpha\Phi} \partial_\mu \Phi_s \partial_\nu \Phi_s) - V(\Phi, \mu) \right] + \dots \tag{3.1}$$

In this Lagrangian, the “...” denote all the other degrees of freedom, while the effective potential V depends on Φ and the renormalization scale μ . We are looking for gravitational solutions based on isotropic and homogeneous FRW space–time metrics,

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 d\Omega_3^2, \tag{3.2}$$

where Ω_3 is a 3-dimensional compact space with constant curvature k , such as a sphere or an orbifold of hyperbolic space. This defines an effective one-dimensional action, the so-called “mini-super-space” (MSS) action [13–16].

A way to include into the MSS action the quantum fluctuations of the full metric and matter degrees of freedom (and thus taking into account the back-reaction on the space–time metric), is to switch on a thermal bath at temperature T [14–16]. In this way, the remaining degrees of freedom are parameterized by a pressure $P(T, m_i)$ and a density $\rho(T, m_i)$, where m_i are the non-vanishing masses of the theory. Note that P and ρ have an implicit dependence on Φ , through the mass $m(\Phi)$ defined in Eq. (1.2) [6]. The presence of the thermal bath modifies the effective MSS action, including the corrections due to the quantum fluctuations of the degrees of freedom whose masses are below the temperature scale T . The result, together with the fields Φ and Φ_s , reads

$$\begin{aligned} S_{\text{eff}} = & -\frac{|k|^{-\frac{3}{2}}}{6} \int dt a^3 \left(\frac{3}{N} \left(\frac{\dot{a}}{a} \right)^2 - \frac{3kN}{a^2} - \frac{1}{2N} \dot{\Phi}^2 - \frac{1}{2N} e^{2\alpha\Phi} \dot{\Phi}_s^2 \right. \\ & \left. + NV - \frac{1}{2N}(\rho + P) + \frac{N}{2}(\rho - P) \right), \end{aligned} \tag{3.3}$$

where a “dot” denotes a time derivation. $N(t)$ is a gauge dependent function that can be arbitrarily chosen by a redefinition of time. We will always use the gauge $N \equiv 1$, unless it is explicitly specified.

The variation with respect to N gives rise to the Friedman equation,

$$3H^2 = -\frac{3k}{a^2} + \rho + \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}e^{2\alpha\Phi}\dot{\Phi}_s^2 + V, \tag{3.4}$$

where $H = (\dot{a}/a)$.

The other gravitational equation is obtained by varying the action with respect to the scale factor a :

$$2\dot{H} + 3H^2 = -\frac{k}{a^2} - P - \frac{1}{2}\dot{\Phi}^2 - \frac{1}{2}e^{2\alpha\Phi}\dot{\Phi}_s^2 + V + \frac{1}{3}a\frac{\partial V}{\partial a}. \tag{3.5}$$

In the literature, the last term $a(\partial V/\partial a)$ is frequently taken to be zero. However, this is not valid due to the dependence of V on μ , when this scale is chosen appropriately as will be seen in Section 3. We thus keep this term and will see that it plays a crucial role in the derivation of the inflationary solutions under investigation.

We find useful to replace Eq. (3.5) by the linear sum of Eqs. (3.4) and (3.5), so that the kinetic terms of Φ and Φ_s drop out,

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P) + V + \frac{1}{6}a\frac{\partial V}{\partial a}. \tag{3.6}$$

The other field equations are the moduli ones,

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial}{\partial\Phi}\left(V - P - \frac{1}{2}e^{2\alpha\Phi}\dot{\Phi}_s^2\right) = 0 \tag{3.7}$$

and

$$\ddot{\Phi}_s + (3H + 2\alpha\dot{\Phi})\dot{\Phi}_s = 0. \tag{3.8}$$

The last equation (3.8) can be solved immediately,

$$K_s \equiv \frac{1}{2}e^{2\alpha\Phi}\dot{\Phi}_s^2 = C_s\frac{e^{-2\alpha\Phi}}{a^6}, \tag{3.9}$$

where C_s is a positive integration constant. It is important to stress here that we insist to keep in Eq. (3.7) both terms $\partial P/\partial\Phi$ and $\partial K_s/\partial\Phi$ that are however usually omitted in the literature. The first term vanishes *only* under the assumption that all masses are taken to be Φ -independent, while the absence of the second term assumes a trivial kinetic term. However, *both assumptions are not valid in string effective supergravity theories!* (see Section 3).

Finally, we display for completeness the total energy conservation of the system,

$$\frac{d}{dt}\left(\rho + \frac{1}{2}\dot{\Phi}^2 + K_s + V\right) + 3H(\rho + P + \dot{\Phi}^2 + 2K_s) = 0. \tag{3.10}$$

Before closing this section, it is useful to derive some extra useful formulas that are associated to the thermal system. The integrability condition of the second law of thermodynamics reaches, for the thermal quantities ρ and P ,

$$T\frac{\partial P}{\partial T} = \rho + P. \tag{3.11}$$

The fact that these quantities are four-dimensional implies

$$\left(m_i\frac{\partial}{\partial m_i} + T\frac{\partial}{\partial T}\right)\rho = 4\rho \quad \text{and} \quad \left(m_i\frac{\partial}{\partial m_i} + T\frac{\partial}{\partial T}\right)P = 4P. \tag{3.12}$$

Then, the second Eq. (3.12) together with Eq. (3.11) implies [6]:

$$m_i \frac{\partial P}{\partial m_i} = -(\rho - 3P). \tag{3.13}$$

Among the non-vanishing m_i , let us denote with “hat-indices” the masses $m_{\hat{i}}$ that are Φ -independent, and with “tild-indices” the masses $m_{\tilde{i}}$ that have the following Φ -dependence:

$$\{m_i\} = \{m_{\hat{i}}\} \cup \{m_{\tilde{i}}\} \quad \text{where } m_{\tilde{i}} = c_{\tilde{i}} e^{\alpha\Phi}, \tag{3.14}$$

for some constants $c_{\tilde{i}}$. Then, utilizing Eq. (3.13), we obtain a very fundamental equation involving the modulus field Φ [6],

$$-\frac{\partial P}{\partial \Phi} = \alpha(\tilde{\rho} - 3\tilde{P}), \tag{3.15}$$

where $\tilde{\rho}$ and \tilde{P} are the contributions to ρ and P associated to the states with Φ -dependent masses $m_{\tilde{i}}$. The above equation (3.15) clearly shows that the modulus field Φ couples to the (sub-)trace of the energy momentum tensor associated to the thermal system [6] $\tilde{\rho}$, \tilde{P} of the states with Φ -dependent masses defined in Eq. (3.14). We return to this point in the next section.

4. Effective potential and thermal corrections

In order to find solutions to the coupled gravitational and moduli equations discussed in the previous section, it is necessary to analyze the structure of the scalar potential V and the thermal functions ρ , P . More precisely, we have to specify their dependence on Φ , T , a and Φ_s . Although this analysis looks hopeless in a generic field theory, it is perfectly under control in the string effective no-scale supergravity theories.

Classically the potential V_{cl} is zero along the moduli directions Φ and Φ_s . At the quantum level, it receives radiative and thermal corrections that are given in terms of the effective potential [11], $V(m_i, \mu)$, and in terms of the thermal function, $-P(T, m_i)$. Let us consider both types of corrections.

4.1. Effective potential

The one loop effective potential has the usual form [11,17],

$$V = V_{cl} + \frac{1}{64\pi^2} \text{Str} \mathcal{M}^0 \Lambda_{co}^4 \log \frac{\Lambda_{co}^2}{\mu^2} + \frac{1}{32\pi^2} \text{Str} \mathcal{M}^2 \Lambda_{co}^2 + \frac{1}{64\pi^2} \text{Str} \left(\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\mu^2} \right) + \dots, \tag{4.1}$$

where V_{cl} is the classical part, which vanishes in the string effective “no-scale” supergravity case. An ultraviolet cut-off Λ_{co} is introduced and μ stands for the renormalization scale.

$$\text{Str} \mathcal{M}^n \equiv \sum_I (-)^{2J_I} (2J_I + 1) m_I^n \tag{4.2}$$

is a sum over the n th power of the mass eigenvalues. In our notations, the index I is referring to both massless and massive states (with eventually Φ -dependant masses). The weights account for the numbers of degrees of freedom and the statistics of the spin J_I particles.

The quantum corrections to the vacuum energy with the highest degree of ultraviolet divergence is the Λ_{co}^4 term, whose coefficient $\text{Str } \mathcal{M}^0 = (n^B - n^F)$ is equal to the number of bosonic minus fermionic degrees of freedom. This term is thus always absent in supersymmetric theories since they possess equal numbers of bosonic and fermionic states.

The second most divergent term in Eq. (4.1) is the Λ_{co}^2 contribution proportional to $\text{Str } \mathcal{M}^2$. In the $N = 1$ spontaneously broken supersymmetric theories, it is always proportional to the square of the gravitino mass-term $m(\Phi)^2$,

$$\text{Str } \mathcal{M}^2 = c_2 m(\Phi)^2. \tag{4.3}$$

The coefficient c_2 is a field independent number. It depends only on the geometry of the kinetic terms of the scalar and gauge manifold, and *not on the details of the superpotential* [11,17]. This property is very crucial in our considerations.

The last term has a logarithmic behavior with respect to the infrared scale μ and is independent of the ultraviolet cut-off Λ_{co} . Following the infrared regularization method valid in string theory (and field theory as well) adapted in Ref. [18], the scale μ is proportional to the curvature of the three-dimensional space,

$$\mu = \frac{1}{\gamma a}, \tag{4.4}$$

where γ is a numerical coefficient chosen appropriately according to the renormalization group equation arguments. Another physically equivalent choice for μ is to be proportional to the temperature scale, $\mu = \zeta T$. The curvature choice (4.4) looks more natural and has the advantage to be valid even in the absence of the thermal bath.

Modulo the logarithmic term, the $\text{Str } \mathcal{M}^4$ can be expanded in powers of gravitino mass $m(\Phi)$,

$$\frac{1}{64\pi^2} \text{Str } \mathcal{M}^4 = C_4 m^4 + C_2 m^2 + C_0. \tag{4.5}$$

Including the logarithmic terms and adding the quadratic contribution coming from the $\text{Str } \mathcal{M}^2$, we obtain the following expression for the effective potential organized in powers of $m(\Phi)$:

$$V = V_4(\Phi, a) + V_2(\Phi, a) + V_0(\Phi, a), \tag{4.6}$$

where

$$V_n(\Phi, a) = m^n(\Phi)(C_n + Q_n \log(m(\Phi)\gamma a)), \tag{4.7}$$

for constant coefficients C_n and Q_n ($n = 4, 2, 0$). These contributions satisfy

$$\frac{\partial V_n(\Phi, a)}{\partial \Phi} = \alpha(n V_n + m^n Q_n) \quad \text{and} \quad a \frac{\partial V_n(\Phi, a)}{\partial a} = m^n Q_n. \tag{4.8}$$

The logarithmic dependence in the effective potential can be derived in the effective field theory by considering the renormalization group equations (RGE). They involve the gauge couplings, the Yukawa couplings and the soft-breaking terms [11,19]. These soft-breaking terms are usually parameterized by the gaugino mass terms $M_{1/2}$, the soft scalar masses m_0 , the trilinear coupling mass term A and the analytic mass term B [11,19]. However, what will be of main importance in this work is that *all soft breaking mass terms are proportional to $m(\Phi)$* [11,17].

4.2. Thermal potential

For bosonic (or fermionic) fluctuating states of masses m_b (or m_f) in thermal equilibrium at temperature T , the general expressions of the energy density ρ and pressure P are

$$\begin{aligned} \rho &= T^4 \left(\sum_{\text{boson } b} I_\rho^B \left(\frac{m_b}{T} \right) + \sum_{\text{fermion } f} I_\rho^F \left(\frac{m_f}{T} \right) \right), \\ P &= T^4 \left(\sum_{\text{boson } b} I_P^B \left(\frac{m_b}{T} \right) + \sum_{\text{fermion } f} I_P^F \left(\frac{m_f}{T} \right) \right), \end{aligned} \tag{4.9}$$

where

$$I_\rho^{B(F)} \left(\frac{m}{T} \right) = \int_0^\infty dq \frac{q^2 E(q, \frac{m}{T})}{e^{E(q, \frac{m}{T})} \mp 1}, \quad I_P^{B(F)} = \frac{1}{3} \int_0^\infty dq \frac{q^4 / E(q, \frac{m}{T})}{e^{E(q, \frac{m}{T})} \mp 1} \tag{4.10}$$

and $E(q, \frac{m}{T}) = \sqrt{q^2 + \frac{m^2}{T^2}}$.

There are three distinct sub-sectors of states:

(i) The subsector of n_0^B bosonic and n_0^F fermionic massless states. From Eqs. (4.9) and (4.10), their energy density ρ_0 and pressure P_0 satisfy

$$\rho_0 = 3P_0 = \frac{\pi^4}{15} \left(n_0^B + \frac{7}{8} n_0^F \right) T^4. \tag{4.11}$$

In particular, we have $\rho_0 - 3P_0 = 0$ and $\partial P_0 / \partial \Phi = 0$.

(ii) The subsector of states with non-vanishing masses *independent* of $m(\Phi)$.

• Consider the \hat{n}_0^B bosons and \hat{n}_0^F fermions whose masses we denote by $m_{\hat{i}_0}$ are below T . The energy density $\hat{\rho}$ and pressure \hat{P} associated to them satisfy

$$\hat{\rho}(T, m_{\hat{i}_0}) = \hat{\rho}(T, m_{\hat{i}_0} = 0) + m_{\hat{i}_0}^2 \frac{\partial \hat{\rho}}{\partial m_{\hat{i}_0}^2} = \frac{\pi^4}{15} \left(\hat{n}_0^B + \frac{7}{8} \hat{n}_0^F \right) T^4 - \sum_{\hat{i}_0} \hat{c}_{\hat{i}_0} m_{\hat{i}_0}^2 T^2, \tag{4.12}$$

$$\hat{P}(T, m_{\hat{i}_0}) = \hat{P}(T, m_{\hat{i}_0} = 0) + m_{\hat{i}_0}^2 \frac{\partial \hat{P}}{\partial m_{\hat{i}_0}^2} = \frac{\pi^4}{45} \left(\hat{n}_0^B + \frac{7}{8} \hat{n}_0^F \right) T^4 - \sum_{\hat{i}_0} \hat{c}_{\hat{i}_0} m_{\hat{i}_0}^2 T^2, \tag{4.13}$$

where the $\hat{c}_{\hat{i}_0}$'s are non-vanishing positive constants. In particular, one has $\partial \hat{P} / \partial \Phi = 0$.

• For the masses $m_{\hat{i}}$ above T , the contributions of the particular degrees of freedom are exponentially suppressed and decouple from the thermal system. We are not including their contribution.

(iii) The subsector with non-vanishing masses *proportional* to $m(\Phi)$. Its energy density $\tilde{\rho}$ and pressure \tilde{P} satisfy

$$\frac{\partial P}{\partial \Phi} = -\alpha(\tilde{\rho} - 3\tilde{P}), \tag{4.14}$$

as was shown at the end of Section 2. This identity is also valid for the massless system we consider in case (i).

According to the scaling behaviors with respect to T and $m(\Phi)$, we can separate

$$\rho = \rho_4 + \rho_2, \quad P = P_4 + P_2, \tag{4.15}$$

where

$$\left(m(\Phi) \frac{\partial}{\partial m(\Phi)} + T \frac{\partial}{\partial T} \right) (\rho_n, P_n) = n(\rho_n, P_n). \tag{4.16}$$

ρ_4 and P_4 are the sums of the contributions of the massless states (case (i)), the T^4 parts of $\hat{\rho}$ and \hat{P} (case (ii)), and $\tilde{\rho}$ and \tilde{P} (case (iii)),

$$\rho_4 = T^4 \left(\frac{\pi^4}{15} \left((n_0^B + \hat{n}_0^B) + \frac{7}{8} (n_0^F + \hat{n}_0^F) \right) + \sum_{\text{boson } \tilde{b}} I_{\rho}^B \left(\frac{m_{\tilde{b}}}{T} \right) + \sum_{\text{fermion } \tilde{f}} I_{\rho}^F \left(\frac{m_{\tilde{f}}}{T} \right) \right), \tag{4.17}$$

$$P_4 = T^4 \left(\frac{\pi^4}{45} \left((n_0^B + \hat{n}_0^B) + \frac{7}{8} (n_0^F + \hat{n}_0^F) \right) + \sum_{\text{boson } \tilde{b}} I_P^B \left(\frac{m_{\tilde{b}}}{T} \right) + \sum_{\text{fermion } \tilde{f}} I_P^F \left(\frac{m_{\tilde{f}}}{T} \right) \right), \tag{4.18}$$

while ρ_2 and P_2 arise from the T^2 parts of $\hat{\rho}$ and \hat{P} (case (ii)):

$$\rho_2 = P_2 = - \sum_{i_0} \hat{c}_{i_0} m_{i_0}^2 T^2 \equiv -\hat{M}^2 T^2. \tag{4.19}$$

5. Critical solution

The fundamental ingredients in our analysis are the scaling properties of the total effective potential at finite temperature,

$$V_{\text{total}} = V - P. \tag{5.1}$$

Independently of the complication appearing in the radiative and temperature corrected effective potential, the scaling violating terms are under control. Their structure suggests to search for a solution where all the scales of the system, $m(\Phi)$, T and $\mu = (1/\gamma a)$, remain proportional during their evolution in time,

$$e^{\alpha\Phi} \equiv m(\Phi) = \frac{1}{\gamma' a} \implies H = -\alpha \dot{\Phi} \quad \text{and} \quad \xi m(\Phi) = T. \tag{5.2}$$

Our aim is thus to determine the constants γ' and ξ in terms of C_s in Eq. (3.9), γ , and the computable quantities C_n , Q_n ($n = 4, 2, 0$) in string theory, such that the equations of motion for Φ , Φ_s and the gravity are satisfied. On the trajectory (5.2), the contributions V_n ($n = 4, 2, 0$) defined in Eq. (4.7) satisfy

$$V_n = m^n C'_n \quad \text{where} \quad C'_n = C_n + Q_n \log \left(\frac{\gamma}{\gamma'} \right), \tag{5.3}$$

and

$$\frac{\partial V_n}{\partial \Phi} = \alpha m^n (nC'_n + Q_n), \quad a \frac{\partial V_n}{\partial a} = m^n Q_n. \tag{5.4}$$

Also, the contributions of Φ and $1/a^6$ in K_s in Eq. (3.9) conspire to give a global $1/a^4$ dependence,

$$K_s = C_s \frac{\gamma'^2}{a^4}. \tag{5.5}$$

Finally, the sums over the full towers of states with Φ -dependent masses behave in ρ_4/T^4 and P_4/T^4 as pure constants (see Eqs. (4.17) and (4.18)),

$$\rho_4 = r_4 T^4 \quad \text{where}$$

$$r_4 = \frac{\pi^4}{15} \left((n_0^B + \hat{n}_0^B) + \frac{7}{8} (n_0^F + \hat{n}_0^F) \right) + \sum_{\tilde{b}} I_{\rho}^B \left(\frac{\tilde{c}_{\tilde{b}}}{\xi} \right) + \sum_{\tilde{f}} I_{\rho}^F \left(\frac{\tilde{c}_{\tilde{f}}}{\xi} \right), \tag{5.6}$$

$$P_4 = p_4 T^4 \quad \text{where}$$

$$p_4 = \frac{\pi^4}{45} \left((n_0^B + \hat{n}_0^B) + \frac{7}{8} (n_0^F + \hat{n}_0^F) \right) + \sum_{\tilde{b}} I_P^B \left(\frac{\tilde{c}_{\tilde{b}}}{\xi} \right) + \sum_{\tilde{f}} I_P^F \left(\frac{\tilde{c}_{\tilde{f}}}{\xi} \right). \tag{5.7}$$

As a consequence, using Eqs. (4.8) and (4.14), the Φ -equation (3.7) becomes,

$$\begin{aligned} \dot{H} + 3H^2 &= \alpha^2 ((4C'_4 + Q_4)m^4 + (2C'_2 + Q_2)m^2 + Q_0 \\ &\quad + (r_4 - 3p_4)\xi^4 m^4 - 2C_s \gamma'^6 m^4). \end{aligned} \tag{5.8}$$

On the other hand, using Eq. (4.8), the gravity equation (3.6) takes the form

$$\begin{aligned} \dot{H} + 3H^2 &= -2k\gamma'^2 m^2 + \frac{1}{2} (r_4 - p_4)\xi^4 m^4 + (C'_4 m^4 + C'_2 m^2 + C'_0) \\ &\quad + \frac{1}{6} (Q_4 m^4 + Q_2 m^2 + Q_0). \end{aligned} \tag{5.9}$$

The compatibility of the Φ -equation and the gravity equation along the critical trajectory implies an identification of the coefficients of the monomials in m . The constant terms determine C'_0 in term of Q_0

$$C'_0 = \frac{6\alpha^2 - 1}{6} Q_0, \tag{5.10}$$

which amounts to fixing γ' ,

$$\gamma' = \gamma e^{\frac{C_0}{Q_0} - \frac{6\alpha^2 - 1}{6}}. \tag{5.11}$$

The quadratic terms determine the parameter k :

$$k = -\frac{1}{\gamma'^2} \left(\frac{2\alpha^2 - 1}{2} C'_2 + \frac{6\alpha^2 - 1}{12} Q_2 \right). \tag{5.12}$$

Finally, the quartic terms relate ξ to the integration constant C_s appearing in K_s ,

$$C_s = \frac{1}{\gamma'^6} \left(\frac{4\alpha^2 - 1}{2\alpha^2} C'_4 + \frac{6\alpha^2 - 1}{12\alpha^2} Q_4 + \frac{2\alpha^2 - 1}{4\alpha^2} r_4 \xi^4 - \frac{6\alpha^2 - 1}{4\alpha^2} p_4 \xi^4 \right). \tag{5.13}$$

At this point, our choice of ansatz (5.2) and constants γ' , ξ allows to reduce the differential system for Φ_s , Φ and the gravity to the last equation. We thus concentrate on the Friedman equation (3.4) in the background of the critical trajectory $\dot{\Phi}^2 = (H^2/\alpha^2)$,

$$\left(\frac{6\alpha^2 - 1}{6\alpha^2} \right) 3H^2 = -\frac{3k}{a^2} + \rho + \frac{1}{2} e^{2\alpha\Phi} \dot{\Phi}_s^2 + V. \tag{5.14}$$

The dilatation factor in front of $3H^2$ can be absorbed in the definition of λ , \hat{k} and C_R , once we take into account Eqs. (5.10), (5.12) and (5.13),

$$3H^2 = 3\lambda - \frac{3\hat{k}}{a^2} + \frac{C_R}{a^4}, \tag{5.15}$$

where

$$3\lambda = \alpha^2 Q_0, \tag{5.16}$$

$$\hat{k} = \frac{\alpha^2}{\gamma'^2} \left(\frac{2}{6\alpha^2 - 1} \xi^2 \hat{M}^2 - C'_2 - \frac{1}{2} Q_2 \right), \tag{5.17}$$

and

$$C_R = \frac{3}{2\gamma'^4} \left((r_4 - p_4) \xi^4 + 2C'_4 + \frac{1}{3} Q_4 \right). \tag{5.18}$$

We note that for $Q_0 > 0$, λ is positive. In that case, the constraint (5.13) allows us to choose a lower bound for the arbitrary constant C_s , so that ξ^4 is large enough to have $\hat{k} > 0$. This means that the theory is effectively indistinguishable with that of a universe with cosmological constant 3λ , uniform space curvature \hat{k} , and filled with a thermal bath of radiation coupled to gravity. This can be easily seen by considering the Lagrangian

$$\sqrt{-\det g} \left[\frac{1}{2} R - 3\lambda \right] \tag{5.19}$$

and the metric ansatz (3.2), with a 3-space of constant curvature \hat{k} . In the action, one can take into account a uniform space filling bath of massless particles by adding a Lagrangian density proportional to $1/a^4$ (see [14–16]) in the MSS form. One finds

$$S_{\text{eff}} = -\frac{|\hat{k}|^{-\frac{3}{2}}}{6} \int dt N a^3 \left(\frac{3}{N^2} \left(\frac{\dot{a}}{a} \right)^2 + 3\lambda - \frac{3\hat{k}}{a^2} + \frac{C_R}{a^4} \right), \tag{5.20}$$

whose variation with respect to N gives (5.15). Actually, the thermal bath interpretation is allowed as long as $C_R \geq 0$, since the $1/a^4$ term is an energy density. However, in the case under consideration, the effective C_R can be negative due to the m^4 contribution of the effective potential. The general solution of the effective MSS action of Eq. (5.20) with $\lambda > 0$, $\hat{k} > 0$ and $C_R > 0$ was recently investigated in Ref. [16]. It amounts to a thermally deformed de Sitter solution, while the pure radiation case where $\lambda = \hat{k} = 0$, $C_R > 0$ was studied in Ref. [6]. In the latter case, the time trajectory (5.2) was shown to be an attractor at late times, giving rise to a radiation evolving universe with

$$a = \left(\frac{4C_R}{3} \right)^{1/4} t^{1/2}, \quad m(\Phi) = \frac{T}{\xi} = \frac{1}{\gamma' a}. \tag{5.21}$$

Following Ref. [16], the general case with $\lambda > 0$, $\hat{k} > 0$ and $C_R > 0$ gives rise to cosmological scenarios we summarize here. Depending on the quantity

$$\delta_T^2 = \frac{4}{3} \frac{\lambda}{\hat{k}^2} C_R, \tag{5.22}$$

a first or second order phase transition can occur:

$$\delta_T^2 < 1 \iff 1^{\text{st}} \text{ order transition}, \quad \delta_T^2 > 1 \iff 2^{\text{d}} \text{ order transition}. \tag{5.23}$$

(i) *The case $\delta_T^2 < 1$*

There are two cosmological evolutions connected by tunnel effect:

$$a_c(t) = \mathcal{N} \sqrt{\varepsilon + \cosh^2(\sqrt{\lambda}t)}, \quad t \in \mathbb{R}, \tag{5.24}$$

and

$$a_s(t) = \mathcal{N} \sqrt{\varepsilon - \sinh^2(\sqrt{\lambda}t)}, \quad t_i \leq t \leq -t_i, \tag{5.25}$$

where

$$\mathcal{N} = \sqrt{\frac{\hat{k}}{\lambda}} (1 - \delta_T^2)^{1/4}, \quad \varepsilon = \frac{1}{2} \left(\frac{1}{\sqrt{1 - \delta_T^2}} - 1 \right),$$

$$t_i = -\frac{1}{\sqrt{\lambda}} \operatorname{arcsinh} \sqrt{\varepsilon}. \tag{5.26}$$

The “cosh”-solution corresponds to a deformation of a standard de Sitter cosmology, with a contracting phase followed at $t = 0$ by an expanding one. The “sinh”-solution describes a big bang with a growing up space till $t = 0$, followed by a contraction till a big crunch arises. The two evolutions are connected in Euclidean time by a Φ -gravitational instanton

$$a_E(\tau) = \mathcal{N} \sqrt{\varepsilon + \cos^2(\sqrt{\lambda}\tau)}, \quad \Phi_E(\tau) = -\frac{1}{\alpha} \log(\gamma' a_E(\tau)). \tag{5.27}$$

The cosmological scenario is thus starting with a big bang at $t_i = -\frac{1}{\sqrt{\lambda}} \operatorname{arcsinh} \sqrt{\varepsilon}$ and expands up to $t = 0$, following the “sinh”-evolution. At this time, performing the analytic continuation $t = -i(\pi/2\sqrt{\lambda} + \tau)$ reaches (5.27) (where τ is chosen in the range³ $-\pi/2\sqrt{\lambda} \leq \tau \leq 0$). At $\tau = 0$, a different analytic continuation to real time exists, $\tau = it$, that gives rise to the inflationary phase of the “cosh”-evolution, for $t \geq 0$ (see Fig. 1).

There are thus two possible behaviors when $t = 0$ is reached. Either the universe carries on its “sinh”-evolution and starts to contract, or a first order transition occurs and the universe enters into the inflationary phase of the “cosh”-evolution. In that case, the scale factor jumps instantaneously from a_- to a_+ at $t = 0$,

$$a_- = \sqrt{\frac{\hat{k}}{2\lambda} \left(1 - \sqrt{1 - \delta_T^2} \right)} \longrightarrow a_+ = \sqrt{\frac{\hat{k}}{2\lambda} \left(1 + \sqrt{1 - \delta_T^2} \right)}. \tag{5.28}$$

An estimate of the transition probability is given by

$$p \propto e^{-2S_{E\text{eff}}}, \tag{5.29}$$

where $S_{E\text{eff}}$ is the Euclidean action computed with the instanton solution (5.27), for $\tau \in [-\pi/2\sqrt{\lambda}, 0]$. Actually, following Refs. [15,16], one has:

$$S_{E\text{eff}} = -\frac{1}{3\lambda} \sqrt{\frac{1 + \sqrt{1 - \delta_T^2}}{2}} \left(E(u) - \left(1 - \sqrt{1 - \delta_T^2} \right) K(u) \right), \tag{5.30}$$

³ It is also possible to consider the instantons associated to the ranges $\sqrt{\lambda}\tau \in [-(2n + 1)\pi/2, 0]$, $n \in \mathbb{N}$, see [16].

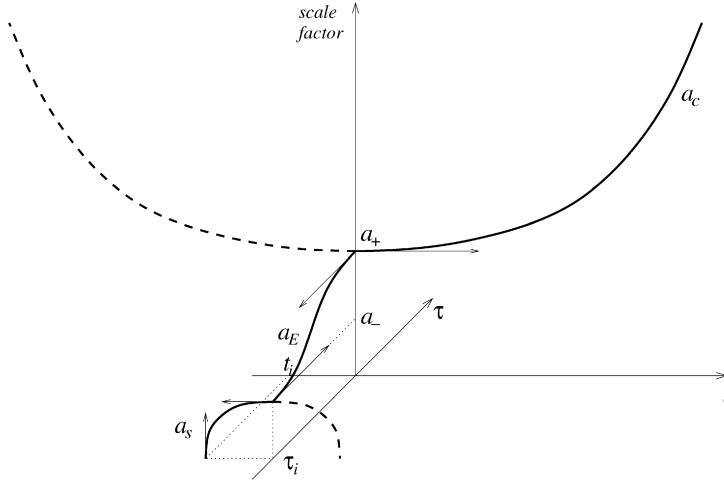


Fig. 1. A first order phase transition can occur. The two cosmological evolutions a_s and a_c are connected by an instanton a_E . The universe starts with a big bang at $t = t_i$ and expands till $t = 0$ along the solution a_s . Then, the scale factor can either contract, or jump instantaneously and enter into the inflationary phase of a_c .

where K and E are the complete elliptic integrals of first and second kind, respectively, and

$$u = \frac{2(1 - \delta_T^2)^{1/4}}{\sqrt{1 + \sqrt{1 - \delta_T^2}}}. \tag{5.31}$$

(ii) The case $\delta_T^2 > 1$

There is a cosmological solution,

$$a(t) = \sqrt{\frac{\hat{k}}{2\lambda}} \sqrt{1 + \sqrt{\delta_T^2 - 1} \sinh(2\sqrt{\lambda}t)}, \quad t \geq t_i, \tag{5.32}$$

where

$$t_i = -\frac{1}{2\sqrt{\lambda}} \operatorname{arcsinh} \frac{1}{\sqrt{\delta_T^2 - 1}}. \tag{5.33}$$

As in the previous case, it starts with a big bang. However, the behavior evolves toward the inflationary phase in a smooth way (see Fig. 2). The transition can be associated to the inflection point arising at t_{inf} , where $a(t_{\text{inf}}) = a_{\text{inf}}$,

$$t_{\text{inf}} = \frac{1}{2\sqrt{\lambda}} \operatorname{arcsinh} \sqrt{\frac{\delta_T - 1}{\delta_T + 1}}, \quad a_{\text{inf}} = \sqrt{\frac{\hat{k}\delta_T}{2\lambda}}. \tag{5.34}$$

Another solution, obtained by time reversal $t \rightarrow -t$, describes a contracting universe that is ending in a big crunch.

(iii) The case $\delta_T^2 = 1$

There is a static solution,

$$a(t) \equiv a_0 \quad \text{where } a_0 = \sqrt{\frac{\hat{k}}{2\lambda}}, \tag{5.35}$$

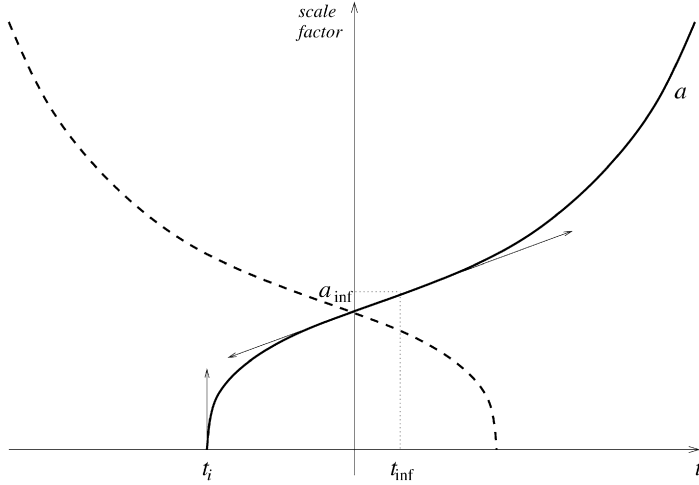


Fig. 2. A second order phase transition occurs. The universe starts with a big bang at $t = t_i$ and evolves smoothly toward the inflationary phase.

corresponding to an S^3 universe of constant radius. This trivial behavior can be reached from the cases (i) and (ii) by taking the limit $\delta_T^2 \rightarrow 1$. Beside it, there are two expanding cosmological evolutions,

$$a_{<}(t) = a_0 \sqrt{1 - e^{-2\sqrt{\lambda}t}}, \quad t \geq 0, \tag{5.36}$$

and

$$a_{>}(t) = a_0 \sqrt{1 + e^{2\sqrt{\lambda}t}}, \quad t \in \mathbb{R}. \tag{5.37}$$

The first one starts with a big bang at $t = 0$, while the second is inflationary. Both are asymptotic to the static one (see Fig. 3). Contracting universes are described by the solutions obtained under the transformation $t \rightarrow -t$.

6. Inclusion of moduli with other scaling properties

We would like to consider generalizations of the previous set up. They are consisting in the inclusion of moduli fields with kinetic terms obeying different scaling properties with respect to Φ . Namely, we take into account the effects of the class of moduli with Lagrangian density

$$\sqrt{-\det g} \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi_6 \partial_\nu \phi_6 + e^{4\alpha\Phi} \partial_\mu \phi_2 \partial_\nu \phi_2 + e^{6\alpha\Phi} \partial_\mu \phi_0 \partial_\nu \phi_0), \tag{6.1}$$

to be added to (3.1). With the metric ansatz (3.2), the MSS action (3.3) is completed by the contributions of the ϕ_w 's ($w = 0, 2, 6$),

$$-\frac{|k|^{-\frac{3}{2}}}{6} \int dt a^3 \left(-\frac{1}{2N} \dot{\phi}_6^2 - \frac{1}{2N} e^{4\alpha\Phi} \dot{\phi}_2^2 - \frac{1}{2N} e^{6\alpha\Phi} \dot{\phi}_0^2 \right). \tag{6.2}$$

The equation of motion for Φ has now terms arising from ϕ_2 and ϕ_0 ,

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial}{\partial\Phi} \left(V - P - \frac{1}{2} e^{2\alpha\Phi} \dot{\phi}_s^2 - \frac{1}{2} e^{4\alpha\Phi} \dot{\phi}_2^2 - \frac{1}{2} e^{6\alpha\Phi} \dot{\phi}_0^2 \right) = 0, \tag{6.3}$$

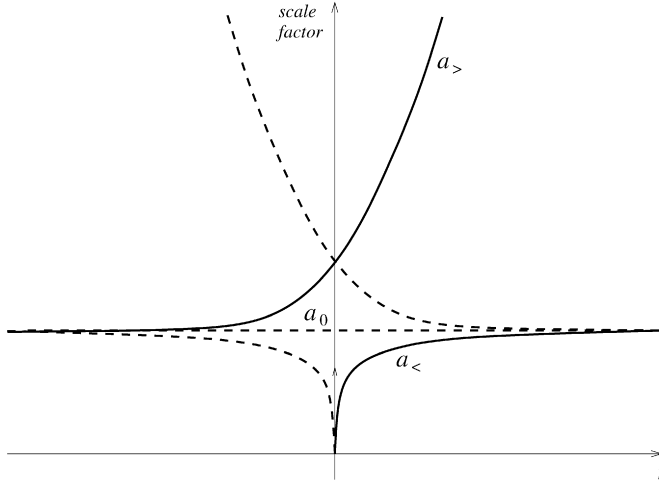


Fig. 3. There are two expanding cosmological evolutions. The first one, $a_<$, starts with a big bang and converges quickly to the static solution. The second one, $a_>$, is almost static for negative time and inflationary for positive time.

while Φ_s and ϕ_w satisfy

$$\ddot{\Phi}_s + (3H + 2\alpha\dot{\Phi})\dot{\Phi}_s = 0, \quad \ddot{\phi}_w + (3H + (6-w)\alpha\dot{\Phi})\dot{\phi}_w = 0. \tag{6.4}$$

Eqs. (6.4) are trivially solved,

$$K_s \equiv \frac{1}{2}e^{2\alpha\Phi}\dot{\Phi}_s^2 = C_s \frac{e^{-2\alpha\Phi}}{a^6}, \quad K_w \equiv \frac{1}{2}e^{(6-w)\alpha\Phi}\dot{\phi}_w^2 = C_{\phi_w} \frac{e^{-(6-w)\alpha\Phi}}{a^6}, \tag{6.5}$$

where C_s and the C_{ϕ_w} 's are positive constants. The equivalent equations of motion for N and the scale factor a have new contributions from the kinetic terms of ϕ_w ($w = 0, 2, 6$). However, these additional terms cancel out from the linear sum of the two equations and thus (3.6) remains invariant.

On the critical trajectory (5.2), one has

$$K_s = C_s \frac{\gamma'^2}{a^4}, \quad K_w = C_{\phi_w} \frac{\gamma'^{(6-w)}}{a^w}. \tag{6.6}$$

This implies that the new contributions arising in the Φ equation (6.3) have dimensions two and zero and thus do not spoil the possible identification between (6.3) and the gravity equation (3.6). In particular, the $1/a^6$ scaling properties of ϕ_6 play no role at this stage. The Φ -equation becomes

$$\dot{H} + 3H^2 = \alpha^2((4C'_4 + Q_4)m^4 + (2C'_2 + Q_2)m^2 + Q_0 + (r_4 - 3p_4)\xi^4 m^4 - 2C_s\gamma'^6 m^4 - 4C_{\phi_2}\gamma'^6 m^2 - 6C_{\phi_0}\gamma'^6), \tag{6.7}$$

to be compared with Eq. (5.9). The identification of the constant terms implies

$$C'_0 + 6\alpha^2 C_{\phi_0} \gamma'^6 = \frac{6\alpha^2 - 1}{6} Q_0, \tag{6.8}$$

which is an equation for γ' . For $Q_0 \leq 0$, there is always a unique solution for γ' . However, it is interesting to note that for $Q_0 > 0$, there is a range for $C_{\phi_0} > 0$ where there are always two solutions for γ' . This case is thus giving rise to two different critical trajectories. The m^2

contributions impose

$$k = \frac{1}{\gamma'^2} \left(2\alpha^2 C_{\phi_2} \gamma'^6 - \frac{2\alpha^2 - 1}{2} C'_2 + \frac{6\alpha^2 - 1}{12} Q_2 \right). \tag{6.9}$$

This fixes the value of k for any arbitrary C_{ϕ_2} . Finally, the equation implied by the quartic mass terms is identical to the one of the previous section, and relates ξ to C_s . We repeat it here for completeness,

$$C_s = \frac{1}{\gamma'^6} \left(\frac{4\alpha^2 - 1}{2\alpha^2} C'_4 + \frac{6\alpha^2 - 1}{12\alpha^2} Q_4 + \frac{2\alpha^2 - 1}{4\alpha^2} r_4 \xi^4 - \frac{6\alpha^2 - 1}{4\alpha^2} p_4 \xi^4 \right). \tag{6.10}$$

We would like to stress again that for C_s sufficiently large, ξ^4 can take any value other some bound we may wish.

The Friedman equation in the presence of the extra moduli becomes,

$$3H^2 = -\frac{3k}{a^2} + \rho + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} \dot{\phi}_6^2 + \frac{1}{2} e^{2\alpha\Phi} \dot{\phi}_s^2 + \frac{1}{2} e^{4\alpha\Phi} \dot{\phi}_2^2 + \frac{1}{2} e^{6\alpha\Phi} \dot{\phi}_0^2 + V. \tag{6.11}$$

On the critical trajectory where $\dot{\phi}^2 = (H^2/\alpha^2)$, and taking into account Eqs. (6.8), (6.9) and (6.10), one obtains

$$3H^2 = 3\lambda - \frac{3\hat{k}}{a^2} + \frac{C_R}{a^4} + \frac{C_M}{a^6}, \tag{6.12}$$

where

$$3\lambda = \alpha^2 (Q_0 - 6C_{\phi_0} \gamma'^6), \tag{6.13}$$

$$\hat{k} = \frac{\alpha^2}{\gamma'^2} \left(2C_{\phi_2} \gamma'^6 + \frac{2}{6\alpha^2 - 1} \xi^2 \hat{M}^2 - C'_2 - \frac{1}{2} Q_2 \right), \tag{6.14}$$

$$C_R = \frac{3}{2\gamma'^4} \left((r_4 - p_4) \xi^4 + 2C'_4 + \frac{1}{3} Q_4 \right), \tag{6.15}$$

and

$$C_M = \frac{6\alpha^2}{6\alpha^2 - 1} C_{\phi_6}. \tag{6.16}$$

Some observations are in order:

- C_{ϕ_0} gives rise to a negative contribution to the cosmological term 3λ .
- As previously, it is possible to have C_R positive if one wishes, by considering large enough values for ξ^4 . This condition can always be satisfied due to the freedom on C_s in Eq. (6.10). To reach positive values of \hat{k} , one can either consider a large enough ξ^4 or utilize C_{ϕ_2} as a parameter.
- C_M is always positive and is determined by the modulus ϕ_6 only.
- Here also the system can be described by an effective MSS action similar to the one examined in [16],

$$S_{\text{eff}} = -\frac{|\hat{k}|^{-\frac{3}{2}}}{6} \int dt N a^3 \left(\frac{3}{N^2} \left(\frac{\dot{a}}{a} \right)^2 + 3\lambda - \frac{3\hat{k}}{a^2} + \frac{C_R}{a^4} + \frac{C_M}{a^6} \right), \tag{6.17}$$

whose associated Friedman equation is precisely (6.12). Thus, once taking into account the thermal and quantum corrections as well as the effects of the moduli we are considering here, the system admits solutions that cannot be distinguished from the de Sitter cosmology deformed by

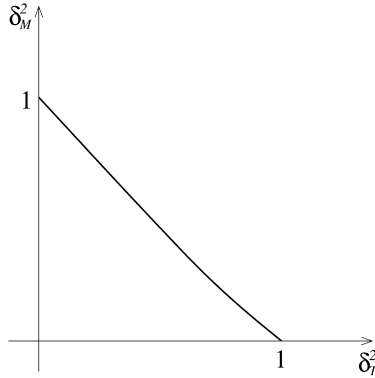


Fig. 4. Phase diagram in the (δ_T^2, δ_M^2) -plane. Inside the “almost triangular” domain (6.19), a first order transition can connect two cosmological solutions by tunnel effect. Outside the domain, there is a single expanding solution (and one contracting) that describes a second order transition. On the boundary, there are two expanding (and two contracting) solutions, beside a static one.

the presence of thermal radiation and time dependent ϕ_6 -moduli fields [16]; this interpretation is valid only when C_R is positive.

Assuming λ, \hat{k}, C_R positive and utilizing the equivalence of the effective MSS action to the thermally and moduli deformed de Sitter action studied in [16], we can immediately derive the general solution of the system under investigation. Our results are summarized as follows (more details can be found in [16]).

For convenience, we choose rescaled parameters δ_T^2, δ_M^2 ,

$$\delta_T^2 = \frac{4}{3} \frac{\lambda}{\hat{k}^2} C_R, \quad \delta_M^2 = \frac{9}{4} \frac{\lambda^2}{\hat{k}^3} C_M, \tag{6.18}$$

and define the domain

$$\delta_T^2 \leq 1 \quad \text{and} \quad \delta_M^2 \leq \frac{1}{2} \left(\sqrt{1 - \frac{3}{4} \delta_T^2} + 1 \right)^2 \left(\sqrt{1 - \frac{3}{4} \delta_T^2} - \frac{1}{2} \right) \tag{6.19}$$

in the (δ_T^2, δ_M^2) -plane, as shown in Fig. 4. The Friedman equation (6.12) admits solutions that involve a first order phase transition inside this domain and a second order one outside it. It is convenient to express these conditions in terms of κ , the only real positive root of the polynomial equation

$$\kappa^3 + \kappa^2 + \frac{\delta_T^2}{4} \kappa - \frac{4}{27} \delta_M^2 = 0, \tag{6.20}$$

by defining

$$\Delta \equiv \frac{4\kappa^2 + 4\kappa + \delta_T^2}{(1 + \kappa)^2} = \frac{16}{27} \frac{\delta_M^2}{\kappa(1 + \kappa)^2}, \tag{6.21}$$

where the second equality in Eq. (6.21) is just a consequence of Eq. (6.20):

$$\Delta < 1 \iff 1^{\text{st}} \text{ order transition}, \quad \Delta > 1 \iff 2^{\text{d}} \text{ order transition}. \tag{6.22}$$

(i) *The case $\Delta < 1$*

There are two cosmological solutions connected via tunneling. The first one takes a simple form in terms of a function $\tilde{t}(t)$,

$$a_c(\tilde{t}(t)) = \mathcal{N} \sqrt{\varepsilon + \cosh^2(\sqrt{\lambda} \tilde{t}(t))}, \tag{6.23}$$

where

$$\mathcal{N} = \sqrt{\frac{\hat{k}(1+\kappa)}{\lambda}} (1-\Delta)^{1/4}, \quad \varepsilon = \frac{1}{2} \left(\frac{1}{\sqrt{1-\Delta}} - 1 \right), \tag{6.24}$$

and $\tilde{t}(t)$ is found by inverting the definition of t as a function of \tilde{t} ,

$$t = \int_0^{\tilde{t}} dv \sqrt{\frac{\cosh^2(\sqrt{\lambda} v) + \varepsilon}{\cosh^2(\sqrt{\lambda} v) + \varepsilon + \tilde{\varepsilon}}} \quad \text{where } \tilde{\varepsilon} = \frac{\kappa}{(1+\kappa)\sqrt{1-\Delta}}. \tag{6.25}$$

In a_c , the variables t and \tilde{t} are arbitrary in \mathbb{R} . The second cosmological evolution is

$$a_s(\tilde{t}(t)) = \mathcal{N} \sqrt{\varepsilon - \sinh^2(\sqrt{\lambda} \tilde{t}(t))} \quad \text{where}$$

$$t = - \int_{\tilde{t}}^0 dv \sqrt{\frac{\varepsilon - \sinh^2(\sqrt{\lambda} v)}{\varepsilon + \tilde{\varepsilon} - \sinh^2(\sqrt{\lambda} v)}}, \tag{6.26}$$

with the range of time

$$\tilde{t}_i \equiv -\frac{1}{\sqrt{\lambda}} \operatorname{arcsinh} \sqrt{\varepsilon} \leq \tilde{t} \leq -\tilde{t}_i, \quad \text{i.e.}$$

$$t_i = - \int_{\tilde{t}_i}^0 dv \sqrt{\frac{\varepsilon - \sinh^2(\sqrt{\lambda} v)}{\varepsilon + \tilde{\varepsilon} - \sinh^2(\sqrt{\lambda} v)}} \leq t \leq -t_i. \tag{6.27}$$

As in the previous section, the “cosh”-solution describes a contracting phase followed by an expanding one and approaches a standard de Sitter cosmology for positive or negative large times. The “sinh”-solution starts with a big bang, ends with a big crunch, while the scale factor reaches its maximum at $t = 0$. The two cosmological solutions are connected by a Φ -gravitational instanton

$$a_E(\tilde{\tau}(\tau)) = \mathcal{N} \sqrt{\varepsilon + \cos^2(\sqrt{\lambda} \tilde{\tau}(\tau))},$$

$$\Phi_E(\tilde{\tau}(\tau)) = -\frac{1}{\alpha} \log(\gamma' a_E(\tilde{\tau}(\tau))), \tag{6.28}$$

where $\tilde{\tau}(\tau)$ is the inverse function of

$$\tau = - \int_{\tilde{\tau}}^0 dv \sqrt{\frac{\cos^2(\sqrt{\lambda} v) + \varepsilon}{\cos^2(\sqrt{\lambda} v) + \varepsilon + \tilde{\varepsilon}}}, \tag{6.29}$$

and the range of Euclidean time is⁴

$$-\frac{\pi}{2\sqrt{\lambda}} \leq \tilde{\tau} \leq 0 \quad \text{i.e.} \quad \tau_i \leq \tau \leq 0 \quad \text{where}$$

$$\tau_i = - \int_{-\frac{\pi}{2\sqrt{\lambda}}}^0 dv \sqrt{\frac{\cos^2(\sqrt{\lambda}v) + \varepsilon}{\cos^2(\sqrt{\lambda}v) + \varepsilon + \tilde{\varepsilon}}}. \tag{6.30}$$

The cosmological scenario starts with an initial singularity at t_i and follows the “sinh”-expansion till $t = 0$. At this time, the solution can be analytically continued to the instantonic one by choosing $\tilde{t} = -i(\pi/2\sqrt{\lambda} + \tilde{\tau})$ i.e. $t = -i(-\tau_i + \tau)$. When the Euclidean time $\tau = 0$ is reached, a second analytic continuation to real time, $\tilde{\tau} = i\tilde{t}$ i.e. $\tau = it$, gives rise to the inflationary phase of the “cosh”-solution, for later times $t \geq 0$ (see Fig. 1). At $t = 0$, the universe has thus two different possible behaviors. It can carry on its evolution along the “sinh”-solution i.e. enter in a phase of contraction. Or, a first order phase transition occurs and the trajectory switches to the “cosh”-evolution. In that case, the scale factor jumps instantaneously from a_- to a_+ ,

$$a_- = \sqrt{\frac{\hat{k}(1 + \kappa)}{2\lambda}(1 - \sqrt{1 - \Delta})} \rightarrow a_+ = \sqrt{\frac{\hat{k}(1 + \kappa)}{2\lambda}(1 + \sqrt{1 - \Delta})}. \tag{6.31}$$

The transition probability is controlled by the Euclidean action, $p \propto e^{-2S_{E \text{ eff}}}$, where $S_{E \text{ eff}}$ has been computed in [16]. For $\sqrt{\lambda}\tilde{\tau} \in [-\pi/2, 0]$, one has

$$S_{E \text{ eff}} = -\frac{1}{3^{5/4}\lambda}(4 - 3\delta_T^2)^{1/4} \sqrt{\sin\left(\frac{\theta + \pi}{3}\right)}$$

$$\times \left(E(u) - \frac{\sqrt{4 - 3\delta_T^2} \cos(\frac{\theta}{3}) - 1 + \frac{3}{2}\delta_T^2}{\sqrt{3}\sqrt{4 - 3\delta_T^2} \sin(\frac{\theta + \pi}{3})} K(u) \right), \tag{6.32}$$

where

$$u = \sqrt{\frac{\sin(\frac{\theta}{3})}{\sin(\frac{\theta + \pi}{3})}}, \quad \theta = \arccos\left(\frac{16\delta_M^2 + 9\delta_T^2 - 8}{(4 - 3\delta_T^2)^{3/2}}\right). \tag{6.33}$$

We have displayed the scale factors a_c and a_s in terms of δ_T^2 and δ_M^2 . However, it is interesting to express the solutions with more intuitive quantities, namely the temperatures T_+ at $t = 0_+$ (along the “cosh”-evolution) and T_- at $t = 0_-$ (along the “sinh”-evolution). Using the fact that $a_{\pm}T_{\pm} = \xi/\gamma'$, one finds

$$T_{\pm}(\delta_T^2, \delta_M^2) = T_m \frac{\sqrt{\delta_T}}{\sqrt{(1 + \kappa)(1 \pm \sqrt{1 - \Delta})}} \quad \text{where} \quad T_m = \frac{\xi}{\gamma'} \sqrt{\frac{2\lambda}{\hat{k}\delta_T}}. \tag{6.34}$$

It is shown in [16] that the cosmological solutions can be written as

$$a_c(\tilde{t}(t)) = \frac{\sqrt{\hat{k}}}{\sqrt{2\lambda(1 - \mathcal{A})}} \sqrt{1 + \frac{T_-^2 - T_+^2}{T_-^2 + T_+^2} \cosh(2\sqrt{\lambda}\tilde{t}(t))}, \quad t > 0 \tag{6.35}$$

⁴ Actually, one can also consider the instantons associated to the ranges $\sqrt{\lambda}\tilde{\tau} \in [-(2n + 1)\pi/2, 0]$, $n \in \mathbb{N}$, see [16].

and

$$a_s(\tilde{t}(t)) = \frac{\sqrt{\hat{k}}}{\sqrt{2\lambda(1-\mathcal{A})}} \sqrt{1 + \frac{T_+^2 - T_-^2}{T_+^4 + T_-^2} \cosh(2\sqrt{\lambda}\tilde{t}(t))}, \quad t < 0, \tag{6.36}$$

where we have defined

$$\mathcal{A} = \frac{1 - T_+^2 T_-^2 / T_m^4}{(T_+ / T_- + T_- / T_+)^2}. \tag{6.37}$$

We note that there is a temperature duality $T_+ \leftrightarrow T_-$ that switches the two cosmological solutions $a_c(\tilde{t}(t))$ for $t > 0$ and $a_s(\tilde{t}(t))$ for $t < 0$ into each other:

$$(T_+ \leftrightarrow T_-) \iff (a_c(\tilde{t}(t)) \text{ for } t > 0 \leftrightarrow a_s(\tilde{t}(t)) \text{ for } t < 0). \tag{6.38}$$

Once quantum and thermal corrections are taken into account, the no-scale supergravities we are considering share a common effective behavior with the thermally and moduli deformed de Sitter evolution (see Eq. (6.12)). This means that the temperature T_m can be defined in both contexts. In Eq. (6.34), T_m is expressed in terms of critical trajectory quantities. However, one can consider the effective $1/a^4$ -radiation energy density in Eq. (6.12) to define the “would-be number” of massless bosonic (fermionic) degrees of freedom, $n_{\text{eff}}^{B(F)}$, of the equivalent deformed de Sitter point of view:

$$\rho_R \equiv \frac{C_R}{a^4} \equiv \frac{\pi^4}{15} \left(n_{\text{eff}}^B + \frac{7}{8} n_{\text{eff}}^F \right) T^4, \tag{6.39}$$

where we have applied the relations (4.9) and (4.10) for massless states. Using Eq. (6.39) and the fact that $aT \equiv a_{\pm} T_{\pm}$, one can rewrite T_m as

$$T_m = \left(\frac{45}{\pi^4} \frac{\lambda}{n_{\text{eff}}^B + \frac{7}{8} n_{\text{eff}}^F} \right)^{1/4}. \tag{6.40}$$

(ii) *The case $\Delta > 1$*

There is an expanding solution we briefly describe (a contracting one is obtained by time reversal $t \rightarrow -t$),

$$a(\tilde{t}(t)) = \sqrt{\frac{\hat{k}(1+\kappa)}{2\lambda}} \sqrt{1 + \sqrt{\Delta - 1} \sinh(2\sqrt{\lambda}\tilde{t}(t))}, \tag{6.41}$$

where t as a function of \tilde{t} is given by

$$t = \int_0^{\tilde{t}} dv \sqrt{\frac{\sqrt{\Delta - 1} \sinh(2\sqrt{\lambda}v) + 1}{\sqrt{\Delta - 1} \sinh(\sqrt{2}\lambda v) + 1 + \frac{2\kappa}{1+\kappa}}} \tag{6.42}$$

and we consider

$$\begin{aligned} \tilde{t} \geq \tilde{t}_i &\equiv -\frac{1}{2\sqrt{\lambda}} \operatorname{arcsinh} \frac{1}{\sqrt{\Delta - 1}} \quad \text{i.e.} \\ t \geq t_i &\equiv -\int_{\tilde{t}_i}^0 dv \sqrt{\frac{\sqrt{\Delta - 1} \sinh(2\sqrt{\lambda}v) + 1}{\sqrt{\Delta - 1} \sinh(\sqrt{2}\lambda v) + 1 + \frac{2\kappa}{1+\kappa}}}. \end{aligned} \tag{6.43}$$

This cosmological solution describes a smooth evolution from a big bang to an inflationary era (see Fig. 2). It has a single inflection point arising when $a = a_{\text{inf}}$, which is defined by the following condition,

$$\frac{\lambda a_{\text{inf}}^2}{\hat{k}^2} > 0 \quad \text{satisfies} \quad x^3 - \frac{\delta_T^2}{4}x - \frac{8}{27}\delta_M^2 = 0. \tag{6.44}$$

(iii) *The case $\Delta = 1$*

Beside the following static solution,

$$a(t) \equiv a_0 \quad \text{where} \quad a_0 = \sqrt{\frac{\hat{k}(1 + \kappa)}{2\lambda}} \equiv \sqrt{\frac{\hat{k}}{3\lambda} \left(1 + \sqrt{1 - \frac{3}{4}\delta_T^2} \right)}, \tag{6.45}$$

the two expanding cosmological evolutions we found for vanishing δ_M^2 are generalized by,

$$a_{<}(\tilde{t}(t)) = a_0 \sqrt{1 - e^{-2\sqrt{\lambda}\tilde{t}(t)}} \quad \text{where} \quad t = \int_0^{\tilde{t}} dv \sqrt{\frac{1 - e^{-2\sqrt{\lambda}v}}{1 + \frac{2\kappa'}{1+\kappa'} - e^{-2\sqrt{\lambda}v}}}, \tag{6.46}$$

for $\tilde{t} \geq 0$ i.e. $t \geq 0$, and

$$a_{>}(\tilde{t}(t)) = a_0 \sqrt{1 + e^{2\sqrt{\lambda}\tilde{t}(t)}} \quad \text{where} \quad t = \int_0^{\tilde{t}} dv \sqrt{\frac{1 + e^{2\sqrt{\lambda}v}}{1 + \frac{2\kappa'}{1+\kappa'} + e^{2\sqrt{\lambda}v}}}, \tag{6.47}$$

for arbitrary \tilde{t} and t . They are monotonically increasing as in the pure thermal case: the first one starts with a big bang and the second one is inflationary (see Fig. 3). Two other solutions obtained under $t \rightarrow -t$ are decreasing. These time-dependent solutions are asymptotic to the static one.

Before concluding let us signal that the number $\hat{n}_0^{B(F)}$ of states with Φ -independent masses below $T(t)$ is not strictly speaking a constant. It is actually lowered by one unit each time the temperature $T(t)$ passes below the mass threshold $m_{\tilde{t}}$ of a boson (fermion). Our critical solutions for the scale factor are thus well defined in any range of time where \hat{n}_0^B and \hat{n}_0^F are constant. The full cosmological scenario is then obtained by gluing one after another these ranges. Each time a mass threshold is passed, the values of $\hat{n}_0^{B(F)}$, \hat{M}^2 , r_4 and p_4 decrease (see Eqs. (4.19), (5.6) and (5.7)), and the parameters of the critical trajectory have to be evaluated again. However, the constraint (6.10) implies that $(\hat{n}_0^B + \frac{7}{8}\hat{n}_0^F)\xi^4$ remains constant, due to the fact that C_s (and any other C_{ϕ_w}) is constant along the full cosmological evolution. This implies that C_R defined in Eq. (6.15) is also invariant. However, the term $\xi^2\hat{M}^2$ and thus \hat{k} can decrease (see Eq. (6.14)). The positivity of \hat{k} can nevertheless be guaranteed by the modulus term $C_{\phi_2}\gamma^6$. Actually, this procedure that is consisting in gluing time intervals as soon as an energy threshold is reached is identical to what is assumed in Standard Cosmology. In the latter case, the full time evolution is divided in different phases (e.g. radiation dominated, matter dominated, and so on).

7. String perspectives and conclusions

At the classical string level, it is well known that it is difficult to construct exact cosmological string solutions. It is even more difficult to obtain de Sitter like inflationary evolutions, even in less than four dimensions.

The main difficulty comes from the fluxes and torsion terms which are created via non-trivial field strength (kinetic terms) and have the tendency to provide negative contributions to the cosmological term, thus anti-de Sitter like vacua. To illustrate a relative issue, consider for instance the Euclidean version of the de Sitter space in three dimensions, dS^3 , which is nothing but the 3-sphere S^3 . Although S^3 can be represented by an exact conformal field theory based on an $SU(2)_k$ WZW model, the latter does not admit any analytic continuation to real time. This is due to the existence of a non-trivial torsion $H_{\mu\nu\rho}$ that becomes imaginary under an analytic continuation [20–22].

This obstruction in string and M-theory is generic and follows from the kinetic origin of the flux terms. A way to bypass this fact is to take into account higher derivative terms [23]. Another strategy is to assume non-trivial effective fluxes coming from negative tension objects such as orientifolds. This idea was explored in the field theory approximation in Ref. [24]. To go further, it is necessary to work with string cosmological backgrounds based on exact conformal field theories. However, the only known exact cosmological solution without the torsion problem described above is that of $SL(2, \mathbb{R})/U(1)_{-|k|} \times K$ [25]. Its Euclidean version is also well defined by the parafermionic T-fold [26,27].

In this work we have implemented a more revolutionary approach. We start with a classical superstring background with spontaneously broken $N = 1$ supersymmetry defined on a flat space–time. The effective field theories associated to these cases are nothing but the $N = 1$ string induced “no-scale supergravity models”. Working at the field theory level, we have shown that the quantum and thermal corrections create dynamically universal effective potential terms that give rise to non-trivial cosmological accelerating solutions.

The main ingredients we have used are the scaling properties of the effective potential at finite temperature in the “no-scale $N = 1$ spontaneously broken supergravities”, once the backgrounds follow critical trajectories where all fundamental scales have a similar evolution in time. Namely, the supersymmetry breaking scale $m(\Phi)$, the inverse of the scale factor a , the temperature T the infrared scale μ remain proportional to each other:

$$e^{\alpha\Phi} \equiv m(\Phi) = \frac{1}{\gamma'a} = \frac{T}{\xi} = \frac{\gamma}{\gamma'}\mu.$$

The “no-scale modulus Φ ” is very special in the sense that it is the superpartner of the goldstino and couples to the trace of the energy momentum of a subsector of the theory. It also provides non-trivial dependences in the kinetic terms of other special moduli of the type:

$$K_w \equiv -\frac{1}{2}e^{(6-w)\alpha\Phi} (\partial\phi_w)^2 \quad (w = 0, 2, 4, 6),$$

where $\phi_4 \equiv \Phi_s$ in the text. The quantum and thermal corrections, together with the non-trivial motion of the special moduli, allow to find thermally and moduli deformed de Sitter evolutions. The cosmological term $3\lambda(am)$, the curvature term $\hat{k}(am, aT)$ and the radiation term $C_R(am, aT)$ (see Eqs. (5.15) or (6.12)), are dynamically generated in a *controllable* way and are effectively constant. Obviously, as stated in the introduction, these solutions are valid below Hagedorn-like scales associated to the temperature as well as the supersymmetry breaking scale m , where instabilities would occur in the extension of our work in a stringy framework. These restrictions on m are supported by the analysis of the string theory examples considered in Ref. [8].

When the deformation of the de Sitter evolution is below some critical value, there are two cosmological solutions which are connected by tunnel effect and interchanged under a temperature duality. The first one describes a big bang with a growing up space till $t = 0$, followed by a

contraction that ends with a big crunch. The second corresponds to a deformation of a standard de Sitter evolution, with a contracting phase followed at $t = 0$ by an expanding one. The universe starts on the big bang cosmological branch and expands up to $t = 0$ along the first solution. At this time, two distinct behaviors can occur. Either the universe starts to contract, or a first order phase transition arises via a Φ -gravitational instanton, toward the inflationary phase of the deformed de Sitter evolution. The transition probability p can be estimated.

If on the contrary the induced cosmology corresponds to a de Sitter-like universe with deformation above the critical value, the previous big bang and inflationary behaviors are smoothly connected via a second order phase transition.

It is of main importance that the field theory approach we developed here can easily be adapted at the string level [8], following the recent progress in understanding the stringy wave-function of the universe [27,28]. This will permit us to go beyond the Hagedorn temperature and understand better the very early “stringy” phase of our universe [29]. At this point, we insist again on a highly interesting question that can be raised concerning the common wisdom which states that all radii-like moduli should be large to avoid Hagedorn-like instabilities. If this statement was true, then the quantum and thermal corrections should be considered in a 10 rather than in a 4-dimensional picture. However, the recent results of Ref. [8] where explicit string models are considered show that this is only valid for the radii-moduli which are participating to the supersymmetry breaking mechanism. The remaining ones are free from any Hagedorn-like instabilities and can take very small values, even of the order of the string scale.

Acknowledgements

We are grateful to Constantin Bachas, Ioannis Bakas, Adel Bilal, Gary Gibbons, John Iliopoulos, Jan Troost and Nicolas Toumbas for discussions.

The work of C.K. and H.P. is partially supported by the EU contract MRTN-CT-2004-005104 and the ANR (CNRS-USAR) contract 05-BLAN-0079-01. C.K. is also supported by the UE contract MRTN-CT-2004-512194, while H.P. is supported by the UE contracts MRTN-CT-2004-503369 and MEXT-CT-2003-509661, INTAS grant 03-51-6346, and CNRS PICS 2530, 3059 and 3747.

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