Failure by void coalescence in metallic materials containing primary and secondary voids subject to intense shearing

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Failure under intense shearing at close to zero stress triaxiality is widely observed for ductile metallic materials, and is identified in experiments as smeared-out dimples on the fracture surface. Numerical cell-model studies of equal sized voids have revealed that the mechanism governing this shear failure mode boils down to the interaction between primary voids which rotate and elongate until coalescence occurs under severe plastic deformation of the internal ligaments. The objective of this paper is to analyze this failure mechanism of primary voids and to study the effect of smaller secondary damage that co-exists with or nucleation in the ligaments between larger voids that coalesce during intense shearing. A numerical cell-model study is carried out to gain a parametric understanding of the overall material response for different initial conditions of the two void populations, subject to shear dominated loading. To account for both length scales involved in this study, a continuum model that includes the softening effect of damage evolution in shear is used to represent the matrix material surrounding the primary voids. Here, a recently extended Gurson-type model is used, which represents the effect of the small secondary voids under the low triaxiality loading conditions considered. This work suggests a failure mechanism for materials that contain voids on two different length scales, subject to intense shearing, in terms of; (i) the interaction of the primary voids, and (ii) the material softening of the ligaments due to the evolution of secondary damage. It is found that coalescence of primary voids under shear loading is severely affected by the presence of smaller secondary voids or defects in the ligaments. The change in overall ductility is presented for a wide range of initial material conditions, and an empirical correlation with the peak load is reported.

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1. Introduction

Ductile material behavior and failure at loading conditions dominated by shearing, where the hydrostatic tension is zero or even negative, have received a great deal of attention in recent years. In particular due to the lack of micro-mechanics based models that can describe failure under such conditions (Barsoum and Faleskog, 2007a,b; Scheyvaerts, 2008; Leblond and Mottet, 2008; Nahshon and Hutchinson, 2008; Tvergaard, 2008, 2009; Xue et al., 2010; Jodlowski, 2009; Tvergaard and Nielsen, 2010). Barsoum and Faleskog (2007b) presented a full 3D numerical analysis of double notched specimens under combined twist and tension with focus on matching their experimental findings (Barsoum and Faleskog, 2007a). The use of a simple shear deformation criterion to determine failure was demonstrated and its physical relevance discussed. The shape evolution of primary voids and their rotation in a shear field have been analyzed by Scheyvaerts (2008) in a numerical cell-model study in full 3D. The first stage of the void deformation was of particular interest and their analysis contributed to a further extension to the coalescence criterion by Thomason (1990), Pardo-en and Hutchinson (2000). Leblond and Mottet (2008) proposed a theoretical approach to account for coalescence by void growth as well as by the void sheet mechanism. A comparison with 3D numerical cell-model predictions showed a good agreement. The micro-mechanism governing ductile shear failure was brought out in a recent study by Tvergaard (1982a, 2009) using a 2D plane strain numerical cell-model of a single row of equal sized circular cylindrical voids under shearing. As a first, Tvergaard (2008) demonstrated that a maximum load carrying capacity for a ductile material is attained in a shear field due to micro-voids interaction. It was shown that during shearing the voids are flattened out to micro-cracks, which rotate and elongate until interaction with neighboring micro-cracks gives coalescence (Anderson et al., 1990). The failure mechanism in shear is thereby very different from that at moderate or high stress triaxiality, where the voids grow until necking of the internal ligaments between neighboring voids gives coalescence. The contact problem arising as the discretely modeled voids are flattened to micro-cracks...
in shear was not fully resolved in Tvergaard (2008), and the contact procedure was therefore adapted by Tvergaard (2009) to account more realistically for the voids surface contact.

To mimic the ductile material behavior in shear, Nahshon and Hutchinson (2008) recently suggested an extension to the micro-mechanics based Gurson-type models (Gurson, 1977; Tvergaard, 1990; Gologanu et al., 1997), which otherwise cannot predict void growth to coalescence at zero mean stress. The model by Nahshon and Hutchinson (2008) allows for failure under intense shearing by letting the damage parameter increase continuously during plastic loading at zero mean stress. A softening effect from existing damage is thereby obtained and traditional coalescence models can be reached (Tvergaard and Needleman, 1984; Thomason, 1990; Nielsen, 2010). The model by Nahshon and Hutchinson (2008) is, however, purely phenomenological and the damage evolution is no longer tied to the void volume fraction. Instead, it must be regarded as an effective void volume fraction. Nevertheless, the modification by Nahshon and Hutchinson (2008) has received a great deal of attention among researchers, and it is key to the study presented in this paper. Tvergaard and Nielsen (2010) recently compared the predictions of the shear-extended Gurson model to cell-model results. This comparison showed that the trends of cell-model results. This comparison showed that the trends of cell-model results agree well with the overall material response for the range of stress states, initial void volume fractions and strain hardening considered.

As discussed by Tvergaard (1982a, 1989), Faleskog and Shih (1997), Fabrègue and Pardoen (2008), the growth and coalescence of large primary voids (1–100 μm) are severely affected by the nucleation and growth of much smaller secondary voids (typically on the order 0.1–3 μm) at sufficiently high hydrostatic tension. It is well-known that the growth of secondary voids in the material surrounding primary voids weakens the ligaments and enhances the localization process, which accelerates the coalescence of the primary voids. Thus, the presence of a secondary void population can significantly lower the critical strain at which coalescence takes place at moderate to high stress triaxiality. One question that remains is: How does the presence of smaller secondary voids affect the interaction of primary voids when subject to intense shearing? The objective of this paper is to study this interaction and to bring out the softening effect of secondary voids on the coalescence of primary voids in a shear field, with no or very limited hydrostatic tension. Using a 2D plane strain finite element cell-model, the shear-extended Gurson model by Nahshon and Hutchinson (2008) is adopted to represent the softening effect of the secondary void population in the matrix material. By restricting the present study to the loading and material conditions considered in Tvergaard and Nielsen (2010), a rather accurate representation of the matrix material surrounding the primary voids is ensured, despite the phenomenological origin of the model by Nahshon and Hutchinson (2008). Since the study by Tvergaard and Nielsen (2010) involved no length scale, their findings can be scaled to any void size (neglecting size effects), and is easily interpreted as the material response of the matrix material containing smaller secondary voids. Obviously, this is an approximation since small voids are prone to size effects as plastic strain gradients toughen the surrounding material and thereby lowers the growth rate of small voids, when compared to the growth rate of large voids under same loading conditions (Liu et al., 2003; Niordson, 2008). Thus, directly applying the results from Tvergaard and Nielsen (2010) in the present work should be seen as upper bounds regarding the influence of secondary voids on coalescence.

The paper is structured as follows. The material model is presented in Section 2, where the boundary value problem and the numerical modelling approach is described in Sections 3 and 4, together with a comparison of two approaches to the contact problem discussed by Tvergaard (2008, 2009). Results are presented in Section 5, where the effect of a secondary void population in the ligaments between primary voids is illustrated. The concluding remarks are given in Section 6.

2. Material model

The employed material model is formulated in a convected coordinate Lagrangian framework and accounts for finite strain deformation. A general tensor notation is adopted, where ( ) and ( ) denote the covariant and contravariant components of a second order tensor, respectively, and denotes covariant differentiation in the reference frame. The incremental rates of the field quantities are denoted by ( ) (Budiansky, 1964; Hutchinson, 1973; Tvergaard, 1990).

2.1. The extended Gurson model

The shear-extended model by Nahshon and Hutchinson (2008) relies on the framework of the well-established micro-mechanics based Gurson model. Thus, the yield surface is given by

\[ \Phi = \frac{\sigma_M^2}{\sigma_M^2} + 2q_1 \cosh \left( \frac{q_2 \sigma_M}{2 \sigma_M} \right) - 1 + (q_3 f^r)^2 = 0 \]  

where the current state is characterized by \( \sigma_M \) the microscopic reference stress in the damage free material, \( \sigma^r \) the macroscopic Cauchy stresses describing the average stress field over the material in the convecting frame, \( f^r \) the damage parameter that represents the softening effect of an evolving void population (here, the second population), \( q_1 \) and \( q_3 \) the yield surface constants (Tvergaard, 1990). The Gurson model is formulated as isotropic where \( \sigma_r = \sqrt{3\sigma_{ij} \sigma_{ij}^r} \) is the effective Mises stress with \( \sigma_{ij} \) being the stress deviators of the Cauchy stress, given on the convected base vectors. Here, \( G^r \) and \( G^r_{ij} \) are the metric tensors of the convecting frame. In the present study, no coalescence of the (secondary) voids is accounted for. Instead, the calculation is terminated when damage parameter, \( f^r \), has reached unrealistic high values (close to one) in a sufficiently high volume of the material. All damage parameter values are shown in Table 1.

Using the shear-extended Gurson model, the damage growth rate is given by

\[ \dot{f} = (1 - f) G^r_{ij} \dot{e}^p_{ij} + \dot{e}^m_{ij} f_s \sqrt{2\pi} \exp \left[ -\frac{1}{2} \left( \frac{\dot{e}^m_{ij} - \varepsilon_0}{\varepsilon_0} \right)^2 \right] + k_s f \omega_0 \frac{S^v_i}{\sigma_r} \]  

where \( \omega_0 = 1 - \frac{27J_3}{2\sigma_r^2} \), \( J_3 = \frac{1}{3} G^r_{ij} s_is_js_j \) is the plastic strain increment, \( \dot{e}^m_{ij} \) is the increment of the macroscopic effective plastic strain, \( f_s \) is the secondary porosity to nucleate

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Youngs modulus</td>
<td>E</td>
<td>210 GPa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>ν</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield stress</td>
<td>σ_y</td>
<td>420 MPa</td>
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<tr>
<td>Strain hardening</td>
<td>N</td>
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<tr>
<td>Initial porosity</td>
<td>f_s</td>
<td>0.0–0.02</td>
</tr>
<tr>
<td>Porosity to nucleate</td>
<td>f_n</td>
<td>0.0–0.02</td>
</tr>
<tr>
<td>Mean nucleation strain</td>
<td>e_n</td>
<td>0.3–0.9</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>k_s</td>
<td>0.05</td>
</tr>
<tr>
<td>Yield surface constants</td>
<td>q_i, q_j</td>
<td>1.5, 1</td>
</tr>
<tr>
<td>Shear coefficient</td>
<td>k_w</td>
<td>0–2</td>
</tr>
</tbody>
</table>
be nucleated, \( \varepsilon_n \) is the main nucleation strain, and \( \varepsilon_s \) is the standard deviation of the mean nucleation strain.

In Eq. (2), the first term represents the growth of existing void, the second term accounts for void nucleation, and the third term introduced by Nahshon and Hutchinson (2008) allows for damage growth during low triaxiality shearing. Nahshon and Hutchinson (2008) introduced only one additional model parameter, \( k_v \in [0,3] \), which sets the magnitude of the damage growth rate in pure shear, while \( c_v = 0 \) in Eqs. (2) and (3) is formulated to vanish at an axi-symmetric stress state so that the modified model coincides with the original Gurson model assumptions. Consequently, it can be shown that, \( c_v \in [0,1] \) with \( c_v = 0 \) for an axi-symmetric stress state and \( c_v = 1 \) for all stress states combined by shear and hydrostatic pressure (for further discussions see Nahshon and Hutchinson, 2008; Nielsen and Tvergaard, 2009, 2010; Tvergaard and Nielsen, 2010).

The shear extension by Nahshon and Hutchinson (2008) is formulated to be consistent with the material softening due to micro-void deformation and interaction in shear. The modification is, however, purely phenomenological and the damage parameter, \( f \), is no longer tied to the void volume fraction, but must be regarded as an effective void volume fraction. Nevertheless, the model has recently been shown consistent with the void softening mechanism in shear based on cell model predictions (Tvergaard and Nielsen, 2010).

As for the original Gurson model, normality implies that the plastic strain increment is given by

\[
\dot{\varepsilon}_{pl} = \frac{1}{E_t} m_y \dot{u}_y \sigma^M
\]

with

\[
m_y = \frac{3}{2} \frac{S_\sigma}{\sigma_M} + \frac{1}{2} \dot{f}_y q_2 \sinh \left( \frac{q_2 \sigma^2}{2 \sigma_M} \right) \dot{G}_y
\]

where \( \sigma^M \) is the Jaumann stress rate, while the shear term in Eq. (2) leads to a slight change in the hardening modulus, \( h \), used in Eq. (4) (see Nahshon and Hutchinson (2008) for further details). The macroscopic and microscopic quantities in the model are coupled by assumed equality of the plastic work rate on the two levels: \( (1 - f) \sigma_M \dot{\varepsilon}_M - \sigma^M \dot{\varepsilon}_M \). Employing this together with: \( \dot{\varepsilon}_M = \left( \frac{1}{1 - f} \right) \dot{\sigma}_M \), the microscopic reference stress rate takes the form

\[
\dot{\sigma}_M = EE_t \sigma^M \frac{\dot{u}_y}{E_t \dot{u}_y + (1 - f) \dot{\sigma}_M}
\]

where \( E \) is Young’s modulus and \( E_t \) is the tangent modulus. The true stress–logarithmic strain curve in uni-axial tension of undamaged material is taken as

\[
E = \begin{cases} \frac{\sigma}{\dot{x}} & \text{for } \sigma_M < \sigma_y \\ \sigma_y \left( \frac{\sigma}{\sigma_y} \right)^{1/N} & \text{for } \sigma_M \geq \sigma_y \end{cases}
\]

where \( N \) is the strain hardening exponent and \( \sigma_y \) is the initial yield stress. All material parameter values are given in Table 1. The simple power hardening law is chosen to limit the number of model parameters, but more realistic hardening laws (e.g. the Voce law typically used for aluminum (Simar et al., 2010)) could equally well be used for the analysis.

### 3. Finite strain, finite element formulation

A convected coordinate Lagrangian framework is used for the finite strain formulation with the undeformed body described in a Cartesian reference denoted by \( \chi \). The principle of virtual work for the incremental problem can be written in the undeformed configuration as

\[
\int_V \left( \dot{\varepsilon}^{pl} \delta h_y + t^T \dot{u} \delta u_y \right) dV = \int_S T^T \delta u dS - \int_S T^T \delta u dS
\]

where \( V \) and \( S \) are the volume and surface, respectively, in the reference configuration. While, \( u^i \) and \( u_\ell \) are the contravariant and covariant components of the displacements vector, \( T \) are the contravariant components of the surface tractions, and \( f = f^M + f^V \) are the contravariant components of the Kirchhoff stress rate, work conjugate to the Lagrangian strain rate, \( \eta_v \). The total strain rate is here assumed to be the sum of an elastic and a plastic part, \( \eta_v = \eta_v^E + \eta_v^p \). The last term in square brackets in Eq. (8) is included as a means to eliminate residual equilibrium errors in the finite element formulation (see also Tvergaard, 1990).

Using the finite element method, the spatial domain governed by the field equations is discretized by 8 node isoparametric 2D elements, using reduced Gauss quadrature (2 by 2 Gauss points) for the integration. The numerical results are obtained using a linear incremental forward Euler procedure with limitations enforced on the microscopic plastic strain rate and the damage rate, so that the simulation remains rather stable up to failure. The numerical difficulties in the Gurson model when the material loses its stress carrying capacity \((f - 1/q_2)\) are treated by an element vanishing technique (Tvergaard, 1982b; Besson et al., 2003). Furthermore, the employed shear extended model inherits the well-known mesh dependency for the Gurson type models, displayed when plastic localization occurs. The role of the finite element mesh will be elaborated on in the following section on the model formulation.

### 4. Problem formulation

The boundary value problem considered in this study is that also considered by Tvergaard (2008, 2009), Tvergaard and Nielsen (2010), where coalescence of a single row of voids in a bulk material, subject to intense shearing, has been analyzed. In the present study, the focus is on the effect of a second void population that either co-exists with or nucleates during shear localization between larger discretely modeled voids. As in Tvergaard (2008, 2009), assuming plane strain conditions, the discretely modeled voids are initially circular cylindrical of radius, \( R_0 \), and are arranged in a periodic array with the void spacing \( 2A_0 \) in the \( x^1 \)-direction and \( 2B_0 \) in the \( x^2 \)-direction, according to Fig. 1 \((B_0/A_0 = 4 \) for all calculations). A representative volume element of unit thickness is considered in the numerical analysis (highlighted in Fig. 1(a)), with periodic boundary conditions applied along the left \((x^1 = -A_0)\) and right \((x^1 = A_0)\) edge so that \( u^i(-A_0,x^2) = u^i(A_0,x^2) \) and \( u^i(-A_0,x^2) = u^i(A_0,x^2) \). An incremental load is applied along the top \((x^2 = B_0)\) and bottom \((x^2 = -B_0)\) of the volume considered so that

\[
\dot{u}^1 = \dot{U}_1, \quad \dot{u}^2 = \dot{U}_2 \quad \text{for } x^2 = B_0
\]

\[
\dot{u}^1 = -\dot{U}_1, \quad \dot{u}^2 = -\dot{U}_2 \quad \text{for } x^2 = -B_0
\]

where \( \dot{U}_1 \) is a constant prescribed deformation rate in the \( x^1 \)-direction, while \( \dot{U}_2 \) is continuously corrected so that the average stress ratio \( \kappa = \sum_{-12} / \sum_{12} \) is maintained throughout the calculations.

The average stresses at the top or bottom surface are calculated as

\[
\sum_{12} = \frac{1}{2A_0} \int_{-A_0}^{A_0} T^2 \, dx^1, \quad \sum_{-12} = \frac{1}{2A_0} \int_{-A_0}^{A_0} T^1 \, dx^1, \quad \text{for } x^2 = \pm B_0
\]
overall shear angle, $\psi$, is in the following defined by; $\tan(\psi) = U_l/(B_0 + U_l)$. The shear deformation makes the primary voids collapse into micro-cracks, and contact at the voids/cracks surface is therefore expected. To deal with this, the pseudo-contact algorithm used in Tvergaard (2008, 2009) is adopted. In Tvergaard (2008), an internal pressure is applied to the void surface if the average aspect ratio $p = W/l$ reaches a critical level, $\rho_c$. Here, $l$ is the maximum length of the primary void as it deforms into a micro-crack and $W = V_p/l$ is the average void width calculated from the current void volume, $V_p$, per unit dept in the $x^2$-direction. As discussed in Tvergaard (2008), a hydrostatic pressure loading can be applied to the void surface as

$$T^1 = -pa n_r, \quad \text{with} \quad x^r = \frac{1}{2} \epsilon_{ijk} e^{lmr} (g_{jl} + u_{jl})(g_{km} + u_{km})$$

(12)

where $n_r$ is the normal to the reference void surface, $g_{jl}$ is the metric tensor for the reference coordinate system (see Section 3) and $\epsilon_{ijk}$ the alternating tensor (or Levi-Civita tensor, Sewell, 1965). Applying a hydrostatic pressure to the void surface to avoid contact is, however, shown by Tvergaard (2009) to affect the onset of coalescence (see also Fig. 2). Thus, Tvergaard (2009) adapted the approach so that only a loading transverse to the line segment of length $l$ between the two end points of the deformed void is introduced as the $\rho < \rho_c$. The loads applied to the void surface in contact is thereby given as

$$P^1 = -(T^1 \sin(\phi) + T^2 \cos(\phi)) \sin(\phi)$$

(13)

$$P^2 = (T^1 \sin(\phi) + T^2 \cos(\phi)) \cos(\phi)$$

(14)

where $\phi$ is the angle of the void inclination from the $x^1$-direction to the line element, $l$. Employing Eqs. (13) and (14) a small additional contribution should be added to ensure moment equilibrium (see Tvergaard (2009) for further details). An example of applying both types of contact algorithms is shown in Fig. 2, while only the transverse loading approach will be used in the following calculations as it is thought to more closely resemble the contact problem.

The boundary value problem posed above and its solution possess 180° rotational symmetry about the $x^1$-axis such that only the region above the $x^1$-axis needs to be considered in the numerical model. Consistent with the rotational symmetry, the boundary conditions along $x^2 = 0$ for the upper part of the finite element mesh in Fig. 1(c) are: $u^1(x^1,0) = -u^1(-x^1,0)$ and $u^2(x^1,0) = -u^2(-x^1,0)$. These boundary conditions are applicable to strictly symmetric and anti-symmetric deformations, as-well as the present mixed problem (see also Nielsen and Hutchinson, 2010), and are imposed in the finite element code using a standard penalty approach (Zienkiewicz and Taylor, 2000). To verify the cell-model predictions, the material response is compared to the results in Tvergaard (2009) for different parameter settings (with no secondary voids). Fig. 2 shows an example of the material response for both the full mesh and the rotational symmetric mesh, which compares well to the predictions by Tvergaard (2009). The predictions from the two mesh types are, furthermore, seen to coincide. Thus, the rotational symmetric mesh is employed in the remaining study to lower the calculation time.

As discussed previously, the Gurson model displays mesh sensitivity when localization occurs. Thus, the effect of the finite element mesh on the results of interest is brought out by Fig. 3 where the material response (average shear stress vs. average shear angle) is presented for one typical case of material parameters and for three meshes with close to square elements of dimensions $L_e$ near the discretely modeled void. Prior to localization in the ligament between the primary voids, there is essentially no
mesh dependence because all three meshes are fine enough to predict the shear deformation of the void. But, the subsequent growth of the localization and failure in the ligament is directly tied to the element size (see also discussion in Nielsen and Hutchinson, 2010). Fig. 3 clearly indicates that the final stage of the material response curves depends on the element size – the larger element gives thicker localization band and higher overall ductility of the material. In particular this well-known effect is seen for combined shear and tension ($j = 0.6$), where no void surface contact is observed. However, in the case of simple shear ($j = 0$), this model setup displays rather limited mesh sensitivity, which is ascribed to the interruption of the shear localization in the ligament as contact comes into play at the void surface. In the following, the finest mesh (with $L_e/R_0 = \pi/64$) is used throughout the analysis (see Fig. 1(b)).

5. Results: shear failure mechanism and overall material response

Tvergaard (2008, 2009), Tvergaard and Nielsen (2010) studied shear failure of ductile metallic material containing one population of voids and reported the following failure mechanism; as shearing of the material takes place voids are flattened out to micro-cracks, which rotate and elongate until interaction with neighboring micro-cracks gives coalescence and a peak load is attained. The failure mechanism in shear is thereby very different from that under loadings dominated by hydrostatic tension (Tvergaard, 1989; Fabrègue and Pardoen, 2008). In the present study, a similar overall shear failure mechanism is predicted for materials that contain two populations of voids on different length scales; one population of large primary voids, and one population of much smaller secondary voids which are present initially or nucleate between the primary voids during the deformation.

5.1. Effect of secondary voids which are present initially

For a comparison, the interaction of the large primary voids is shown in Fig. 4 for a damage free matrix (no second population), and in Fig. 5 for a matrix with a secondary void population denoted by the damage parameter, $f$. Here, using material properties similar to those in Tvergaard and Nielsen (2010), the ductile failure process is strongly affected by the existence of smaller voids in terms of the plastic flow localization in the ligaments. Fig. 4(a) and Fig. 5(a) show the microscopic plastic strain, $\varepsilon_p$, in an advanced deformation stage for simple shear loading ($j = 0$), while the corresponding evolution of the secondary damage, $f$, is illustrated in
In general, materials with co-existing void populations on different length scales experience the softening effect of the secondary damage in the following way: as shearing takes place a high-value band of secondary damage develops in the ligament between the primary voids until the void surfaces contact ($q > q_c$) for the larger voids (see Fig. 5(b)). During contact, the evolution of this softer band is temporarily slowed down while additional secondary damage is concentrated around the primary voids which continuously rotate and elongate (see Fig. 5(c)). After localization in the ligaments has occurred, the secondary damage continues to evolve in the soft band, while two additional high-value bands of secondary damage start to form (see Fig. 5(d)). Final failure (and element killing) in the ligaments occur as a combination between the different high-value bands (see Fig. 5(e)). This evolution of secondary damage significantly weakens the ligaments and enhances plastic flow localization near the tip of the micro-cracks as well as in the shear bands that span the entire width of the ligaments (compare Fig. 4(a) and Fig. 5(a)). During the collapse and the subsequent localization in the ligaments, a change in the local stress state near the primary voids occurs, which makes the local stress triaxiality differ substantially from that applied at the outer boundaries. This is easily seen from Fig. 4(b), where the local stress triaxiality is shown for the simple shear case ($\psi = 0.576$). Here, the local stress triaxiality reaches values ($T > 0.3$) that are far from the applied $T = 0$. Prior to the collapse, the stress state in the ligament more closely matches that at the boundaries, which...
under stationary conditions ($\dot{\epsilon}_c = 0$) can be approximated by $T \approx \kappa/\sqrt{3}$ (see also Appendix A).

It is seen from Fig. 5, that the addressed failure mechanism for ductile materials to some extent resembles what is already well-known as the formation of “wing cracks” in brittle materials, where existing micro-cracks grow during compressive loadings (Nemat-Nasser and Horii, 1982; Ashby and Hallam, 1986). Brittle materials that contain micro-cracks which initially are rotated relative to the compressive loading direction (the cracks are in shear), will experience sliding along the crack surfaces. This sliding makes the crack kink and grow in an angle relative to its original direction, which is very similar to the high-value bands of secondary damage seen from Fig. 5(e).

Considering Fig. 5(b)–(e), it should be kept in mind that the damage parameter, $f$, is not strictly tied to the volume fraction of secondary voids, but only reflects its softening effect. The enhanced localization due to the second population naturally affects the overall material response, both in terms of a highly reduced ductility and a much steeper drop in the load carrying capacity during the localization process in the ligaments. Figs. 6, 7 and 9 show the overall material response in terms for the average shear stress vs. average shear angle for a range of loading situations, initial material conditions and shear model settings. In the following, the onset of primary voids coalescence is identified to occur were the peak load, $\sigma_c$, is attained. The corresponding shear angle is denoted, $\psi_c$, which directly reflects the overall material ductility.

Fig. 6 brings out the effect of initially co-existing secondary voids for two loading conditions dominated by shearing ($\kappa = 0$ and 0.6). Here, using a shear-coefficient of $k_{\psi} = 1$ in the extended Gurson model. By increasing the initial volume fraction of secondary voids, $f_0$, a noticeable effect on the onset of coalescence, $\psi_c$, is predicted. In particular for $\kappa = 0.6$ (combined shear and tension) where the damage evolution is governed partly by void growth and partly by the softening effect from the shear-term introduced by Nahshon and Hutchinson (2008) in Eq. (2). By including even a small amount of the secondary voids a rather significant change in ductility for the damage free matrix (see Fig. 8).

To gain a parametric understanding of the shear extension to the Gurson model in Eq. (2), a similar study has been carried out for different values of the shear-coefficient, $k_{\psi}$. Examples of the overall material response are shown from Fig. 9 for different loading situations and different values of $k_{\psi}$, while Fig. 10 presents a more systematic study of the change in material ductility when altering the shear-coefficient, $k_{\psi}$, and the initial volume fraction of the secondary voids, $f_0$. It is seen from Fig. 10 that even the original Gurson model ($k_{\psi} = 0$) predicts a small change in the overall ductility for the simple shear case. This is due to the change in the local stress state near the collapsed primary voids (see Fig. 4(b) or the discussion in Appendix A).

The damage contribution from the shear-term in Eq. (2) is known to depend highly on the current stress state, thus Fig. 9 presents the material response for three different loading situations. The most significant change in material ductility is found for

![Fig. 6. Average shear stress vs. average shear angle for $f_0 = [0.0005, 0.01], f_\psi = 0, R_0/A_0 = 0.25, k_{\psi} = 1, \rho_1 = 0.15$ and $\kappa = [0.0, 0.6]$.](attachment:image.jpg)
simple shear ($\kappa = 0$), when altering $k_x$ from 1 to 2, for a secondary population with $f_0 > 0.005$. This dramatic decrease in ductility is strongly coupled to the change in surface contact conditions for the primary voids (see Fig. 10). As discussed previously, the secondary damage evolution in the ligaments temporarily slows down during contact (see Fig. 5), which increases the overall material ductility. Thus, the combined high amplification of the shear term (high $k_{\omega}$) and the absence of contact at the primary void surface significantly decreases the ductility. Similar effects are seen for combined shear and tension (e.g. $\kappa = 0.6$). However, a more
A moderate change in the overall ductility is predicted for $\kappa = 0.6$, while the non-contact condition occurs at a slightly lower initial volume fraction of the secondary voids ($0 < f_0 < 0.005$).

Combining the findings for the change in contact conditions with the failure mechanism revealed in Fig. 5(b)–(e), it seems that the contact between the primary void surfaces and the corresponding delay of the secondary damage evolution in a single band helps to increase the overall ductility of the material. This is also evident from Fig. 10, where a substantially lower ductility is found whenever no contact is predicted.

Compared to the two load cases $\kappa = 0$ and 0.6, the effect of including secondary damage is somewhat smaller in the case of combined shear and compression ($\kappa = 0.6$ in Fig. 9). For $k_x = 1$ only a very limited change in the tensile curves is observed, while the change is much more evident for $k_x = 2$. This is due to the competition between: (i) the inherited void closure in compression from the original Gurson model, and (ii) the continuous damage increase coming from the shear-term in Eq. (2). For sufficiently high compression the void closure will take over in the extended Gurson model, leading to a negative damage growth ($f < 0$), and no effect or a very limited effect of the second void population will be noticeable. This mechanism is elaborated on in Appendix A.

5.2. Effect of secondary damage that nucleates during deformation

The study of the softening effect coming from an initially present second population of voids is repeated in the following for the case where the smaller secondary voids nucleate during intense plastic deformation of the matrix material. The nucleation of smaller voids is often reported in the literature for metallic alloys. In general it is found that, for materials containing a nucleating second void population, the predicted failure mechanism is rather similar to the case of initially co-existing secondary voids; as the primary voids collapse during shearing, the ligaments undergo intense plastic deformation which makes the smaller voids nucleate in a rather narrow band until the surface of the primary voids come in contact ($\rho \approx \rho_c$) (see Fig. 11(a)). During contact, the evolution of this softer band is temporarily slowed down while additional

Fig. 10. Critical average shear angle at maximum load (coalescence) for $f_0 = [0,0.02]$, $f_n = 0$, $R_0/A_0 = 0.25$, $k_x = [0,1,2]$, $\rho_c = 0.15$ and $\kappa = [0,0.6]$.

Fig. 11. Curves of constant (a)–(d) secondary damage, $f$, ($f_0 = 0$, $f_n = 0.01$), and (e) microscopic effective plastic strain, $\epsilon_{ei}$, near primary voids when subject to simple shear, $\kappa = 0$, ($R_0/A_0 = 0.25$, $\rho_0 = 0.15$, $k_x = 1$, $\lambda_n = 0.6$ and $\lambda_s = 0.05$).
secondary damage is concentrated near the crack-tips of the primary voids that continuously rotate and elongate (see Fig. 11(b)). During localization in the ligaments, the secondary damage continues to evolve in the soft band, while two additional high-value bands are formed (see Fig. 11(c)). Failure in the ligaments occurs as a combination between the different high-value bands (see Fig. 11(d)). As previously mentioned, the damage parameter, \( f \), in Fig. 11(a)–(d) is not strictly tied to the volume fraction of second-

![Fig. 12. Average shear stress vs. average shear angle for; \( f_0 = 0, f_n = [0, 0.01, 0.02], R_0/A_0 = 0.25, \rho_v = 0.15, k_m = 1, \varepsilon_n = 0.6 \) and \( \kappa = [0, 0.6] \).](image1)

![Fig. 13. Average shear stress vs. average shear angle for \( f_0 = 0, f_n = 0.01, R_0/A_0 = 0.25, \rho_v = 0.15, k_m = 1, \varepsilon_n = [0.3, 0.6, 0.9, \infty] \) and \( \kappa = [0, 0.6] \).](image2)

![Fig. 14. Critical average shear angle at maximum load (coalescence) for; \( f_0 = 0.01, f_n = 0, \varepsilon_n = [0, 0.6, 0.9, \infty], R_0/A_0 = 0.25, k_m = 1, \rho_v = 0.15 \) and \( \kappa = [0, 0.6] \).](image3)
ary voids. The microscopic plastic strain, \( e_{\text{pl}} \), corresponding to the deformation stage just before failure is shown in Fig. 11(d).

Fig. 12 shows the predicted overall material response when altering the secondary void volume fraction to be nucleated from \( f_n = 0 \) to 0.01. Here shown for simple shearing (\( \kappa = 0 \)) and combined shear and tension (\( \kappa = 0.6 \)), respectively. As for an initially co-existing secondary population of voids, the softening effect of nucleating smaller secondary voids is obvious. But, the reduction in overall ductility is seen to be more moderate, and the drop in the load carrying capacity is less abrupt compared to the case of initially co-existing secondary voids (compare Figs. 6 and 12). In particular, a significant change in the final drop of the load carrying capacity is seen for combined shear and tension when \( f_0 \geq 0.005 \) or \( f_n \geq 0.005 \). The softening effect of a nucleating second population is, however, rather dependent on the mean nucleation strain, \( e_{\text{nm}} \), in Eq. (2). Fig. 13 shows the material response for different values of \( e_{\text{nm}} \) for a void volume fraction of \( f_n = 0.01 \). By increasing the mean nucleation strain, the overall ductility is increased due to the later nucleation of the majority of the smaller voids, thus the softening of the ligaments takes place much later.

The results presented in Figs. 12 and 13 are combined in Fig. 14 to illustrate the change in the critical shear angle at coalescence, \( \psi_c \), (thus ductility) for a wider range of volume fractions of secondary damage that can nucleate during the deformation. It is seen that, for both loading cases (\( \kappa = 0 \) and 0.6), a drop in the overall ductility occurs as the porosity to be

**Fig. 15.** Correlation between the critical average shear angle, \( \psi_c \), and the corresponding effective stress, \( \sigma_c = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3 \), at maximum load. All cell-model calculations carried out in the remaining paper are considered.

**Fig. 16.** Curves of zero damage growth (\( \dot{f} = 0 \)) at combined shear loading and compression (\( \kappa < 0 \)). Here, assuming stationary conditions (\( \dot{e}_{\text{pl}} = 0 \)) for the deformation of a homogeneous material with the applied loadings specified in Section 4 (\( \sigma_{12} = \sigma \) and \( \sigma_{11} = \sigma_{22} = \sigma_{33} = \kappa \sigma \)).
nucleated increases. Similarly, it is seen that the largest change in ductility occurs if the secondary voids nucleate early in the deformation process. For \( m_0 \to \infty \), no influence of the second population is predicted, since the voids never nucleate, and the material response therefore corresponds to that of a damage free matrix material.

A similar analysis has been carried out for the standard deviation, \( s_n \), that governs the range in which the secondary voids will nucleate (see Eq. (2)). It was found that for materials with; \( s_n \in [0.025, 0.1] \), the overall response almost coincided.

5.3. Correlation between peak load, \( \sigma_c \), and overall ductility, \( \psi_c \)

Despite the large change in the overall ductility, \( \psi_c \), when introducing a second population of voids that initially co-exist with or nucleates between primary voids only a limited effect on the corresponding peak load, \( \sigma_c \), is predicted. From Figs. 6, 7, 9, 12, and 13 it can be seen that regardless of the secondary voids, the material response tends to follow the same behavior, with only little deviation. Thus, an empirical correlation between the peak load, \( \sigma_c \), and the onset on coalescence, \( \psi_c \), can be estimated. Fig. 15 shows the predicted peak load, \( \sigma_c \), vs. the critical shear angle at coalescence, \( \psi_c \), for all simulations carried out during the present study, together with a least square fit of a functional on the form;

\[
\frac{\sigma_c}{\sigma_y} = \frac{1}{\sqrt{3}} \left( 1 + \psi_c/\tilde{\psi} \right)^{1/3} \quad \text{. The least square fit gives a reference shear angle } \frac{\tilde{\psi}}{\psi_c} = 0.0251 \text{ of and a hardening of } \tilde{N} = 0.157 \text{. Based on this empirical correlation, the change in the peak load, } \sigma_c \text{, can easily be backed out from the wide range of results for } \psi_c \text{ presented in Figs. 9, 10 and 13.}
\]

6. Concluding remarks

The numerical studies presented in Tvergaard (2008, 2009), Tvergaard and Nielsen (2010) are combined in the present study to analyze the overall response of ductile metallic materials that contain primary and secondary voids, subject to intense shearing. The predicted shear failure mechanism for such materials is found to be very different from that in tension (Fabrègue and Pardoen, 2008), but are in general displaying the same influence of a matrix enriched by smaller secondary voids. Thus, the overall material ductility is found to be widely affected by the presence of a secondary void population. More specifically, the key findings of this study are:

- **The failure mechanism** for ductile materials that contain two populations of voids on different length scales is illustrated for the two cases where secondary voids either co-exist with or nucleate between the primary voids during intense shear deformation. The contact condition at the primary void surfaces is found to play a major role in the evolution of the secondary damage and hence on the softening of the internal ligaments (see Figs. 4, 5 and 11). It is found that the addressed ductile failure resembles the mechanism in play during the formation of so-called “wing cracks” during failure of brittle materials at compressive loadings (Nemat-Nasser and Hori, 1982; Ashby and Hallam, 1986).

- **The presence of secondary voids** is found to significantly lower the overall critical shear angle at coalescence, \( \psi_c \), (thus the material ductility), even for a rather low volume fraction of the smaller voids. For both co-existing and nucleating secondary voids, the overall ductility decreases with increasing volume fraction (see Figs. 6, 8, 10, 12). An increase in the mean nucleation strain increases the material ductility (see Figs. 13 and 14).

- **The relative size of the primary voids** is found to increase the material ductility when the primary voids are made smaller (the primary void volume fraction decreases, see Figs. 7 and 8). The change in ductility is, however, dependent on the applied loading and the volume fraction of secondary voids.

- **The shear-coefficient in the extended Gurson model** is found to lower the material ductility, when increased (as intended by Naishon and Hutchinson, 2008). However, the shear extension is highly dependent on the current stress state, and a transition to negative damage growth \( \left( f < 0 \text{ for } f > 0 \right) \) can still occur in the shear extended Gurson model, e.g. for combined shear and compression (see Figs. 9 and 10 and Appendix A).

As shown in Fig. 3, the shear extended Gurson model inherits the well-known mesh dependency, thus the width of the localization band between the primary voids is determined by the element size (see also Nielsen and Hutchinson, 2010). Finer meshes (smaller elements) therefore enhance the plastic flow localization in the softer region of ligaments, and lead to more localized secondary damage. Similarly, the model predictions are sensitive the critical void aspect ratio, \( \psi_c \), used in the contact algorithm (Tvergaard (2008, 2009)). Thus, the results presented should not be regarded as quantitative predictions. Nevertheless, the present study gives an insight to how the presence of small secondary voids or defects affects the interaction between primary voids when subject to intense shearing.

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Appendix A

By neglecting void nucleation in Eq. (2), the damage growth rate consists of two parts; (i) the first governs ordinary void growth, and (ii) the second governs the softening effect in shear. In combined shear and compression \( (k < 0) \), the first term will try to close the voids (as in the original Gurson model), while the second term continuously tries to increase the void volume fraction to mimic the softening effect of voids that collapse and rotate. These competing mechanisms lead to the following requirement to the shear-coefficient, \( k_{\psi} \), for damage growth \( (f > 0 \text{ when } f > 0) \) to occur;

\[
k_{\psi} > 3 \alpha (1 - f) \sigma_{\text{ REF}} C_\psi C_\sigma, \quad \text{with } x = q_1 q_2 f \sinh \left( q_2 r_1^2 / (2 r_{\text{REF}}) \right) / 2.
\]

For a homogeneous material loaded in combined shear and compression \( (k < 0) \), and enforced with the periodic boundary conditions specified in Section 4, the stress state is given by: \( \sigma_{12} = \sigma \text{ and } \sigma_{11} = \sigma_{22} = \sigma_{33} = k \sigma \) for stationary conditions \( (i_\psi = 0) \). The requirement to \( k_{\psi} \), for damage growth and thereby failure to occur is illustrated in Fig. 16 for this specific stress field. In case of \( k = -0.6 \) (as in Fig. 9), Section 5.1), it is seen that a transition from negative \( (f < 0) \) to positive \( (f > 0) \) damage growth occurs when \( k_{\psi} \) is between 1 and 2. This explains the rather significant change in the material response seen from Fig. 9, when increasing the shear-coefficient, \( k_{\psi} \), from 1 to 2. However, for \( k_{\psi} = 1 \) a small effect of the shear-term is predicted by finite element cell-model when \( k = -0.6 \). This is due to the applied loading, \( \kappa = \sum_{22}/\sum_{12} \), of the unit-cell, which is an average value for the stresses on any
cross-section parallel to the $x^3$-axis in Fig. 1. The local stress state near the collapsed primary voids can thereby differ from the overall condition (see also Fig. 4(b)). Thus, Fig. 16 cannot give an exact $\kappa_{x^3}$-value for where the transition, $f = 0$, will occur in the cell-model, but only highlight the mechanisms in play.

References