City Logistics and Traffic Management: Modelling the Inner and Outer Urban Transport Flows in a Two-Tiered System

Rodrigo Rezende Amaral, El-Houssaine Aghezzaf *

Abstract

City logistics deals with the logistical aspects involved in urban freight transport and related operations. These include, in addition to transport, handling and storage of goods, the management of inventory, and all related pickup and delivery operations. Traffic management and control deals with all operational aspects related to the traffic flow and (re)design of infrastructure. These aspects include the possibility to turn normal traffic lanes into reserved bus lanes or to define special routes to be operated only by freight transport. In this paper we discuss and model one of the effective management strategies for city logistics and traffic management in a metropolitan area. This strategy is based on a two-tiered system for which we propose an optimization model, in natural variables, and discuss some of its related challenges and solution approaches. We then propose an instance generator for this optimization model and show the solutions of one of these instances.

© 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
Selection and/or peer-review under responsibility of the Organizing Committee of ISTS'14

Keywords: city logistics; optimization of traffic networks; logistics and traffic flow modelling

* Corresponding author: E.-H. Aghezzaf - Tel.: +33-9-2645500
E-mail address: ElHoussaine.Aghezzaf@ugent.be
1. Introduction

City logistics aims at optimally planning, managing and controlling, in an integrated and coordinated manner, the freight movement within a logistical network in a metropolitan area. The logistical activities and aspects involved in this process include, in addition to transport, handling and storage of goods, the management of inventory and related pickup and delivery operations. An in-depth discussion of the challenges and perspectives in city logistics can be found in Crainic et al. (2009), Benjelloun and Crainic (2008), Benjelloun et al. (2008), Dablanc (2007), Russo and Comi (2004), and Taniguchi et al. (2001). The typical basic optimization problem underlying the city logistics system can be formulated as an enriched inventory routing problem including facility location-allocation features. Variants of this problem attempting to integrate traffic information were formulated as dynamic routing problems with time windows and time dependent travel times (Taniguchi and Shimamoto, 2004). Impact on the environment is also considered, and models have been proposed to measure and minimize CO2 emissions (Taniguchi and Van Der Heijden, 2000, Taniguchi and Thompson, 2002).

Traffic management and control in a metropolitan traffic network is typically formulated as a bi-level optimization problem (Patriksson and Rockafeller, 2002). In the upper level of this non-cooperative Stackelberg game, the traffic manager sets controls (e.g. allowed driving directions, traffic signals and restrictions, speed limits) in order to optimize a given objective such as minimizing the total time spent by all travellers. In the lower level, each traveller is assumed to selfishly optimize his travel choices (mainly route of travel), leading to the well-known Wardrop user equilibrium conditions (Wardrop, 1958). Total traffic demand is usually assumed as given and fixed (i.e. travellers set out for their trip regardless of the traffic conditions or control). Special user classes (e.g. emergency vehicles, public transportation vehicles) can be given higher weights in the objective of the controller, or in practice are usually given full priority. This is however known to substantially increase the delays incurred by other user classes (Furth and Muller, 2002).

Many strategies or systems were proposed for city logistics and traffic management, to efficiently and effectively manage urban freight transport and other traffic flows, with a main objective of achieving an optimal trade-off between ensuring optimum supply network productivity, reliable customer service and reducing environmental impacts, air pollution emissions, energy consumption and traffic congestion. A first strategy involves a single-tier system, in which inbound freight is consolidated at one or several city distribution centres (CDC), located at the city limits, and is then distributed to the customers inside the city (Benjelloun et al., 2009). Single-tier systems, suitable for small to medium-sized cities, implement direct-distribution strategies, serving customers in the city centre by vehicles operating tours starting and finishing at some CDC facility. A second strategy, involving two-tiered systems (Crainic et al., 2004, 2009), typically appropriate for large cities, are based on a so-called consolidation-distribution strategy, which uses a second level of facilities and different vehicle fleets in order to avoid the presence of large vehicles in the city centre, and improve the ratio of load to travelled distance.

This paper considers a two-tiered system and proposes a two-layered optimization model for the urban freight transport, taking delivery time-windows and traffic conditions into account, as well as the replenishment of satellite warehouses and coordination of both flows. We then discuss some properties of the model and possible solution approaches for it.

2. Modelling the urban traffic in the two-tier strategy

The model described in this paper is formulated as a Time Dependent Vehicle Routing Problem with Time Windows (TDVRPTW) composed of two layers and enriched with constraints which reflect the city environment. The outer layer deals with the urban freight transport from the CDCs, situated on the outskirts of the city and therefore referred as external zones, to depots or satellites, which are intermediate facilities strategically located in the interior of the city. The transportation in this layer is performed by urban vehicles, which have relatively high capacities but are not allowed in all areas of the city and cannot directly serve the customers.

The inner layer deals with the transport of the goods from the intermediate facilities to the customers. This is performed by smaller vehicles called city freighters. Urban vehicles and city freighters meet at appointed times at
depots or satellites, where the goods are transferred from the former to the latter vehicles. Intermediate storage may or may not be allowed at a specific depot or satellite. After loaded, city freighters set on delivery routes in order to fulfil the customer demands. Both classes of vehicles consist of heterogeneous fleets, i.e. the urban vehicles may have different costs, capacities etc., and the same holds for the city freighters.

Customers specify the type of products, the demanded volumes and the time windows in which they wish to be served. A customer may specify different types of products and different volumes to be delivered within different time windows, and the volumes may be split and assigned to several city freighters. If splitting occurs, the loads of different city freighters may be delivered at different moments (as long as each moment lies within the specified time window).

The travel time between two points of interest, i.e. customers, depots, satellites and external zones, depends mainly on the distance between the facilities, on the type of the vehicle and the time of the day. For modelling purposes, the planning horizon is discretized into time intervals and the travel times are measured in terms of these intervals. Travel times are not necessarily symmetric and the triangle-inequality conditions cannot be assumed. Loading and unloading times are considered to be smaller than one time interval and are therefore neglected.

In general, urban vehicles are allowed to wait at external zones and at depots when they are not travelling, while city freighters are only allowed to wait at depots. Moreover, vehicles are not allowed to remain at satellites or at customers: in the first case, they must depart as soon as a transfer is completed, and in the second case, they must depart as soon as the products are delivered. External zones, depots, satellites and customers may have limitations on the number of urban vehicles or city freighters simultaneously present on it, and this can vary along the day. It is assumed that the urban vehicles start and end the operations at external zones, while the city freighters start and end at depots.

The objective of the model is to minimize the logistics expenses related to the vehicles, which include fixed and variable operating costs. The fixed operating cost of a vehicle is a constant amount to be charged if the vehicle is assigned for at least one trip during the planning horizon. This amount stands for the costs of making the vehicle available for operation and can include the acquisition, leasing or rental prices; equipment costs; personnel wages and training; depreciation costs; taxes; etc. Variable operating costs are directly related to the use of the vehicle on the roads. In order to model these costs, a certain amount is charged each time a vehicle travels from a point to another in the network. This amount comprises expenses related to fuel; toll fees; distance-related maintenance costs; charges for emissions and other pollution or nuisance issues; etc. The variable operating costs depend on the vehicle and on the road; can be time-dependent; and are not necessarily proportional to the travel time.

2.1. Model description

We consider a supply network with a set of product types \( P = \{p\} \), a set of external zones \( E = \{e\} \), a set of depots \( G = \{g\} \), a set of satellites \( S = \{s\} \), a set of retailers or customers \( C = \{c\} \) and a set of customer demands \( D = \{d\} \), which must be delivered to their respective customers using a set of urban vehicles \( \Omega = \{\omega\} \) and a set of city freighters \( V = \{v\} \). A customer \( c \) may have multiple demands \( d \). The products are always available at any external zone \( e \).

The period available for operations is divided into \( t = 1, \ldots, T_{\text{max}} \) time intervals. Travel times for urban vehicles are represented by \( \delta_{ii}(t) \), which can be interpreted as the duration of a trip (in number of time intervals) from point \( i \) to \( j \) when the urban vehicle \( \omega \in \Omega \) departs from \( i \) at the time interval \( t \). The points \( i \) and \( j \) can be external zones, depots or satellites \( (i, j \in E \cup G \cup S) \). In the same fashion, travel times for city freighters are indicated by \( \delta_{ij}(t) \), where \( i \) and \( j \) are depots, satellites or customers \( (i, j \in G \cup S \cup C) \) and \( v \in V \). The minimum value which \( \delta_{ii}(t) \) and \( \delta_{ij}(t) \) can assume is 1.

In practical situations, it often happens that the urban vehicles and city freighters are individually assigned to cover specific areas of the city. Thus, if a point \( i \in E \cup G \cup S \) is not in the area covered by the urban vehicle \( \omega \in \Omega \), then \( \delta_{ii}(t) = \delta_{ii}(t) = \infty \) for any \( j \in E \cup G \cup S \). Similarly, if a point \( i \in G \cup S \cup C \) is not in the area covered by the city freighter \( v \in V \), then \( \delta_{ii}(t) = \delta_{ii}(t) = \infty \) for any \( j \in G \cup S \cup C \).
If the urban vehicle \( \omega \in \Omega \) is allowed to remain at point \( i \) between \( t \) and \( t + 1 \), then \( \delta_\omega^\omega(t) \) is, by definition, equal 1; otherwise it is infinity. The same approach holds for the city freighters. In general, it follows from these definitions that 
\[
\delta_\omega^\omega(t) = \delta_\omega^\omega(t) = \delta_\omega^\omega(t) = 1 \quad \text{and} \quad \delta_\omega^\omega(t) = \delta_\omega^\omega(t) = \delta_\omega^\omega(t) = \infty
\]
for any \( \omega \in \Omega, v \in V, e \in E, g \in G, s \in S, c \in C \) and \( t \). However, the model allows some flexibility here. For instance, if a certain urban vehicle \( \omega_0 \in \Omega \) is allowed to remain at a certain satellite \( s_0 \) between \( t \) and \( t + 1 \), \( \delta_\omega^\omega_0(t) = 1 \). Similar approaches can be applied for city freighters and customers too.

The maximum number of urban vehicles which are simultaneously allowed at point \( i \in E \cup G \cup S \) at time interval \( t \) is given by \( \pi_\omega^\omega(t) \). For the city freighters, this number is denoted by \( \pi_v^v(t) \) where \( i \in G \cup S \cup C \).

A demand \( d \in D \) is characterized by its customer \( c(d) \in C \), the product type \( p(d) \in P \), the demanded volume \( \text{vol}(d) \), and a time window \([a(d), b(d)]\) in which the customer wishes to receive the product. A urban vehicle \( \omega \in \Omega \) has a capacity \( u^\omega \), a fixed operating cost \( \psi^\omega \) and a set of product types which it may transport \( P(\omega) \). The analogue parameters for a city freighter \( v \in V \) are \( u^v \), \( \psi^v \) and \( P(v) \). The variable operating costs for the two classes of vehicles are denoted by \( k^\omega_{ij}(t) \) and \( k^v_{ij}(t) \), which respectively account for the amount to be charged if the urban vehicle \( \omega \) or the city freighter \( v \) is assigned to travel from point \( i \) to \( j \) starting the trip at the time interval \( t \). In the cases where the vehicles are allowed to remain at a point \( i \) between \( t \) and \( t + 1 \), \( k^\omega_{ij}(t) \) and \( k^v_{ij}(t) \) account for the amount to be charged if it happens (i.e. parking fees, etc.), except when \( t = T_{\text{max}} \). In this case, by definition, \( k^\omega_{ij}(T_{\text{max}}) = k^v_{ij}(T_{\text{max}}) = 0 \).

**Model parameters**

The list below summarises the model parameters:

- \( \Omega = \{ \omega \} \): Set of urban vehicles which take the goods from the external zones (CDCs) and bring them to depots or satellites, where they meet the city freighters and transfer the loads.
- \( V = \{ v \} \): Set of city freighters which receive the goods from the urban vehicles at depots or satellites and deliver them to customers or retailers.
- \( E = \{ e \} \): Set of external zones where the goods are available.
- \( G = \{ g \} \): Set of depots where the city freighters may be parked and where transfers from urban vehicles to city freighters may happen.
- \( S = \{ s \} \): Set of satellites where urban vehicles and city freighters may meet at appointed times in order to transfer goods.
- \( P = \{ p \} \): Set of product types.
- \( C = \{ c \} \): Set of retailers or customers to set which the goods must be delivered.
- \( D = \{ d \} \): Set of customer demands. Each demand \( d \) consists of a product type \( p(d) \in P \) and a volume \( \text{vol}(d) \) which should be delivered to its customer \( c(d) \in C \) during the time window \([a(d), b(d)]\).
- \( P(\omega) \): Set of product types which can be transported by urban vehicle \( \omega \).
- \( u^\omega \): Capacity of the urban vehicle \( \omega \).
- \( \psi^\omega \): Fixed operating cost of the urban vehicle \( \omega \).
- \( k^\omega_{ij}(t) \): Variable operating cost of the urban vehicle \( \omega \), which should be charged when it travels from a point \( i \) to \( j \) at the time interval \( t \) (\( i, j \in E \cup G \cup S \)). Remark: if \( i = j \), it is the cost which should be
charged when \( \omega \) remains at \( i \) for one period starting at time interval \( t \).

- **\( \delta_{ij}^{\omega}(t) \)**: Number of time intervals spent by the urban vehicle \( \omega \) to travel from a point \( i \) to \( j \) departing at the time interval \( t \) (\( i, j \in E \cup G \cup S \)). By definition, when \( i = j \) \( \delta_{ii}^{\omega}(t) = 1 \) if \( \omega \) is allowed to remain at \( i \) and \( \delta_{ij}^{\omega}(t) = \infty \) if it is not.

- **\( \pi_{i}^{\omega}(t) \)**: Maximum number of urban vehicles allowed at a point \( i \in E \cup G \cup S \) during time interval \( t \).

- **\( P(v) \)**: Set of product types which can be transported by the city freighter \( v \).

- **\( u^v \)**: Capacity of the city freighter \( v \).

- **\( \psi_{v} \)**: Fixed operating cost of the city freighter \( v \).

- **\( k_{ij}^{v}(t) \)**: Variable operating cost of the city freighter \( v \), which should be charged when it travels from a point \( i \) to \( j \) at the time interval \( t \) (\( i, j \in G \cup S \cup C \)). Remark: if \( i = j \), it is the cost which should be charged when \( v \) remains at \( i \) for one period starting at time interval \( t \).

- **\( \delta_{ij}^{v}(t) \)**: Number of time intervals spent by the city freighter \( v \) to travel from a point \( i \) to \( j \) departing at the time interval \( t \) (\( i, j \in G \cup S \cup C \)). By definition, when \( i = j \) \( \delta_{ij}^{v}(t) = 1 \) if \( v \) is allowed to remain at \( i \) and \( \delta_{ij}^{v}(t) = \infty \) if it is not.

- **\( \pi_{i}^{v}(t) \)**: Maximum number of city freighters allowed at a point \( i \in G \cup S \cup C \) during time interval \( t \).

### Decision Variables

In order to build the model, the following decision variables were considered:

- **\( y^{v} \)**: A binary variable set to 1 if the city freighter \( v \in V \) is used; and 0 otherwise.

- **\( x_{ij}^{v}(t) \)**: A binary variable set to 1 if the city freighter \( v \in V \) departs from \( i \) towards \( j \) at the time interval \( t \) (\( i, j \in G \cup S \cup C \)); 0 otherwise.

- **\( w_{d}^{v}(t) \)**: A continuous variable representing the quantity of the product from the type \( p(d) \in P \) which is delivered by the city freighter \( v \in V \) to customer \( c(d) \in C \) at the time interval \( t \) in order to satisfy its demand \( d \in D \).

- **\( \lambda_{lp}^{v}(t) \)**: A continuous variable representing the quantity of the product from the type \( p \in P(v) \) which is loaded into the city freighter \( v \in V \) at depot or satellite \( i \in G \cup S \) at time interval \( t \).

- **\( y^{\omega} \)**: A binary variable set to 1 if the urban vehicle \( \omega \in \Omega \) is used; and 0 otherwise.

- **\( x_{ij}^{\omega}(t) \)**: A binary variable set to 1 if the urban vehicle \( \omega \in \Omega \) departs from \( i \) towards \( j \) at the time interval \( t \) (\( i, j \in E \cup G \cup S \)); 0 otherwise.

- **\( q_{pe}^{\omega}(t) \)**: Volume of product \( p \) which is loaded in urban vehicle \( \omega \) at external zone \( e \) during the time interval \( t \).

- **\( \lambda_{lp}^{\omega}(t) \)**: A continuous variable representing the quantity of the product from the type \( p \in P(\omega) \) which is unloaded from the urban vehicle \( \omega \in \Omega \) at depot or satellite \( i \in G \cup S \) at time interval \( t \).
Decision variables containing indices $i$, $j$ and $t$ for which the corresponding $\delta_i^j(t)$ or $\delta_i^o(t)$ is infinity can be eliminated from the problem (except for $t = T_{\text{max}}$). Furthermore, it is worth mentioning that setting a variable $x_i^j(t)$ (or $x_i^o(t)$) to 1 does not mean that a real departure will take place. Instead, it indicates that the vehicle $\alpha$ (or $v$) will remain at point $i$ between $t$ and $t + \delta_i^j(t)$ (or $t + \delta_i^o(t)$) at the cost of $k_i^j(t)$ (or $k_i^o(t)$). In practice, it will only be possible if the vehicle is allowed to remain at point $i$, i.e. if $\delta_i^j(t) = 1$ (or $\delta_i^o(t) = 1$).

The variables $w_i^d(t)$ for which $t < a(d)$ or $t > b(d)$ may be eliminated, since the customer demands must be fulfilled within its time window. We assume urban vehicles and city freighters start and end the operations empty, and therefore no transfers or deliveries can happen during the first and the last periods. As a consequence, the variables $\lambda_i^a(1), \lambda_i^o(T_{\text{max}}), \lambda_i^v(1), \lambda_i^v(T_{\text{max}}), w_i^a(1)$ and $w_i^v(T_{\text{max}})$ may be eliminated too.

**Objective Function**

The objective is to minimize the sum of fixed and variable costs for operating the vehicles.

$$\text{Minimize } Z = \sum_{v \in V} \psi^v y^v + \sum_{\alpha \in E} \psi^o v^o + \sum_{t \in T} \sum_{i \in C} \sum_{j \in C^+} \sum_{\alpha \in E} k_i^j(t)x_i^j(t) + \sum_{t \in T} \sum_{i \in C^+} \sum_{j \in C} k_i^o(t)x_i^o(t)$$

where $C^+ = G \cup S \cup C, E^+ = E \cup G \cup S$

**Constraints**

- **Set of constraints (1V)**

$$\sum_{i \in C^+} \sum_{j \in C} x_i^j(t_0) \leq y^v$$

for every $v_0 \in V$, $t_0 = 1, ..., T_{\text{max}}$

A city freighter can only depart from one point at a given time interval.

- **Set of constraints (2V)**

$$\sum_{g \in G} \sum_{i \in C^+} x_i^g(1) = y^v$$

for every $v_0 \in V$

$$\sum_{g \in G} x_i^g(T_{\text{max}}) = y^v$$

for every $v_0 \in V$

The city freighters must start and end their operation at depots. Note that the first group of equations do not force the city freighter $v_0$ to actually leave its initial depot, since $i \in C^+$ can equal $g$ (if $i = g$, the vehicle remains at $g$). Note also that the second group of equations do not indicate real trips starting at $T_{\text{max}}$, but simply require that the city freighter is at one of the depots at $T_{\text{max}}$.

In some situations it may be desired to fix the initial or the final depot where a city freighter must start its operation. In this case, the above-mentioned constraints should be replaced by:
if \( v_0 \in V \) must start at \( g_0 \in G \)

\[
\sum_{i \in C^+} x^v_{g_0i}(1) = y^v_0
\]

if \( v_0 \in V \) must end at \( g_0 \in G \)

\[
x^v_{g_0g_0}(T_{\text{max}}) = y^v_0
\]

- **Set of constraints (3V)**

\[
\min\left(\delta^v_{i_0j_0}(t_0) - 1, T_{\text{max}} - t_0\right) \\
\sum_{i \in C^+} \sum_{j \in C^+} x^v_{ij}(t_0 + \Delta) - \left[\delta^v_{i_0j_0}(t_0) - 1\right] \left[1 - x^v_{i_0j_0}(t_0)\right]
\]

for every \( v_0 \in V, j_0 \in C^+, i_0 \in C^+, t_0 = 1, \ldots, T_{\text{max}} \)

If a city freighter \( v_0 \in V \) departs from certain point \( i_0 \) towards \( j_0 \) (\( i_0, j_0 \) can be customers, depots or satellites) at the time interval \( t_0 \), it cannot depart from anywhere else during the trip, which comprehends the time interval between \( t_0 + 1 \) until \( t_0 + \delta^v_{i_0j_0}(t_0) - 1 \).

- **Set of constraints (4V)**

\[
\sum_{i \in C^+} \sum_{\theta \in \Theta^v_{ij_0}(t_0)} x^v_{ij}(\theta) = \sum_{k \in C^+} x^v_{ij_k}(t_0)
\]

for every \( v_0 \in V, j_0 \in C^+, t_0 = 2, \ldots, T_{\text{max}} \)

where \( \Theta^v_{ij_0}(t) = \{\theta | 1 \leq t \leq T_{\text{max}} \} | \delta^v_{ij_0}(\theta) = t - \theta \} \)

\( \Theta^v_{ij_0}(t) \) is the set of time intervals in which the city freighter \( v \) can leave \( i \) towards \( j \) (\( i, j \in C^+ \)) and arrive at the destination at time interval \( t \). If a city freighter \( v_0 \in V \) reaches a point \( j_0 \in C^+ \) at \( t_0 \), either the variable \( x^v_{i_0j_0}(t_0) \) must be set to 1 indicating that \( v_0 \) will remain at \( j_0 \) until \( t_0 + \delta^v_{i_0j_0}(t_0) \), or a variable \( x^v_{i_0j_k}(t_0) \) with \( k \in C^+, k \neq j_0 \) must be set to 1 indicating that \( v_0 \) will immediately leave \( j_0 \) towards \( k \) at \( t_0 \). These constraints ensure the continuity of city freighters’ tours. They do not apply for \( t_0 = 1 \), since \( \Theta^v_{ij_0}(1) = 0 \) for any \( i, j \) and \( v \), but they apply for \( t_0 = T_{\text{max}} \), although the variable \( x^v_{i_0j_0}(T_{\text{max}}) \) does not indicate a real trip, but simply determine that the vehicle ends the operations at \( i \).

- **Set of constraints (5V)**

\[
\sum_{v \in V} \sum_{i \in C^+} x^v_{g_0i}(1) \leq \pi^v_{g_0}(1)
\]

for every \( g_0 \in G \)

\[
\sum_{v \in V} \sum_{i \in C^+} \sum_{\theta \in \Theta^v_{ij_0}(t_0)} x^v_{ij_0}(\theta) \leq \pi^v_{ij_0}(t_0)
\]

for every \( j_0 \in C^+, t_0 = 2, \ldots, T_{\text{max}} \)
These constraints ensure that the maximum number of city freighters which are allowed to be at a depot, satellite or customer is respected.

**Set of constraints (6V)**

\[
\sum_{d \in D(c_0, t_0)} w^v_{d}(t_0) \leq u^v \left( \sum_{i \in C^+} \sum_{\theta \in \Theta^v_{ic_0}(t_0)} x^v_{ic_0}(\theta) \right)
\]

for every \(v_0 \in V, c_0 \in C, t_0 = 1, ..., T_{\text{max}}\)

where \(D(c_0, t_0) = \{d \in D|c(d) = c_0, a(d) \leq t_0 \leq b(d)\}\)

\[
\sum_{p \in P(v_0)} \lambda^v_{jp_0}(t_0) \leq u^v \left( \sum_{i \in C^+} \sum_{\theta \in \Theta^v_{jp_0}(t_0)} x^v_{jp_0}(\theta) \right)
\]

for every \(v_0 \in V, j_0 \in G^+, t_0 = 1, ..., T_{\text{max}}\)

where \(G^+ = G \cup S\)

A city freighter \(v_0 \in V\) can only deliver some volume \(\sum_{d \in D(c_0,t_0)} w^v_{d}(t_0) > 0\) to a customer \(c_0 \in C\) at \(t_0\) if it is there at that time interval. It means that \(v_0\) must have departed from some point \(i \in C^+\) towards \(c_0\) at some \(\theta \in \Theta^v_{ic_0}(t_0)\). By the same token, \(v_0\) can only be loaded with some volume \(\sum_{p \in P(v_0)} \lambda^v_{jp_0}(t_0) > 0\) if it is at the depot or satellite \(j_0 \in G^+\) at the time interval \(t_0\), which means it must have departed from some point \(i \in C^+\) towards \(j_0\) at some \(\theta \in \Theta^v_{jp_0}(t_0)\).

**Set of constraints (7V)**

\[
0 \leq \sum_{\Delta = 0}^{t_o-1} \sum_{p \in P(v_0), j \in G^+} \lambda^v_{jp_0}(t_0 - \Delta) - \sum_{\Delta = 0}^{t_o-1} \sum_{d \in D(t_0-\Delta)} w^v_{d}(t_0 - \Delta) \leq u^v
\]

for every \(v_0 \in V, t_0 = 1, ..., T_{\text{max}}\)

where \(D(t) = \{d \in D|a(d) \leq t \leq b(d)\}\)

The city freighter load at a time interval is equal sum of all volumes which may have been previously loaded into it minus the sum of all volumes which may have been unloaded from it. This value must always be between zero and its capacity.

**Set of constraints (8V)**

\[
\sum_{d \in D(p_0,t_0)} \sum_{\Delta = 0}^{t_o-a(d)} w^v_{d}(t_0 - \Delta) \leq \sum_{\Delta = 0}^{t_o-1} \sum_{j \in G^+} \lambda^v_{jp_0}(t_0 - \Delta)
\]

for every \(p_0 \in P, v_0 \in V(p_0), t_0 = 1, ..., T_{\text{max}}\)

where \(D(p_0, t_0) = \{d \in D|p(d) = p_0, t_0 \leq b(d)\}\), \(V(p_0) = \{v \in V|p_0 \in P(v)\}\)
A city freighter can only deliver some volume in order to fulfill a demand (entirely or partially) if it is sufficiently loaded with the corresponding product.

- *Set of constraints (9V)*

\[
\sum_{t=a(d_0)}^{b(d_0)} \sum_{v \in V} w_{d_0}^v(t) = \text{vol}(d_0)
\]

for every \(d_0 \in D\)

The sum of the quantities delivered by all city freighters to fulfill a demand \(d_0 \in D\) in the interval \([a(d_0), b(d_0)]\) must equal \(\text{vol}(d_0)\).

- *Set of constraints (1Ω)*

\[
\sum_{l \in E^+} \sum_{e \in E^+} x_{ij}^{\omega_0}(t_0) \leq y^{\omega_0}
\]

for every \(\omega_0 \in \Omega, t_0 = 1, ..., T_{\text{max}}\)

A urban vehicle can only depart from one point at a given time interval.

- *Set of constraints (2Ω)*

\[
\sum_{e \in E} \sum_{i \in E^+} x_{i0}^{\omega_0}(1) = y^{\omega_0}
\]

for every \(\omega_0 \in \Omega\)

\[
\sum_{e \in E} x_{ee}^{\omega_0}(T_{\text{max}}) = y^{\omega_0}
\]

for every \(\omega_0 \in \Omega\)

The urban vehicles must start and end their operation at external zones. Note that the first group of equations do not force the urban vehicle \(\omega_0\) to actually leave its initial external zone, since \(i \in E^+\) can equal \(e\) (if \(i = e\), the vehicle remains at \(e\)). Note also that the second group of equations do not indicate real trips starting at \(T_{\text{max}}\), but simply require that the urban vehicle is at one of the external zones at \(T_{\text{max}}\).

In some situations it may be desired to fix the initial or the final external zone where a urban vehicle must start its operation. In this case, the above-mentioned constraints should be replaced by:

\[
\sum_{i \in E^+} x_{i0}^{\omega_0}(1) = y^{\omega_0}
\]

if \(\omega_0 \in \Omega\) must start at \(e_0 \in E\)

\[
x_{e0}^{\omega_0}(T_{\text{max}}) = y^{\omega_0}
\]

if \(\omega_0 \in \Omega\) must end at \(e_0 \in E\)
Set of constraints (3Ω)

$$
\min(\delta^{\omega_0}_{l_0l_0}(t_0) - 1, T_{\text{max}} - t_0)
\sum_{\Delta=1}^{\sum_{i \in E^+} \sum_{j \in E^+}} x^{\omega_0}_{ij}(t_0 + \Delta) \leq \left[ \delta^{\omega_0}_{l_0l_0}(t_0) - 1 \right] \left[ 1 - x^{\omega_0}_{l_0l_0}(t_0) \right]
$$

for every $\omega_0 \in \Omega$, $i_0 \in E^+$, $j_0 \in E^+$, $t_0 = 1, ..., T_{\text{max}}$

If a urban vehicle $\omega_0 \in \Omega$ departs from certain point $i_0$ towards $j_0$ ($i_0, j_0$ can be external zones or depots) at the time interval $t_0$, it cannot depart from anywhere else during the trip, which comprehends the time interval between $t_0 + 1$ until $t_0 + \delta^{\omega_0}_{l_0l_0}(t_0) - 1$.

Set of constraints (4Ω)

$$
\sum_{i \in E^+} \sum_{\theta \in \Theta^{\omega_0}_{ij}(t_0)} x^{\omega_0}_{ij}(t_0) = \sum_{k \in E^+} x^{\omega_0}_{ik}(t_0)
$$

for every $\omega_0 \in \Omega$, $j_0 \in E^+$, $t_0 = 2, ..., T_{\text{max}}$

where $\Theta^{\omega_0}_{ij}(t) = \{ \theta \in \mathbb{1} : T_{\text{max}} | \delta^{\omega_0}_{ij}(\theta) = t - \theta \}$

$\Theta^{\omega_0}_{ij}(t)$ is the set of time intervals in which vehicle $\omega_0$ can leave $i$ towards $j$ ($i, j \in E^+$) and arrive at the destination at time interval $t$. $\Theta^{\omega_0}_{ij}(t)$ is directly derived from $\delta^{\omega_0}_{ij}(t)$. If a urban vehicle $\omega_0 \in \Omega$ reaches a point $j_0 \in E^+$ at $t_0$, either the variable $x^{\omega_0}_{l_0j_0}(t_0)$ must be set to 1 indicating that $\omega_0$ will remain at $j_0$ until $t_0 + \delta^{\omega_0}_{l_0l_0}(t_0)$, or a variable $x^{\omega_0}_{l_0k}(t_0)$ with $k \in E^+: k \neq j_0$ must be set to 1 indicating that $\omega_0$ will immediately leave $j_0$ towards $k$ at $t_0$. These constraints ensure the continuity of urban vehicle’ tours. They do not apply for $t_0 = 1$, since $\Theta^{\omega_0}_{ij}(1) = \emptyset$ for any $i, j$ and $\omega_0$, but they apply for $t_0 = T_{\text{max}}$, although the variable $x^{\omega_0}_{ij}(T_{\text{max}})$ does not indicate a real trip, but simply determine that the vehicle ends the operations at $i$.

Set of constraints (5Ω)

$$
\sum_{\omega_0 \in \Omega} \sum_{i \in E} x^{\omega_0}_{ie}(1) \leq \pi^{\omega_0}_{e}(1)
$$

for every $e_0 \in E$

$$
\sum_{\omega_0 \in \Omega} \sum_{i \in E^+} \sum_{\theta \in \Theta^{\omega_0}_{ij}(t_0)} x^{\omega_0}_{ij}(t_0) \leq \pi^{\omega_0}_{j}(t_0)
$$

for every $j_0 \in E^+$, $t_0 = 2, ..., T_{\text{max}}$

These constraints ensure that the maximum number of urban vehicles which are allowed to be at an external zone, depot or satellite is respected.

Set of constraints (6Ω)

$$
\sum_{p \in F(\omega_0)} q^{\omega_0}_{ep}(t_0) \leq u^{\omega_0} \left( \sum_{i \in E^+} \sum_{\theta \in \Theta^{\omega_0}_{ie}(t_0)} x^{\omega_0}_{ie}(\theta) \right)
$$

for every $\omega_0 \in \Omega$, $e_0 \in E$, $t_0 = 1, ..., T_{\text{max}}$
for every $\omega_0 \in \Omega, j_0 \in G^+, t_0 = 1, ..., T_{\text{max}}$

A urban vehicle $\omega_0 \in \Omega$ can only be loaded with some volume $\sum_{p \in P(\omega_0)} q_{e_0p}(t_0) > 0$ at an external zone $e_0 \in E$ at $t_0$ if it is there at that time interval. It means that $\omega_0$ must have departed from some point $i \in E^+$ towards $e_0$ at some $\theta \in \Theta^0_{\omega_0}(t_0)$. By the same token, $\omega_0$ can only unload some volume $\sum_{p \in P(\omega_0)} \lambda^0_{ip_0}(t_0) > 0$ if it is at the depot or satellite $j_0 \in G^+$ at the time interval $t_0$, which means it must have departed from some point $i \in E^+$ towards $j_0$ at some $\theta \in \Theta^0_{j_0}(t_0)$.

– Set of constraints (7$\Omega$)

\[
0 \leq \sum_{\Delta=0}^{t_0-1} \sum_{p \in P(\omega_0)} \sum_{e \in E} q_{e_0p}(t_0 - \Delta) - \sum_{\Delta=0}^{t_0-1} \sum_{p \in P(\omega_0)} \sum_{i \in G^+} \lambda^0_{ip_0}(t_0 - \Delta) \leq u^0_{\omega_0}
\]

for every $\omega_0 \in \Omega, t_0 = 1, ..., T_{\text{max}}$

The urban vehicle load at a time interval is equal sum of all volumes which may have been previously loaded into it minus the sum of all volumes which may have been unloaded from it. This value must always be between zero and its capacity.

– Set of constraints (8$\Omega$)

\[
\sum_{\Delta=0}^{t_0-1} \sum_{j \in G^+} \lambda^0_{jp_0}(t_0 - \Delta) \leq \sum_{e \in E} \sum_{\Delta=0}^{t_0-1} q_{e_0p}(t_0 - \Delta)
\]

for every $p_0 \in P, \omega_0 \in \Omega(p_0), t_0 = 1, ..., T_{\text{max}}$

A urban vehicle can only transfer some volume of a product to a city freighter if it is sufficiently loaded.

– Set of constraints (9V&$\Omega$)

\[
\sum_{v \in V(\omega_0)} \lambda^V_{j_0p_0}(t_0) = \sum_{\omega \in \Omega(p_0)} \lambda^\omega_{j_0p_0}(t_0)
\]

for every $j_0 \in G^+, p_0 \in P, t_0 = 1, ..., T_{\text{max}}$, if no intermediate storage is allowed at $j_0$

\[
\sum_{\Delta=0}^{t_0-1} \sum_{v \in V} \lambda^V_{j_0p_0}(t_0 - \Delta) \leq \sum_{\Delta=0}^{t_0-1} \sum_{\omega \in \Omega(p_0)} \lambda^\omega_{j_0p_0}(t_0 - \Delta)
\]

for every $j_0 \in G^+, p_0 \in P, t_0 = 1, ..., T_{\text{max}}$, if intermediate storage is allowed at $j_0$

where $\Omega(p_0) = \{\omega \in \Omega | p_0 \in P(\omega)\}$
If no intermediate storage is allowed at a depot or satellite, the quantities which are unloaded from all urban vehicles must equal the quantities which are loaded into all city freighters there at each time interval. If intermediate storage is allowed, then we must ensure that the quantity loaded into every city freighter from the beginning of the operations until each time interval does not exceed the amount which it has been unloaded from all urban vehicles there in the same period.

2.2. Model analysis and discussion

The formulation proposed in this section includes service network design and vehicle routing with time windows (VRPTW) within a time-dependent framework. It also includes the replenishment layer of the satellite warehouses as well as the related coordination and synchronisation constraints. This model includes the coordination and the synchronization of the multi-echelon transportation operations as suggested in Crainic et al., (2009). It is well known that Network Design and Routing Problems are NP-hard problems. In the simplest case the problem described in section 2.1 reduces to either Network Design or Routing Problem, which make the problem at hand NP-hard. The preliminary tests carried out with the model have shown some limitations. Only small instances could be solved to optimality. It is thus intended to assist us in generating lower and upper bounds for the medium and large size instances. These bounds will be used to measure the performance of some MILP based heuristics that we are currently investigating.

Fig. 1. Instance Generator (first screen).
3. Towards Practical Applications

We have written the model in AMPL system modelling tool and, in order to perform some tests, we developed two tools in MATLAB: an Instance Generator and an Output Visualizer. The first one has an user interface which allows generating the input of all model parameters and creates a data file that can be read in AMPL. The last one has a graphical interface that enables us to display the solution (routes, etc.) on a map. In this section, we briefly present these tools, together with the solution of a small instance of the problem.

3.1. The Instance Generator

A careful reader may have observed that even a simple instance of the problem will have a huge number of parameters. For instance, the travel times and costs are time and vehicle-dependent. This means that the number of travel times for all city freighter is $|\mathcal{V}| \times |\mathcal{G}| \times |\mathcal{S}| \times |\mathcal{C}| \times T_{\text{max}}$. In a problem with 3 city freighters, 10 customers, 2 depots, 2 satellites and 16 time intervals, this results in 1920 parameters. The same number of parameters would be needed for the travel costs of those city freighters. Finally, we would also need a number of similar magnitude for the corresponding urban vehicles’ parameters. This clearly points out the need for a tool to generate and manipulate instances of the problem.

The Instance Generator has been developed to fulfil this need. The user interface of this tool enables the user to input the parameters in two steps. In the first step, depicted in Fig. 1, some general information of the instance must be input, such as the number of customers, depots, satellites, external zones, urban vehicles, city freighters, product types and time intervals. The map of the city can be used as background image and the points indicating the installations can be positioned (drag-and-drop) where they are actually located.

![Instance Generator](image)
Based on the information entered on the first screen, a second screen is presented to the user containing all the required model parameters, as shown in Fig. 2. It includes the demanded volumes, time windows and product types; the vehicles’ costs and capacities; the travel times and costs for each vehicle in each time interval; etc. Some of these parameters are randomly generated, while others are suggested based on quick calculations. For instance, the travel times and costs are estimated based on the Euclidian distances between the installations. The user is allowed to change all the parameters, and some consistency checks are made on the changes (e.g., the time windows for the demands should be valid). Some special actions can be performed to change multiple parameters at once. For example, it is known that in a real situation some vehicles will be from a same type, and therefore their travel times and costs will be the same. So it was made possible for the user to adjust the travel times for one vehicle and, in one click, copy it to all similar ones.

After the user has input all parameters, a data file (.dat) for AMPL can be generated. A MATLAB file can also be saved, and it has two purposes. The first purpose is to let the user quickly load the parameters in the Instance Generator and edit it to create a new instance. The second purpose is to export some information to the Output Visualizer, such as the file used as background image (city map) and the coordinates of the installations (which is not a parameter for the optimization model).

![City Logistics Viewer](image)

Fig. 3. Output Visualizer.
3.2. The Output Visualizer

Due to the large number of variables involved in any instance of the problem, it is not easy to interpret a solution just by checking their numeric values. The route of a city freighter, for example, is determined by the binary variables $x_{ij}(t)$ which are set to 1, but finding these variables in their multi-dimensional tables and regarding them as real departures from a point to another in the city is not an easy task for humans. This consideration has driven us towards the development of the Output Visualizer, a graphical tool which interprets the output of the model obtained using AMPL/CPLEX and displays the vehicles’ routes on the city map, together with some compiled information concerning the loaded, unloaded and delivered products. The Output Visualizer is shown in Fig. 3.

The user can scroll across the time intervals and check the expected position of each vehicle at the selected time interval. The arcs previously travelled by the vehicles may be highlighted, allowing the user to visualize the routes which are taken until the selected interval. On the bottom of the screen, some additional data is displayed in a table (vehicle load, quantity being delivered at the selected time interval, etc.).

4. Concluding remarks

This paper proposes and discusses a preliminary model, in natural variables, for the two-tiered system described in Crainic et al., (2009). The model is implemented in AMPL and two supporting tools were developed using MATLAB: an instance generator and a graphical interface for visualizing the output. The first experiments show that this complete set has potential to be applied for real cases with few adaptations.

The objective is to start from a natural formulation of the problem, investigate its properties and its possible known sub-problems, then later develop possible reformulations which would allow us to efficiently solve practical instances. Promising solution approaches to realistic instances of this two-tiered system seem to go through decomposition. The coordination and synchronization constraints related to the multi-echelon transportation operations seem to offer some algorithmic possibilities. This is being currently instigated within an on-going research project.

Acknowledgements

This research work is support by CAPES Foundation (Proc. n° 9369-13-9), Ministry of Education of Brazil, Brasília - DF 70040-020, Brazil.

References


Taniguchi, E. & Thompson, R. Modeling City Logistics. Transportation Research Record: Journal of the Transportation Research Board 1790, 45–51 (2002).