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Modeling Steady State Creep Behavior of Functionally Graded Thick Cylinder in the Presence of Residual Stress

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Abstract

The steady state creep behavior in a functionally graded thick composite cylinder subjected to internal pressure has been investigated in the presence of residual stress. Hoffman’s yield criterion is used, to describe the yielding of the cylinder material in order to account for residual stress. The results obtained are compared with a similar cylinder but yielding according to von Mises criterion. The cylinder is made of functionally graded material containing linearly varying silicon carbide whiskers in a matrix of 6061 aluminum. The study shows that in the presence of residual stress in composite cylinder the radial stress decreases slightly over the entire cylinder, whereas the tangential and axial stresses increase significantly near the inner radius but show considerable decrease towards the outer radius. The effective stress in the cylinder increases in the presence of residual stress and the maximum increase near the outer radius. The study also reveals that the creep rates are strongly influenced by the presence of residual stress in the cylinder material.

Keywords: Steady state creep; thick cylinder; functionally graded material

1. Introduction

The residual stress is induced in the composite materials during processing when these are cooled from higher temperature. Therefore, due to presence of this thermal residual stress the composite materials possess difference in yield stresses in tension and compression. The von-Mises yield criterion for isotropic material assumes that yielding under both uniaxial tension and compression starts at the same level of stress. However, the experimental results often indicate that even in isotropic metal matrix composite yielding does not begins at the same level of tensile and compressive stresses under uniaxial loading [1]. Badani [2] has conducted experiments to determine the compressive and tensile yield strengths in longitudinal direction of the extruded bars made of 6061 Al-SiC w. The results indicate that the compressive yield strength is greater than the tensile yield strength.

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The study carried out in this paper puts forward a framework for the analysis of steady state creep behaviour of a thick-walled cylinder made of functionally graded material (FGM) in the presence of thermal residual stress. The problem undertaken has significant practical importance.

2. Analysis and solution

Consider a long, closed end, thick-walled FG 6061Al-SiCw composite cylinder with \( r_1 \) and \( r_2 \) as inner and outer radii respectively. The cylinder is subjected to internal pressure \( p \). The whiskers content (SiCw) in the cylinder is assumed to decrease linearly from the inner to outer radius. The content (vol \%) of silicon carbide \( V(r) \) at any radius \( r \) of the cylinder is given by;

\[
V(r) = V_{\text{max}} - \frac{(r - r_1)}{(r_2 - r_1)} \left[ V_{\text{max}} - V_{\text{min}} \right]
\]

where, \( V_{\text{max}} \) and \( V_{\text{min}} \) are the maximum and minimum whisker content in the cylinder at inner and outer radius respectively.

The creep behavior of FG composite in this study is described by well documented creep law based on threshold stress (\( \sigma_0 \)),

\[
\dot{\epsilon}_e = \left[ M(r) \{ \sigma_e - \sigma_0 (r) \} \right]^5
\]

The creep parameters \( M(r) \) and \( \sigma_0 (r) \) given in eq. (2) are dependent on particle size (\( P \)), particle content (\( V \)) and operating temperature (\( T \)). The values of creep parameters required in this study have been calculated respectively from following regression equations,

\[
M(r) = 0.03324 - \frac{0.00879}{P} - \frac{14.02666}{T} + \frac{0.03224}{V(r)}
\]

\[
\sigma_0 (r) = -0.084P - 0.0232T + 1.1853V(r) + 51.317
\]

The cylinder is assumed to operate under following boundary conditions,

\text{At inner radius (}r_1\text{): Radial Stress (}\sigma_r\text{) = }-p\)

\text{At outer radius (}r_2\text{): Radial Stress (}\sigma_r\text{) = 0}

By considering equilibrium of forces acting on an element of the cylinder, in the radial direction, we get,

\[
r \frac{d\sigma_r}{dr} = \sigma_\theta - \sigma_r
\]

The material of the cylinder is assumed to be incompressible, \( i.e. \)

\[
\dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = 0
\]
The yielding of cylinders is determined by Hoffman’s yield criterion [3, 4], which accounts for different yield strengths in tension and compression as given by:

\[
2f(\sigma_{ij}) = \frac{1}{2f_c f_t} \left[ (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{11} - \sigma_{22})^2 \right] + \frac{(f_c - f_t)}{f_c f_t} (\sigma_{11} + \sigma_{22} + \sigma_{33}) = 1
\]  

(9)

where, \( f_c \) and \( f_t \) are respectively the uniaxial compressive and tensile yield strengths. The constitutive equations for a thick cylinder are also given below:

\[
\frac{d\dot{u}_r}{dr} = \dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2a - 1 - b + T]\sigma_\theta
\]  

(10)

\[
\frac{\dot{u}_r}{r} = \dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2 - b - a + T]\sigma_\theta
\]  

(11)

\[
\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e} [2b - a - 1 + T]\sigma_\theta
\]  

(12)

where, \( \frac{\sigma_r}{\sigma_\theta} = a ; \frac{\sigma_\theta}{\sigma_\theta} = b ; \frac{f_c - f_t}{\sigma_\theta} = T \); and \( \dot{\varepsilon}_e \) is effective strain rate. The yield stresses in compression (\( f_c \)) and tension (\( f_t \)) have been taken from the experimental results reported by Badani [2] for extruded bars made of 6061Al-15 vol % SiCw composite. It is reported by Badani that \( f_c = 218 \text{MPa} \) and \( f_t = 186 \text{MPa} \).

Following Hoffman’s yield criterion, the effective stress, \( \sigma_e \) in an isotropic FG cylinder may be expressed as:

\[
\sigma_e = \sigma_\theta \left[ 1 + a^2 + b^2 - a - b - ab + T(1 + a + b) \right]^{1/2}
\]  

(13)

For a closed end cylinder made of incompressible material, the plane strain condition exists \( i.e \) the axial strain rate (\( \dot{\varepsilon}_z \)) is zero. Therefore, eq. (12) yields,

\[
\sigma_z = \frac{\sigma_r + \sigma_\theta - (f_c - f_t)}{2}
\]  

(14)

Dividing eq. (10) by eq. (11), and integrating between limits \( r_1 \) to \( r \), we obtain

\[
\dot{u}_r = \ddot{u}_r \left[ \exp \left( \frac{r}{r_1} \right) \right] \text{where, } \lambda(r) = \frac{2a - 1 - b + T}{2 - b - a + T}
\]  

(15)

where, \( \ddot{u}_r \) is radial displacement at inner radius (\( r_1 \)) of the cylinder.

Dividing eq. (15) by \( r \), equating it to eq. (11), substituting \( \dot{\varepsilon}_e \) and \( \sigma_e \) respectively from eq. (2) and (13) and simplify, we get
\[ \sigma_\theta = \frac{(\mu_0)^{1/5}}{M} K_1(r) + K_2(r) \]  

(16)

where, \( K_1(r) = \left[ \frac{2(a^2 + b^2 - a - b - ab + T(1 + a + b))^{-2}}{r[2 - b - a + T]} \right]^{1/5} \left[ \exp \int_{r_1}^r \frac{\lambda(r)}{r} \, dr \right] \)

and \( K_2(r) = \frac{\sigma_o}{\sqrt{a^2 + b^2 - a - b - ab + T(1 + a + b)}} \)

Integrating equilibrium eq. (7) between limits \( r_1 \) to \( r_2 \):

\[ \int_{r_1}^{r_2} \sigma_\theta \, dr = r_1 p \]  

(17)

Substituting \( \sigma_\theta \) from eq. (16) into above equation, we get the value of \( \frac{(\mu_0)^{1/5}}{M} \), which is substituted back in eq. (16), we obtain,

\[ \sigma_\theta = \frac{r_1 p - \int_{r_1}^{r_2} K_2(r) \, dr}{\int_{r_1}^{r_2} K_1(r) \, dr} K_1(r) + K_2(r) \]  

(18)

From eq. (16), the average tangential stress, \( \sigma_{\theta_{avg}} \), in the cylinder may be given as,

\[ \sigma_{\theta_{avg}} = \frac{1}{(r_2 - r_1)} \int_{r_1}^{r_2} \sigma_\theta \, dr = \frac{r_1 p}{(r_2 - r_1)} \]  

The above equation yields \( r_1 p = (r_2 - r_1)\sigma_{\theta_{avg}} \), which on substituting in eq. (18) gives,

\[ \sigma_\theta = \frac{(r_2 - r_1)\sigma_{\theta_{avg}} - \int_{r_1}^{r_2} K_2(r) \, dr}{\int_{r_1}^{r_2} K_1(r) \, dr} K_1(r) + K_2(r) \]  

(19)

Integrating again eq. (7), but between limit \( r_1 \) to \( r \), we get,

\[ \sigma_r = \frac{1}{r} \int_{r_1}^{r} \sigma_\theta \, dr - \frac{r_1 p}{r} \]  

(20)
In order to find the first approximation of \(a\) and \(b\) i.e. \(a_1\) and \(b_1\) respectively, we assume \(\sigma_\theta = \sigma_{\theta_{\text{avg}}}\) in eq. (20) to obtain the first approximation of \(\sigma_r\) i.e. \(\sigma_{r_1}\);

\[
\sigma_{r_1} = \frac{r_1 P}{r} \left[ \frac{r - r_2}{r_2 - r_1} \right]
\]

The above values of \(a_1\) and \(b_1\) are used to calculate the first approximation of \(K_1(r)\), \(K_2(r)\), \(\lambda(r)\), which are substituted in eq. (19) to estimate the first approximation of \(\sigma_\theta\) i.e. \(\sigma_{\theta_1}\). The value of \(\sigma_\theta\) thus estimated is used in eq. (20) to obtain the second approximation of \(\sigma_r\) i.e. \(\sigma_{r_2}\). Following procedure described above the second approximation of \(\sigma_\theta\) i.e. \(\sigma_{\theta_2}\) is calculated. The iteration process is continued till the error between the previous and present value of tangential stress becomes less than 0.01. Knowing the distributions of \(\sigma_r, \sigma_\theta\) and \(\sigma_\sigma\), we may get \(\sigma_r\) and \(\dot{\varepsilon}_r\) respectively from eq. (13) and (2). Afterwards, the strain rates are estimated respectively from eq. (10), (11) and (12).

3. Results and discussion

The current analysis and software is validated by using it to obtain the tangential strain rates in an isotropic copper cylinder, as reported in the study of Johnson et al [5]. The tangential strain rates estimated agree well with the experimental values reported by Johnson et al [5], similar to those observed in Fig. 1.

The steady state creep stresses and strain rates have been obtained for FG 6061Al-SiCw composite cylinder and having residual thermal stress. The yielding is described by Hoffman yield criterion.

The results obtained are compared with that estimated for a similar cylinder but without residual stress i.e. \(f_c = f_t\).
The FG cylinder is assumed to contain linearly decreasing whisker content \([V(r)]\) from the inner to outer radius with maximum of 15 vol% SiCw at the inner radius and an average of 10 vol% SiCw, Fig. 2. The dimensions of the FG cylinder taken in this study are similar to those used by Johnson et al [5] for copper cylinder as reported in Table 1.

<table>
<thead>
<tr>
<th>Material of the Cylinder</th>
<th>Copper</th>
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<tbody>
<tr>
<td>Internal Radius ((r_1)) = 25.4 mm, External Radius ((r_2)) = 50.8 mm</td>
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</tr>
<tr>
<td>Internal pressure ((p)) = 23.25 MPa, External pressure ((q)) = 0</td>
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<tr>
<td>Operating temperature ((T)) = 250 °C</td>
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<tr>
<td>Creep parameters: (M = 3.271 \times 10^{-4} s^{-1/5} MPa, \sigma_0 = 11.32 MPa)</td>
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The variation of creep parameters \(M\) and \(\sigma_0\) with radial distance in FG cylinders is shown in Figs. 3(a)-(b). The values of creep parameters in both the cylinders are same due to similar variation of SiCw content. The creep parameter \(M\) increases with increasing radial distance, Fig. 3(a). The increase observed in \(M\) may be attributed to decrease in particle content, on moving from the inner to outer radius of the cylinder. On the other hand, the threshold stress \((\sigma_0)\) shown in Fig. 3(b) decreases linearly on moving from the inner to outer radius of cylinder. The value of threshold stress is higher in regions having more amount of SiCw compared to regions having lower SiCw content.

The radial stress, Fig. 4(a), remains compressive throughout the cylinder, with maximum value at the inner radius and zero at the outer radius, under the imposed boundary conditions. In the presence of thermal residual stress the magnitude of radial stress changes a little in the functionally graded cylinder. The tangential stress shown in Fig. 4(b) remains tensile throughout and is observed to increase with increasing radius, reaches maximum somewhere towards the outer radius followed by a decrease with further increase in radius. In the presence of residual stress, the tangential stress increases near the inner radius of the cylinder but decreases towards the outer radius when compared with cylinder FG cylinder (C1) without thermal residual stress. The axial stress in FG cylinder changes its nature from compressive to tensile on moving from inner radius to outer radius as shown in Fig. 4(c). Due to presence of residual stress, the magnitude of tensile axial stress (at outer radius) decreases; however, the magnitude of compressive stress (at the inner radius) increases. The effective
stress, in the cylinder decreases with increasing radial distance, Fig. 4(d). The presence of residual stress in FG cylinder increases the value of effective stress in the FG cylinder except for a slight decrease observed near the inner radius.

The effective strain rates in FG cylinder is higher in cylinder C2 than C1, except near the inner radius where cylinder C1 has slightly higher value of effective stress due to higher value of stress difference \( (\sigma_e - \sigma_o) \) as shown in Fig. 5(a). The difference observed in effective strain rate in cylinder C1 and C2 increases as on moves towards the outer radius. The tangential strain rate distribution in FG cylinder with thermal residual stress (C2) is always remains higher that observed in FG cylinder without residual stress (C1), Fig. 5(b). The maximum difference in tangential strain rate between both the cylinders is observed at the outer radius.
4. Conclusions

The steady state creep response has been obtained for FG 6061Al-SiCw composite cylinder and having residual thermal stress. The presence of residual stress in thick FG composite cylinder has a marginal effect on the distribution of radial stress; however, it shows noticeable effect on the tangential stress, axial stress and effective stress. The effect of residual stress on effective strain rate is similar to those observed for effective stress. The maximum difference in the values of strain rates are obtained at the outer radius of the cylinder.

References