



# Is cosmic parity violation responsible for the anomalies in the WMAP data?

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## Abstract

In this Letter, I argue that a parity violating extension to general relativity can simultaneously explain the observed loss in power and provides a first step for explaining the alignment at a preferred axis ('Axis of Evil') in the low multipole moments of the WMAP data. This observational possibility also provides an experimental window for an inflationary leptogenesis mechanism arising from large-scale parity violation. Similar to the arguments of Contaldi, Peloso, Kofman and Linde, large scale power is suppressed from the backreaction of the parity violating term by inducing a velocity dependent potential. We also argue that this modification can suppress power of odd parity multipoles on large scales, a pattern that is observed in the WMAP alignment anomaly.

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## 1. Introduction

Parity (P) violation is one of the cornerstones of the current standard model of particle interactions. Nonetheless, this violation only occurs within the weak interaction sector. It is still unknown how parity violation arises from a unified scheme which includes the other forces, in particular, gravity. Over the past two decades, a wealth of cosmological data has confirmed the theoretical predictions of the inflationary paradigm [1–3]. Inflation posits that specific initial conditions encoded in quantum fluctuations are responsible for large scale structure in the universe. Combining these ideas, it is plausible that the initial conditions set by inflation are parity violating. It is interesting to ask what the observational consequences will be.

If parity violation on large scales can coexist with a homogeneous and isotropic universe like ours how can we observe it? This question has been asked in the past and analyzed in the context of the CMB polarization [4,5]. It was found that the direct signal would be undetectable in the most optimistic cases [4]. Based on the work of [4], the authors of [6] found that parity violation sourced by a non-vanishing phase of a pseudo-scalar inflaton field can provide all Sakharov conditions

for leptogenesis. While this mechanism is compelling, it is still necessary to seek its observational consequences, especially in recent and future CMB observations.

In principle, the parity violation in general relativity leads to leptogenesis by transmitting itself into B–L violation through primordial gravitational waves [6,8,9]. This occurs because there is a gravitational Chern–Simons coupling to a pseudo-scalar field which is generated through a Green–Schwarz mechanism [12]. The Chern–Simons operator in the inflationary background is non-vanishing. A key point of this Letter is that the Chern–Simons operator gives a contribution to the energy–momentum tensor leading to a suppression of the odd-parity modes in the power-spectrum.

In this Letter I will show that if parity is violated during the inflationary epoch, the large scale, odd-parity, perturbations of the inflation field will experience a loss of power. This happens because gravitational backreaction induces a velocity dependent potential for the primordial scalar fluctuations. At the same time the gravitational backreaction will produce leptons. The power suppression will cease for large multipoles, which coincides with energy scales comparable to a massive right-handed neutrino. Another goal of this Letter is to identify the origin of the two persistent anomalies in the WMAP data, the loss of power on large scales and an alignment of multipoles along a preferred axis for low  $\ell$ 's (the so-called 'Axis of Evil')

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which has been argued to suppress odd parity modes at  $\ell = 2-5$  [16–19]. I will show that parity violation can account for the loss of power on large angular scales, but will not provide an mechanism which chooses this axis.

## 2. General relativity and parity violation

General Relativity can readily be extended to have parity violation by including a Chern–Simons term. For homogeneous and isotropic space–times, such as de Sitter and FRW, this term vanishes. But in the presence of a rolling pseudo-scalar field this is no longer the case. I will demonstrate this fact and its consequences on the scalar power-spectrum. First, let us consider the extended Gravity action with a Chern–Simons extension. This extension is necessary to cancel the gauge and gravitational anomalies in Heterotic string theory [12].<sup>1</sup>

$$S = \int d^4x \sqrt{-g} \left( R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{\phi}{M_{\text{Pl}}} \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}^{\rho\sigma} \right) \quad (1)$$

integrating the last term in the above equation by parts yields its relation to the three-dimensional Chern–Simons term.

$$\int \phi R \wedge R = - \int d\phi \wedge \Omega_{\text{CS}}, \quad (2)$$

where  $\Omega_{\text{CS}} = d\omega \wedge \omega + \frac{2}{3} \omega \wedge \omega \wedge \omega$ .

The parity violation will be generated by the gravitational wave fluctuations during inflation. We will now show that these gravity waves are sourced by the inflaton field and are amplified on scales where the power suppression is observed in the CMB data. The general form of metric perturbations about an FRW universe can be parameterized as

$$ds^2 = -(1 + 2\varphi) dt^2 + w_i dt dx^i + a^2(t) [(1 + 2\psi)\delta_{ij} + h_{ij}] dx^i dx^j, \quad (3)$$

where  $\varphi$ ,  $\psi$ ,  $w_i$  and  $h_{ij}$  respectively parametrize the scalar-type, vector-type, and tensor-type fluctuations of the metric. It was shown by Choi et al. [7] that the scalar and vector perturbations do not contribute to Chern–Simons term,  $R\tilde{R}$ , so these fluctuations can be ignored in what follows. We can also fix a gauge so that the tensor fluctuation is parametrized by the two physical transverse traceless elements of  $h_{ij}$ . For gravity waves moving in the  $z$  direction without loss of generality, one writes

$$ds^2 = -dt^2 + a^2(t) [(1 - h_+) dx^2 + (1 + h_+) dy^2 + 2h_\times dx dy + dz^2], \quad (4)$$

where  $a(t) = e^{Ht}$  during inflation and  $h_+$ ,  $h_\times$  are functions of  $t$ ,  $z$ . To see the CP violation more explicitly, it is convenient to use a helicity basis

$$h_L = (h_+ - ih_\times)/\sqrt{2}, \quad h_R = (h_+ + ih_\times)/\sqrt{2}. \quad (5)$$

Here  $h_L$  and  $h_R$  are complex conjugate scalar fields. To be very explicit, the negative frequency part of  $h_L$  is the conjugate of the positive frequency part of  $h_R$ , and both are built from wave-functions for left-handed gravitons.

The contribution of tensor perturbations to  $R\tilde{R}$ , up to second order in  $h_L$  and  $h_R$ , is

$$R\tilde{R} = \frac{4i}{a^3} \left[ \left( \partial_z^2 h_R \partial_z \partial_t h_L + a^2 \partial_t^2 h_R \partial_t \partial_z h_L + \frac{1}{2} \partial_t a^2 \partial_t h_R \partial_t \partial_z h_L \right) - (L \leftrightarrow R) \right]. \quad (6)$$

If  $h_L$  and  $h_R$  have the same dispersion relation, this expression vanishes. Thus, for  $R\tilde{R}$  to be nonzero, one needs a ‘cosmological birefringence’ during inflation. Such an effect is induced by the addition of the pseudo-scalar Chern–Simons coupling to the gravitational equations [4]. In the presence of this source the gravitational waves obey the following equation.

$$\square h_L = -2i \frac{\Theta}{a} \dot{h}'_L, \quad \square h_R = +2i \frac{\Theta}{a} \dot{h}'_R, \quad (7)$$

where

$$\Theta = 8 \left( \frac{H}{M_{\text{Pl}}} \right)^2 \dot{\phi} / H M_{\text{Pl}}, \quad (8)$$

dots denote time derivatives, and primes denote differentiation of with respect to  $z$ .

The authors of [6] found exact solutions to the above equations and evaluated the Green’s functions. A one loop effect ensures a VEV for  $\Theta$  and in the presence of the above tensor perturbations this term will not vanish, provided that the pseudo-scalar field which couples to it is dynamical. Let us study the effects of this term on the scalar field evolution. The equation of motion for the pseudo-scalar is:

$$\ddot{\phi} + 3H\dot{\phi} + k^2\phi + \frac{dV}{d\phi} = -\langle R\tilde{R} \rangle. \quad (9)$$

The VEV on the RHS can be calculated from evaluating the Greens function of the gravity waves. This was previously done by [6]; the answer is:

$$\langle R\tilde{R} \rangle = \frac{16}{a} \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3 M_{\text{Pl}}^2} (k\eta)^2 \cdot k^4 \Theta. \quad (10)$$

Plugging this result into Eq. (9) yields,

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + k^2\phi + \frac{dV}{d\phi} &= \frac{16}{a} \int \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3 M_{\text{Pl}}^2} (k\eta)^2 \cdot k^4 \Theta. \end{aligned} \quad (11)$$

In the rest of this Letter we will show that the effect of the right-hand side of (11) is responsible for the loss of power of odd parity modes in the primordial power-spectrum. We will calculate power-spectrum by showing that the Chern–Simons coupling can modify the energy–momentum tensor leading to a modified Friedmann equation. It is through this route that one will be able to calculate the power-spectrum which reflects the odd-parity anomaly.

<sup>1</sup> The 4D gravitational Chern–Simons action is demonstrated to be Lorentz invariant in [13].

### 3. Modified energy–momentum tensor

In a recently work, Contaldi et al. [10] demonstrated that a modified velocity dependent potential operating early during inflation can suppress power on large scales. Our mechanism naturally gives us a velocity dependent potential due to the gravitational backreaction of the Chern–Simons term. I will now show that the Chern–Simons term induces a modification to the energy–momentum tensor. This will give us a velocity dependent contribution to the energy–momentum tensor which will modify the power-spectrum in a specific way to be discussed in the next section. Variation of the Chern–Simons action with respect to the metric yield the correction to the energy–momentum tensor [20].

$$\delta I_{CS} = \delta \int d^4x \phi^* R R = \int d^4x \sqrt{-g} C_{\mu\nu} \delta g^{\mu\nu}, \quad (12)$$

where  $*R_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu} R^{\alpha\beta}$  and

$$C^{\mu\nu} = \frac{1}{-2\sqrt{-g}} [v_\sigma (\epsilon^{\sigma\mu\alpha\beta} D_\alpha R_\beta^\nu + \epsilon^{\sigma\nu\alpha\beta} D_\alpha R_\beta^\mu) + v_{\sigma\tau} (*R^{\tau\nu\sigma\mu})], \quad (13)$$

where  $C_{\mu\nu}$  is the Cotton tensor,  $v_{\sigma\tau}$  is the covariant derivative of the embedding coordinate and in terms of the pseudo-scalar field  $v_{\sigma\tau} = D_\sigma D_\tau \phi$  and  $v_\sigma = D_\sigma \phi$ . Therefore, Einstein's equation is modified in the following way.

$$G^{\mu\nu} = -(8\pi G T^{\mu\nu} + C^{\mu\nu}). \quad (14)$$

The covariant divergence of the Cotton tensor is non-zero.

$$D_\mu C^{\mu\nu} = \frac{1}{8\sqrt{-g}} \partial^\nu \phi^* R R. \quad (15)$$

One can now derive the contribution to the energy–momentum tensor  $T^{00}$  by integrating (15) over time. This is possible in a purely homogeneous and isotropic space–time where the pseudo-scalar field is only time dependent (i.e.,  $\partial_\mu \phi = \dot{\phi}$ ). We obtain

$$T_{CS}^{00} = \frac{1}{H M_{pl}} (\dot{\phi}^* R R - e^{-tH}), \quad (16)$$

where  $T_{CS}^{00} = C^{00}$ . The divergence of the space like components of the Cotton tensor in the presence of the birefringent gravity waves vanish since,

$$C_{ij} = -\frac{\dot{\phi}}{M_{pl}} \epsilon_{(i}^{0kl} \nabla^2 h_{j)l,k} \quad (17)$$

and the divergence of this object is proportional to the gradient of a curl, which is zero.

We see that the energy–momentum tensor, Eq. (16) associated with parity violation only depends on the velocity of the inflaton field and the propagation of the gravity waves in the inflating background,<sup>2</sup> Eqs. (6) and (7). By taking the exact solutions of gravity wave Greens functions and plugging

into  $T_{CS}^{00}$  [6], we can evaluate the effective energy–momentum tensor,  $T_{\text{eff}}^{00} = \dot{\phi}^* R(h_{ij}) R(h_{ij})$ . This procedure employs the methods of Mukhanov et al. [14] for calculating the effective energy–momentum tensor due to gravitational perturbations in an inflationary background. A more systematic derivation of the effective action for Chern–Simons modified gravity is currently in pursuit. With these considerations we are lead to the effective Lagrangian of the inflation perturbation interacting with the parity violating term.

$$\mathcal{L}_\phi = -\partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{H M_{pl}} \dot{\phi}^* R R_{\text{eff}}, \quad (18)$$

where  $*R R_{\text{eff}}$  is the evaluation of the Chern–Simons in terms of the gravitational wave Greens function.

Plugging in the expression (6) for the Chern–Simons term into the rhs of Eq. (18) gives,

$$\mathcal{L}_\phi = -\partial_\mu \phi \partial^\mu \phi + V(\phi) + \dot{\phi}^2 \frac{f(t)}{H M_{pl}}, \quad (19)$$

where  $f(t)$  is Eq. (10) integrated up to a momentum scale  $k_s$ .

$$f(t) = \frac{\mathcal{N}}{(2\pi)^3} a(t)^{-3} \left(\frac{H}{M_{pl}}\right)^4 \left(\frac{k_s}{M_{pl}}\right) \left(\frac{k_s}{H}\right)^3 k_s^2, \quad (20)$$

where  $k_s$  is the UV cutoff in the momentum integral, which in this case is identified with the scale of the right-handed neutrino.

The gravitational backreaction from the coupling of the Chern–Simons term and the pseudo-scalar induces a velocity dependent potential. This is the key to understanding how the power-spectrum can be modified from space–time parity violation.

### 4. Modified power-spectrum and parity violation

Due to arguments in the previous section the effective action for the inflaton, neglecting spatial gradients, gets modified to the following form:

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left( V(\phi) - \frac{1}{2} \left( 1 - \frac{f(t)\dot{\phi}^2}{H M_{pl}} \right) \right). \quad (21)$$

The leptogenesis mechanism discussed by [6] relied on the well-known fact that the lepton number current, and also the total fermion number current, has a gravitational anomaly in the Standard Model. Explicitly,

$$\partial_\mu J_\ell^\mu = \frac{3}{16\pi^2} R \tilde{R}, \quad (22)$$

where

$$J_\ell^\mu = \bar{\ell}_i \gamma^\mu \ell_i + \bar{\nu}_i \gamma^\mu \nu_i, \quad R \tilde{R} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}^{\rho\sigma}. \quad (23)$$

terms are necessarily time dependent and is reflected in the effective Lagrangian due to the time dependence of the inflationary background. I have neglected second order gravitational backreaction contributions to the effective energy momentum tensor, since the tensor-type metric perturbations are suppressed by the Planck mass. The issue of higher order inflationary gravitational backreaction is currently under active research. For a more detailed discussion of the issue and relevant references on effective Lagrangians for inflation see [14].

<sup>2</sup> This effective Lagrangian assumes that the only relevant contribution to the effective energy momentum tensor are the time dependent part of the inflaton scalar field and the first order gauge invariant tensor perturbation. These

The anomaly requires an imbalance of left- and right-handed leptons. In general, (22) will be correct in an effective theory valid below a scale  $\mu$ . Therefore, a non-vanishing  $f$  signifies lepton number production. It is interesting that leptogenesis is directly connected to the modification of the power-spectrum during inflation.

Another important fact is that there will be an additional contribution to the velocity dependent potential which cancels  $f(k)$  when the momentum of the gravity waves approaches the scale  $\mu$  of the right-handed neutrino.

$$f(k, t) = f_l(k = \mu, t) - f_r(k = \mu, t) \rightarrow 0, \quad (24)$$

where  $f_l$  and  $f_r$  are the left- and right-handed contribution to the gravitational anomaly integral, respectively.

For this mechanism to work, the parity violation suppression has to affect the superhorizon modes of the low  $l$  multipole modes. This requires the assumption that the scales where the parity violating effect shuts off corresponds to the scale of comoving wavenumber 60 e-foldings before inflation ends.<sup>3</sup> Given assumption, we will now demonstrate that the suppression of low multipoles occur at approximately 55 e-folds before the end of inflation. A generic feature of scalar field driven inflation is that the choice on the number of e-folds is a theoretical fine tuning on the parameters of the inflationary potential [15].

Given the above modification, one can calculate the primordial power spectrum  $P_H(k(t_f))$  for modes with comoving wavenumber  $k(t_f)$  which re-enter the horizon given the primordial power-spectrum for modes with comoving wavenumber  $k(t_i)$  which exit the horizon at time  $t_i$  with the equation.

$$P(k, t_f)_H = \frac{1 + \omega(t(k)_f)}{1 + \omega(t(k)_i)} P(k, t_i)_H^0, \quad (25)$$

where the equation of state will be modified due to the correction of the kinetic energy. We can compute the modified power-spectrum by using the following identities:

$$\omega = \frac{p}{\rho} \quad (26)$$

and

$$p = \frac{1}{2} \dot{\phi}^2 + V, \quad (27)$$

$$\rho = \frac{1}{2} \dot{\phi}^2 - V. \quad (28)$$

After a little algebra, we obtain:

$$P(k)_H = \frac{P(k)_H^0}{1 + \frac{f(t(k))}{H M_{\text{Pl}}}}, \quad (29)$$

with the above modification for the primordial power-spectrum, the physical mechanism for power suppression becomes clear. Using (29) leads to a modified power-spectrum of the following form:

$$P(k)_H = \frac{P(k)_H^0}{\left(1 + \frac{\mathcal{N}}{(2\pi)^3} a^{-3} \left(\frac{H}{M_{\text{Pl}}}\right)^4 \left(\frac{k_s}{M_{\text{Pl}}}\right)^2 \left(\frac{k_s}{H}\right)^4\right)}. \quad (30)$$

<sup>3</sup> For a similar application of this assumption to the large scale anomalies see [11].

The modified power-spectrum has the following interesting profile. Since  $\ell$  is proportional to the co-moving wavenumber  $k$ , the power-spectrum for all  $\ell$  larger than the scale of the right-handed neutrino ( $k_s$ ) will also remain unmodified because  $f(k \geq k_s)$  vanishes due to anomaly cancellation. This means that the  $C_\ell$ 's for large  $\ell$ 's in the WMAP data will not be affected.

However, the function  $f(t, k < k_s)$  will not vanish for all  $l$ 's less than this characteristic scale. One can estimate this number by setting  $k_s = 10^{14}$  GeV, a sensible upper-limit for the scale of the right-handed neutrino.  $\mathcal{N}$ , is a parameter known as the volume factor in string compactifications, which is related to the ratio of the ten-dimensional string scale and the four-dimensional Planck length. In previous work the authors [6] showed that  $\mathcal{N} \sim 10^3$  is a conservative value to use. This value yields:

$$\frac{f(t(k))}{H M_{\text{Pl}}} \sim 10^{-22} \times e^{N(k)} \quad (31)$$

and the power-spectrum becomes,

$$P(k)_H = \frac{P_H^0}{(1 + 10^{-22} \times e^{3N(k)})}, \quad (32)$$

where  $e^{N(k)} = \frac{a(t_0)}{a(t)}$ . The correction exponentially depends on the number of e-foldings. If we assume that the scale of the right-handed neutrino corresponds to  $l = 2$  (60 e-foldings before the end of inflation) then the power is almost completely suppressed. Moreover, since anomaly cancellation occurs when the right-handed neutrino scale is reached, the power will only be suppressed at low multipole moments less than the scale of the right-handed neutrino. Hence, the scale of the right-handed neutrino can be identified as the turning point in the WMAP data wherein the power spectrum deviates from standard inflationary predictions. It is interesting that the source which suppresses the power arises from the most general parity violating operator in general relativity.

## 5. Discussion

In this Letter, I demonstrate that parity violation in the early universe can tie together the two persistent anomalies in the CMB; loss of power and the alignment of low multipole moments along a preferred axis which has even mirror parity; the so-called 'Axis of Evil'. Recently, Land and Magueijo found a preferred frame in the WMAP data which is significantly aligned for multipoles  $l = 2, 3, 4, 5$  which defines an overall preferred axis [19]. For these multipoles, the  $m$ 's turn out to be preferred such that  $l + m = \text{even}$ . In other words, multipoles are dominated by positive mirror parity in this preferred frame. It still remains to find a mechanism which selects the direction of the 'Axis of Evil'; this requires an understanding of why odd parity modes in this frame are suppressed at low multipole moments. We see this mechanism as a first step toward identifying a possible physical mechanism for correlating the loss of large scale power with the alignment anomaly through parity violation. However, our mechanism does not explain the origin

of plane where even-mirror parity persists. Instead our mechanism will suppress power of odd parity modes once the plane is selected by some as yet unknown mechanism. It is plausible that the selection of the plane is connected to a special three-topology which correlates mirror parity violation with an embedded two-dimensional plane. Such topologies were classified Witt [21].

In conclusion, when general relativity is extended to have large-scale parity violation during the inflationary epoch, by the inclusion of a gravitational Chern–Simons term, the back-reaction of gravitational waves can lead to suppression of the odd parity modes in the primordial scalar power-spectrum at scales less than the right-handed neutrino. Physically this loss of power signifies lepton number production during the inflationary epoch. This is clear since the Chern–Simons term in the inflationary background simultaneously modifies the Energy-Momentum tensor of the scalar field and is proportional to the rate of lepton number density. In a future work, [22] we will report on a more systematic treatment of the data in light of the leptogenesis mechanism of [6].

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