View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Elsevier - Publisher Connector

Physics Letters B 690 (2010) 519–525

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Gauge invariance, causality and gluonic poles

I.V. Anikin *, O.V. Teryaev

Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia

ARTICLE INFO

Article history:
Received 20 March 2010
Received in revised form 27 May 2010
Accepted 29 May 2010
Available online 1 June 2010
Editor: J.-P. Blaizot

Keywords:
Drell–Yan process
Gauge invariance
Gluonic poles

ABSTRACT

We explore the electromagnetic gauge invariance of the hadron tensor of the Drell–Yan process with one transversely polarized hadron. The special role is played by the contour gauge for gluon fields. The prescription for the gluonic pole in the twist 3 correlator is related to causality property and compared with the prescriptions for exclusive hard processes. As a result we get the extra contributions, which naively do not have an imaginary phase. The single spin asymmetry for the Drell–Yan process is accordingly enhanced by the factor of two.

© 2010 Published by Elsevier B.V. Open access under CC BY license.

1. Introduction

The problem of the electromagnetic gauge invariance in the deeply virtual Compton scattering (DVCS) and similar exclusive processes has intensively been discussed during last few years, see for example [1–5]. This development explored the similarity with the earlier studied inclusive spin-dependent processes [6], and the transverse component of momentum transfer in DVCS corresponds to the transverse spin in DIS.

The gauge invariance of relevant amplitudes is ensured by means of twist three contributions and the use of the equations of motion providing a possibility to exclude the three-particle (quark–gluon) correlators from the amplitude. After combining with the two-particle correlator contributions, one gets the gauge invariant expression for the physical amplitude or, in the case of lepton–hadron processes, for the corresponding hadron tensor [6].

This method was originally developed in the case of the particular inclusive processes with transverse polarized hadrons, like structure function g_2 in DIS [6] and Single Spin Asymmetry (SSA) [7] due to soft quark (fermionic poles [8]). At the same time, the colour gauge invariance of the so-called gluonic poles contributions [9] was previously explored [10] by other methods relying on the Wilson exponentials [11–14].

Here we combine the approaches described above and apply them in the relevant case of the Drell–Yan (DY) process where one of hadrons is the transversally polarized nucleon.

The SSA in the DY process was first considered in QCD in the case [15,16] of the longitudinally polarized hadron. This observable is especially interesting if the second hadron is a pion, because of the sensitivity [17,18] to the shape of pion distribution amplitude, being currently the object of major interest [19,20] (see also [21] and references therein).

The imaginary phases in the SSA with longitudinally polarized nucleon are due to the hard perturbative gluon loops [15, 16] or twist 4 contribution of the pion distribution amplitude [17,18,22]. At the same time, the source of the imaginary part, when one calculates the single spin asymmetry associated with $P + P^{\uparrow\downarrow} \to \ell\bar{\ell} + X$ process, is the quark propagator in the diagrams with quark–gluon (twist three) correlators. This leads [23] to the gluonic pole contribution to SSA. It has been reproduced (up to the derivative term, corresponding to the case of single inclusive Drell–Yan process, when only one of the leptons is observed) in the case of the non-zero boundary condition imposed on gluon fields, and the asymmetric boundary conditions have been considered as a privileged ones [24]. The reason is that these boundary conditions provide the purely real quark–gluon function $B^V(x_1, x_2)$ which parameterizes $\langle\bar{\psi}\gamma^+ A^T_\alpha \psi\rangle$ matrix element. By this fact the diagrams with two-particle correlators do not contribute to the imaginary part of the hadron tensor related to the SSA. This property seems quite natural, as the corresponding diagram does not have a cut capable of producing the imaginary phase [25].

In our Letter, we perform a thorough analysis of the transverse polarized DY hadron tensor in the light of the QED gauge invariance, the causality and gluonic pole contributions.

We show that to restore the electromagnetic gauge invariance of the transverse polarized DY hadron tensor, it is mandatory to add the extra diagram contribution (cf. [26] where the similar

* Corresponding author.
E-mail addresses: anikin@theor.jinr.ru (I.V. Anikin), teryaev@theor.jinr.ru (O.V. Teryaev).

0370-2693 © 2010 Published by Elsevier B.V. Open access under CC BY license.
doi:10.1016/j.physletb.2010.05.072

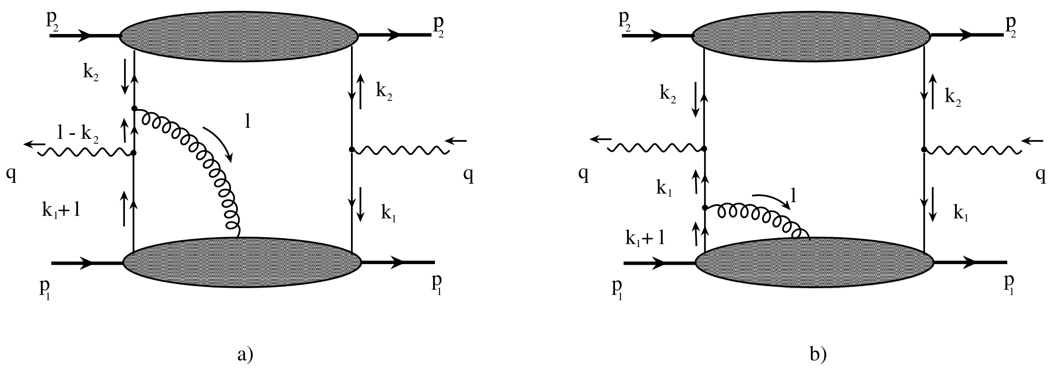

Fig. 1. The Feynman diagrams which contribute to the polarized Drell–Yan hadron tensor.

contribution was associated with the so-called special propagator), also at the twist three level. In contrast to the naive assumption, we demonstrate that this new additional contribution is directly related to the certain complex prescription in the gluonic pole $1/(x_1 - x_2)$ of the quark–gluon function $B^V(x_1, x_2)$. It is essential that this prescription is process-dependent, supporting the idea of effective process-dependent Sivers function (see, e.g., [27] and references therein) related to this correlator.

In more detail, we show that the causal pole prescription in the quark propagator, involved in the hard part of the standard diagram, supports the choice of a contour gauge and, in turn, the representation of the quark–gluon function $B^V(x_1, x_2)$ in the form of the gluonic pole with the mentioned complex prescription. This representation must be extended on the diagram, which naively does not contribute to the imaginary part. They ensure an extra contribution to the imaginary part which is necessary to maintain the electromagnetic gauge invariance. Finally, the account for this extra contributions corrects the SSA formula for the transverse polarized Drell–Yan process by the factor of 2.

Our analysis is also important in view of the recent investigation of DY process within both the collinear and the transverse momentum factorization schemes with hadrons replaced by on-shell parton states [28]. They examined these two factorization approaches and claimed the substantial differences between them in the calculations of the angular asymmetries. This may be compared with the calculations of the angular distribution of the DY lepton pair production in the framework of the transverse momentum-dependent factorization approach [29]. It was found that in the intermediate transverse momentum region the collinear factorization and the transverse momentum-dependent factorization are consistent in the description of the lepton pair angular distributions. This corrected the earlier claims of [30], questioning (like [28]) the unique predictions for the SSAs within collinear and transverse momentum-dependent factorization approaches. Because of these controversies, the properties of transverse momentum integrated SSAs which we are elaborating here are of additional importance.

2. Causality and contour gauge for the gluonic pole

We study the contribution to the hadron tensor which is related to the single spin (left–right) asymmetry measured in the Drell–Yan process with the transversely polarized nucleon: $N^{(\uparrow\downarrow)}(p_1) + N(p_2) \to \gamma^*(q) + X(P_X) \to \ell(l_1) + \bar{\ell}(l_2) + X(P_X)$, where the virtual photon producing the lepton pair ($l_1 + l_2 = q$) has a large mass squared ($q^2 = Q^2$) while the transverse momenta are small and integrated out. The left–right asymmetry means that the transverse momenta of the leptons are correlated with the direction $\mathbf{S} \times \mathbf{e}_z$ where S_μ implies the transverse polarization vector of the nucleon while $\mathbf{e}_z$ is a beam direction [31].

The DY process with the transversely polarized target manifests [23] the gluonic pole contributions. Since we perform our calculations within a *collinear* factorization, it is convenient (see, e.g., [32]) to fix the dominant light-cone directions for the DY process shown in Fig. 1

$$p_1 \approx \frac{Q}{x_B\sqrt{2}} n^*, \qquad p_2 \approx \frac{Q}{y_B\sqrt{2}} n \quad \text{with}$$
$$n^*_\mu = \big(1/\sqrt{2}, \mathbf{0}_T, 1/\sqrt{2}\big), \qquad n_\mu = \big(1/\sqrt{2}, \mathbf{0}_T, -1/\sqrt{2}\big), \tag{1}$$

so that the hadron momenta p_1 and p_2 have the plus and minus dominant light-cone components, respectively. Accordingly, the quark and gluon momenta k_1 and ℓ lie along the plus direction while the antiquark momentum k_2 – along the minus direction.

Focusing on the Dirac vector projection, containing the gluonic pole, let us start with the standard hadron tensor generated by the diagram depicted in Fig. 1(a):

$$\mathcal{W}^{(1)}_{\mu\nu} = \int d^4k_1\, d^4k_2\, \delta^{(4)}(k_1 + k_2 - q) \int d^4\ell\, \Phi^{(A)[\gamma^+]}_\alpha(k_1, \ell) \bar{\Phi}^{[\gamma^-]}(k_2) \times \mathrm{tr}\bigg[\gamma_\mu \gamma^- \gamma_\nu \gamma^+ \gamma_\alpha \frac{\ell^+\gamma^- - k_2^-\gamma^+}{-2\ell^+ k_2^- + i\epsilon}\bigg], \tag{2}$$

where

$$\Phi^{(A)[\gamma^+]}_\alpha(k_1, \ell) \stackrel{\mathcal{F}_2}{=} \big\langle p_1, S^T \big| \bar{\psi}(\eta_1)\gamma^+ g A_\alpha(z)\psi(0) \big| S^T, p_1 \big\rangle,$$
$$\bar{\Phi}^{[\gamma^-]}(k_2) \stackrel{\mathcal{F}_1}{=} \langle p_2 | \bar{\psi}(\eta_2)\gamma^- \psi(0) | p_2 \rangle. \tag{3}$$

Throughout this Letter, $\mathcal{F}_1$ and $\mathcal{F}_2$ denote the Fourier transformation with the measures

$$d^4\eta_2\, e^{ik_2\cdot\eta_2} \quad \text{and} \quad d^4\eta_1\, d^4z\, e^{-ik_1\cdot\eta_1 - i\ell\cdot z}, \tag{4}$$

respectively, while $\mathcal{F}_1^{-1}$ and $\mathcal{F}_2^{-1}$ mark the inverse Fourier transformation with the measures

$$dy e^{iy\lambda} \quad \text{and} \quad dx_1\, dx_2 e^{ix_1\lambda_1 + i(x_2 - x_1)\lambda_2}. \tag{5}$$

Analyzing the γ-structure of (2), we may conclude that the first term in the quark propagator singles out the combination: $\gamma^+\gamma_\alpha\gamma^-$ with $\alpha = T$ which will lead to the matrix element of the twist three operator, $\langle \bar{\psi}\gamma^+ A^T_\alpha \psi \rangle$ with the transverse gluon field. After factorization, this matrix element will be parametrized via the function $B^V(x_1, x_2)$. The second term in the numerator of the quark propagator separates out the combination $\gamma^+\gamma_\alpha\gamma^+$ with $\alpha = -$. Therefore, this term will give $\langle \bar{\psi}\gamma^+ A^+ \psi \rangle$ which, as we will see now, will be exponentiated in the Wilson line $[-\infty^-, 0^-]$. Indeed, this part of the standard hadron tensor is given by

$$\mathcal{W}^{(1)[k_2^-\text{-term}]}_{\mu\nu} = \int d\mu(k_i; x_1, y)\, \mathrm{tr}\big[\gamma_\mu \gamma^- \gamma_\nu \gamma^+ \gamma^- \gamma^+\big] \bar{\Phi}^{[\gamma^-]}(k_2) \times \frac{1}{2}\int dz^- \int d\ell^+ \frac{e^{-i\ell^+ z^-}}{\ell^+ - i\epsilon} \int d^4\eta_1\, e^{-ik_1\cdot\eta_1} \times \langle p_1, S^T \big| \bar{\psi}(\eta_1)\gamma^+ g A^+\big(0, z^-, \vec{\mathbf{0}}_T\big)\psi(0)\big| S^T, p_1\rangle, \tag{6}$$

where

$$d\mu(k_i; x_1, y) = dx_1\, d^4k_1\, \delta\big(x_1 - k_1^+/p_1^+\big)\, dy\, d^4k_2\, \delta\big(y - k_2^-/p_2^-\big) \times \big[\delta^{(4)}(x_1 p_1 + y p_2 - q)\big]. \tag{7}$$

Note that the prescription $-i\epsilon$ in the denominator of this expression directly follows from the standard (see, e.g., [33]) causal prescription for the massless quark propagator in (2).

Integrating over ℓ^+, one can immediately obtain the corresponding θ-function in (6):

$$\mathcal{W}^{(1)[k_2^-\text{-term}]}_{\mu\nu} = \int d\mu(k_i; x_1, y)\, \mathrm{tr}\big[\gamma_\mu \gamma^- \gamma_\nu \gamma^+\big] \bar{\Phi}^{[\gamma^-]}(k_2) \times \int d^4\eta_1\, e^{-ik_1\cdot\eta_1} \langle p_1, S^T \big| \bar{\psi}(\eta_1)\gamma^+ ig \times \int_{-\infty}^{+\infty} dz^-\, \theta\big(-z^-\big) A^+\big(0, z^-, \vec{\mathbf{0}}_T\big)\psi(0)\big| S^T, p_1\rangle. \tag{8}$$

Including all gluon emissions from the antiquark going from the upper blob in Fig. 1(a) (the so-called initial state interactions), we get the corresponding P-exponential in $\Phi^{(A)[\gamma^+]}_\alpha(k_1, \ell)$. The latter is now represented by the following matrix element:

$$\int d^4\eta_1\, e^{-ik_1\cdot\eta_1} \langle p_1, S^T \big| \bar{\psi}(\eta_1)\gamma^+ \big[-\infty^-, 0^-\big]\psi(0)\big| S^T, p_1\rangle, \tag{9}$$

where

$$\big[-\infty^-, 0^-\big] = P\exp\left\{-ig \int_{-\infty}^{0} dz^-\, A^+\big(0, z^-, \vec{\mathbf{0}}_T\big)\right\}. \tag{10}$$

If we include in the consideration the gluon emission from the incoming antiquark (the mirror contributions), we will obtain the Wilson line $[\eta_1^-, -\infty^-]$ which will ultimately give us, together with (10), the Wilson line connecting the points 0 and η_1 in (9). This is exactly what happens, say, in the spin-averaged DY process [34]. However, for the SSA, these two diagrams should be considered individually. Indeed, their contributions to SSAs, contrary to spin-averaged case, differ in sign and the dependence on the boundary point at $-\infty^-$ does not cancel.

To eliminate the unphysical gluons from our consideration and use the factorization scheme [6], we may choose a *contour* gauge [35]

$$\big[-\infty^-, 0^-\big] = 1 \tag{11}$$

which actually implies also the axial gauge $A^+ = 0$ used in [6].

Let us discuss the problem of gauge choice in more detail. In (11), the so-called starting point x_0 (see [35]) is fixed to be at $-\infty^-$ owing to the certain complex prescription $+i\epsilon$ in the quark propagator in (2). If we would change the starting point x_0 on $+\infty^-$, this would correspond to the choice of the "anticausal" complex prescription $-i\epsilon$. On the other hand, the axial gauge $A^+ = 0$ is independent on the choice of x_0 and we are able to eliminate the Wilson line by choosing simply $A^+ = 0$ without referring to the starting point x_0. Nevertheless, since our prescription $+i\epsilon$ in the quark propagator uniquely fixes the starting point x_0 at $-\infty$, the expression for the Wilson line (10) hints the choice of gauge (11).

Imposing this gauge one arrives [35] at the following representation of the gluon field in terms of the strength tensor:

$$A^\mu(z) = \int_{-\infty}^{\infty} d\omega^-\, \theta\big(z^- - \omega^-\big) G^{+\mu}\big(\omega^-\big) + A^\mu(-\infty). \tag{12}$$

Moreover, as we will demonstrate below, if we choose instead an alternative representation for the gluon in the form:

$$A^\mu(z) = -\int_{-\infty}^{\infty} d\omega^-\, \theta\big(\omega^- - z^-\big) G^{+\mu}\big(\omega^-\big) + A^\mu(\infty) \tag{13}$$

(which corresponds to the gauge condition $[+\infty^-, 0^-] = 1$ and also results in $A^+ = 0$) keeping the causal prescription $+i\epsilon$ in (2), the cost of this will be the breaking of the electromagnetic gauge invariance for the DY tensor.

We are now ready to pass to the term with $\ell^+\gamma^-$ in (2) which gives us finally the matrix element of the twist three quark–gluon operator with the transverse gluon field. Let us stop, in more detail, on the parametrization of the relevant matrix elements:

$$\langle p_1, S^T \big| \bar{\psi}(\lambda_1\tilde{n})\gamma_\beta g A^T_\alpha(\lambda_2\tilde{n})\psi(0)\big| S^T, p_1\rangle \overset{\mathcal{F}_2^{-1}}{=} i\varepsilon_{\beta\alpha S^T p_1} B^V(x_1, x_2). \tag{14}$$

Using the representation (12), this function can be expressed as

$$B^V(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\epsilon} + \delta(x_1 - x_2) B^V_{A(-\infty)}(x_1), \tag{15}$$

where the real regular function $T(x_1, x_2)$ ($T(x, x) \neq 0$) parametrizes the vector matrix element of the operator involving the tensor $G_{\mu\nu}$ (cf. [36]):

$$\langle p_1, S^T \big| \bar{\psi}(\lambda_1\tilde{n})\gamma_\beta \tilde{n}_\nu G_{\nu\alpha}(\lambda_2\tilde{n})\psi(0)\big| S^T, p_1\rangle \overset{\mathcal{F}_2^{-1}}{=} \varepsilon_{\beta\alpha S^T p_1} T(x_1, x_2). \tag{16}$$

Owing to the time-reversal invariance, the function $B^V_{A(-\infty)}(x_1)$,

$$i\varepsilon_{\beta\alpha S^T p_1} \delta(x_1 - x_2) B^V_{A(\pm\infty)}(x_1) \overset{\mathcal{F}}{=} \langle p_1, S^T \big| \bar{\psi}(\lambda_1\tilde{n})\gamma_\beta g A^T_\alpha(\pm\infty)\psi(0)\big| S^T, p_1\rangle, \tag{17}$$

can be chosen as

$$B^V_{A(-\infty)}(x) = 0. \tag{18}$$

Indeed, the function $B^V(x_1, x_2)$ is an antisymmetric function of its arguments [6], while the antisymmetrization of the additional term with $B^V_{A(-\infty)}(x_1)$ gives zero.

There is no doubt that the only source of the imaginary part of the hadron tensor is the quark propagator. One may try to realize this property by assumption that matrix elements are purely real,

$$B^V(x_1, x_2) = \frac{\mathcal{P}}{x_1 - x_2} T(x_1, x_2), \tag{19}$$

corresponding to asymmetric boundary condition for gluons [24]:

$$B^V_{A(\infty)}(x) = -B^V_{A(-\infty)}(x). \tag{20}$$

Here we suggest another way of reasoning. The causal prescription for the quark propagator, generating its imaginary part, simultaneously leads to the imaginary part of the gluonic pole. Let us emphasize that this does not mean the appearance of imaginary part of matrix element (which by itself does not have an explicit physical meaning) but rather the prescription of its convolution with hard part. This procedure is in agreement with the prescriptions which were appeared in the exclusive case in the parametrization of the generalized gluon distributions [37,38].

This interplay of large and short distances is especially clear when our result is compared to approach [26] where the similar imaginary part appears in the special propagator formally included to the hard part, while we have a complex soft ingredient generated by its interaction with the hard part.

Note that the fixed complex prescription $+i\epsilon$ in the gluonic pole of $B^V(x_1,x_2)$ (see (15)) is one of our main results and is very crucial for an extra contribution to hadron tensor we are now ready to explore. Indeed, the gauge condition must be the same for all the diagrams, and it leads to the appearance of imaginary phase of the diagram (see Fig. 1(b)) which naively does not have it. Let us confirm this by explicit calculation.

3. Hadron tensor and gauge invariance

We now return to the hadron tensor and calculate the part involving $\ell^+\gamma^-$, obtaining the following expression for the standard hadron tensor (see the diagram in Fig. 1(a)):

$$\begin{aligned}
&\overline{\mathcal{W}}^{(1)[\ell^+\text{-term}]}_{\mu\nu} \\
&\quad = \int d^2\vec{\mathbf{q}}_T\, \mathcal{W}^{(1)}_{\mu\nu} \\
&\quad = -\int dx_1\, dy\, \big[\delta(x_1-x_B)\delta(y-y_B)\big]\bar{q}(y) \\
&\qquad \times \Im\mathrm{m}\int dx_2\, \mathrm{tr}\bigg[\gamma_\mu\gamma_\beta\gamma_\nu\hat{p}_2\gamma^T_\alpha \frac{(x_1-x_2)\hat{p}_1}{(x_1-x_2)ys+i\epsilon}\bigg] \\
&\qquad \times B^V(x_1,x_2)\varepsilon_{\beta\alpha S^T p_1},
\end{aligned} \tag{21}$$

where we used $\ell^+\gamma^- = (x_2-x_1)\hat{p}_1$ and

$$\langle p_2|\bar{\psi}(\lambda\tilde{n}^*)\gamma_\mu\psi(0)|p_2\rangle \overset{\mathcal{F}_1^{-1}}{=} p_{2\mu}\bar{q}(y). \tag{22}$$

We are now in position to check the QED gauge invariance by contraction with the photon momentum q_μ. Calculating the trace

$$\begin{aligned}
&\frac{1}{4}(x_1-x_2)\varepsilon_{\beta\alpha S^T p_1}\,\mathrm{tr}\big[\hat{q}\gamma_\beta\gamma_\nu\hat{p}_2\gamma^T_\alpha\hat{p}_1\big] \\
&\quad = \varepsilon_{\alpha p_2 S^T p_1} g^T_{\alpha\nu}\, y(x_1-x_2)s,
\end{aligned} \tag{23}$$

one gets

$$\begin{aligned}
q_\mu\overline{\mathcal{W}}^{(1)}_{\mu\nu} = &-\int dx_1\, dy\, \big[\delta(x_1-x_B)\delta(y-y_B)\big]\bar{q}(y)\varepsilon_{\nu p_2 S^T p_1} \\
&\times \int_{-1}^{1} dx_2\, \Im\mathrm{m}\frac{x_1-x_2}{x_1-x_2+i\epsilon} B^V(x_1,x_2) \neq 0,
\end{aligned} \tag{24}$$

if the gluonic pole is present. Note that here and below we consider only the imaginary part of the hadron tensor (as for any single spin asymmetry).

Let us analyze this problem from a viewpoint of the so-called ξ-process (see [33], Section 33.2) applied for the partonic subprocess. Generally speaking, the single diagram in Fig. 1(a) cannot give the gauge invariant hadron tensor. One needs the second diagram (cf. [26]) with the gluon insertion in the quark line, see Fig. 1(b).

We would like to emphasize that the diagrams which are analogous to Fig. 1(b) were considered in [26] as well. Note that in the mentioned paper the imaginary parts of these diagrams existed owing to the introduction of the so-called special propagators in the hard part of the hadron tensor. In contrast to that, as we will demonstrate below, our imaginary parts of the diagrams in Fig. 1(b) exist due to the three-particle function $B^V(x_1,x_2)$ working within the standard collinear factorization procedure.

We now focus on the contribution from the diagram depicted in Fig. 1(b). The corresponding hadron tensor takes the form:

$$\mathcal{W}^{(2)}_{\mu\nu} = \int d^4k_1\, d^4k_2\, \delta^{(4)}(k_1+k_2-q)\,\mathrm{tr}\big[\gamma_\mu\mathcal{F}(k_1)\gamma_\nu\bar{\Phi}(k_2)\big], \tag{25}$$

where the function $\mathcal{F}(k_1)$ reads

$$\begin{aligned}
\mathcal{F}(k_1) = S(k_1)\gamma_\alpha &\int d^4\eta_1\, e^{-ik_1\cdot\eta_1} \\
&\times \big\langle p_1, S^T\big|\bar{\psi}(\eta_1)gA^T_\alpha(0)\psi(0)\big|S^T, p_1\big\rangle.
\end{aligned} \tag{26}$$

Performing the collinear factorization, we derive the expression for the factorized hadron tensor which corresponds to the diagram in Fig. 1(b):

$$\begin{aligned}
\overline{\mathcal{W}}^{(2)}_{\mu\nu} = &\int dx_1\, dy\, \big[\delta(x_1-x_B)\delta(y-y_B)\big]\bar{q}(y) \\
&\times \mathrm{tr}\bigg[\gamma_\mu\bigg(\int d^4k_1\, \delta\big(x_1p_1^+-k_1^+\big)\mathcal{F}(k_1)\bigg)\gamma_\nu\hat{p}_2\bigg].
\end{aligned} \tag{27}$$

After some algebra, the integral over k_1 in (27) can be rewritten as

$$\begin{aligned}
&\int d^4k_1\, \delta\big(x_1p_1^+-k_1^+\big)\mathcal{F}^{[\gamma^+]}(k_1) \\
&\quad = \frac{\hat{p}_2\gamma^T_\alpha\gamma_\beta}{2p_2^-p_1^+}\varepsilon_{\beta\alpha S^T p_1}\frac{1}{x_1+i\epsilon}\int_{-1}^{1} dx_2\, B^V(x_1,x_2),
\end{aligned} \tag{28}$$

where the parametrization (14) has been used. Taking into account (28) and calculating the Dirac trace, the contraction of the tensor $\overline{\mathcal{W}}^{(2)}_{\mu\nu}$ with the photon momentum q_μ gives us

$$\begin{aligned}
q_\mu\overline{\mathcal{W}}^{(2)}_{\mu\nu} = &\int dx_1\, dy\, \big[\delta(x_1-x_B)\delta(y-y_B)\big]\bar{q}(y)\varepsilon_{\nu p_2 S^T p_1} \\
&\times \int_{-1}^{1} dx_2\, \Im\mathrm{m} B^V(x_1,x_2).
\end{aligned} \tag{29}$$

From this, one can observe that if the function $B^V(x_1,x_2)$ is the purely real one (see (19)), this part of the hadron tensor, which is associated with the diagram in Fig. 1(b), does not contribute to the imaginary part.

We now study the net effect of the $\overline{\mathcal{W}}^{(1)}_{\mu\nu}$ and $\overline{\mathcal{W}}^{(2)}_{\mu\nu}$ contributions and its role for the QED gauge invariance. Adding the contributions of (24) and (29), one can easily obtain:

$$\begin{aligned}
&q_\mu\overline{\mathcal{W}}^{(1)}_{\mu\nu} + q_\mu\overline{\mathcal{W}}^{(2)}_{\mu\nu} \\
&\quad = \varepsilon_{\nu p_2 S^T p_1}\bar{q}(y_B)\Im\mathrm{m}\int_{-1}^{1} dx_2\, B^V(x_B,x_2)\bigg[\frac{x_B-x_2}{x_B-x_2+i\epsilon}-1\bigg].
\end{aligned} \tag{30}$$

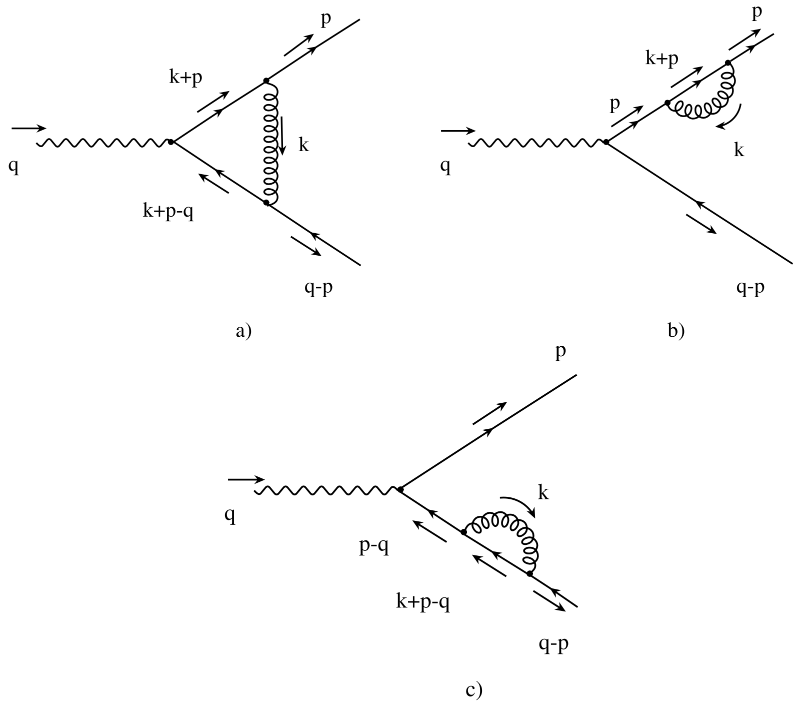

Fig. 2. The Feynman diagrams which contribute to the α^3-order amplitude in QED.

If we tacitly assume that $B^V(x_1, x_2)$ is some real and regular (at $x_1 = x_2$) function that the numerator and denominator in the first term inside the brackets are contracted and, as a result of this, both the first and second terms in (30) do not have an imaginary part. That would mean the electromagnetic gauge invariance for the tensor.

The existence of the gluonic pole changes the situation. Inserting now (15) into (30), one gets

$$
\begin{aligned}
&q_\mu \overline{\mathcal{W}}^{(1)}_{\mu\nu} + q_\mu \overline{\mathcal{W}}^{(2)}_{\mu\nu} \\
&\quad = \varepsilon_{\nu p_2 S^T p_1} \bar{q}(y_B) \\
&\qquad \times \Im\mathrm{m} \int_{-1}^{1} dx_2\, T(x_B, x_2) \left[\frac{x_B - x_2}{(x_B - x_2 + i\epsilon)^2} - \frac{1}{x_B - x_2 + i\epsilon} \right].
\end{aligned}
\tag{31}
$$

Performing the calculation one gets:

$$
q_\mu \overline{\mathcal{W}}^{(1)}_{\mu\nu} + q_\mu \overline{\mathcal{W}}^{(2)}_{\mu\nu} = 0. \tag{32}
$$

This is nothing else than the QED gauge invariance for the imaginary part of the hadron tensor. From (31), we can see that the gauge invariance takes place only if the prescriptions in the gluonic pole and in the quark propagator of the hard part are coinciding. Indeed Eq. (31) with the field (13) takes the form

$$
\begin{aligned}
&q_\mu \overline{\mathcal{W}}^{(1)}_{\mu\nu} + q_\mu \overline{\mathcal{W}}^{(2)}_{\mu\nu} \\
&\quad = \varepsilon_{\nu p_2 S^T p_1} \bar{q}(y_B) \\
&\qquad \times \Im\mathrm{m} \int_{-1}^{1} dx_2\, T(x_B, x_2) \left[\frac{x_B - x_2}{(x_B - x_2 - i\epsilon)(x_B - x_2 + i\epsilon)} \right. \\
&\qquad \left. - \frac{1}{x_B - x_2 + i\epsilon} \right].
\end{aligned}
\tag{33}
$$

It is clear that the first term in the brackets is purely real, and the imaginary part from the second term stays uncompensated. Let us note for completeness, that the treatment of the pole in the principal value sense is equivalent to the mean arithmetic of two discussed prescriptions and also cannot satisfy the gauge invariance. Thus we completed the *reductio ad absurdum* of the hint suggested in Section 2 and found that the contour gauge (11) is a correct one. In other words, it means that the prescription in the quark propagator must agree with the representation of $B^V(x_B, x_2)$. Otherwise, one may face the problem with the gauge invariance.

It is instructive to compare the electromagnetic gauge invariance of the gluonic poles contributions with that of perturbative QCD. In the latter case the imaginary part is provided by hard gluon loops and the QED gauge invariant set consists of 3 diagrams depicted in Fig. 2. At the same time, the imaginary part is due to the single diagram in Fig. 2(a) and it is gauge invariant by itself as the photon line couples to two on-shell (because of the Cutkosky cutting rule) quarks. This reasoning, however, does not happen to work for (non-perturbative) gluonic pole contribution (see Fig. 1(a)) and the contribution of the diagram in Fig. 1(b) should be added to ensure the electromagnetic gauge invariance. This is clearly seen from Eq. (30) where the analog of the contribution of the diagram in Fig. 2(a) is represented by the first term in the brackets. Its imaginary part is zero (i.e. QED gauge invariant) only if gluonic pole is absent at all. This situation corresponds also to the difficulties in the applicability of Ward identities to gluonic poles contributions (see [39] and references therein).

As we have shown, only the sum of two contributions represented by the diagrams in Fig. 1(a) and (b) can ensure the electromagnetic gauge invariance. We now inspect the influence of a "new" contribution 1(b) on the single spin asymmetry and obtain the QED gauge invariant expression for the hadron tensor. It reads

$$
\overline{\mathcal{W}}^{\mathrm{GI}}_{\mu\nu} = \overline{\mathcal{W}}^{(1)}_{\mu\nu} + \overline{\mathcal{W}}^{(2)}_{\mu\nu} = -\frac{2}{q^2} \varepsilon_{\nu S^T p_1 p_2} Z_\mu \bar{q}(y_B) T(x_B, x_B), \tag{34}
$$

where one used $q^2 = s x_B y_B$ and introduced the vector

$$Z_\mu = \hat{p}_{1\mu} - \hat{p}_{2\mu} \equiv x_B p_{1\mu} - y_B p_{2\mu}, \tag{35}$$

which together with the vectors:

$$X_\mu = -\frac{2}{s}\left[(Z\cdot p_2)\left(p_{1\mu} - \frac{q_\mu}{2x_B}\right) - (Z\cdot p_1)\left(p_{2\mu} - \frac{q_\mu}{2y_B}\right)\right],$$
$$Y_\mu = \frac{2}{s}\varepsilon_{\mu p_1 p_2 q} \tag{36}$$

form the mutually orthogonal basis (see [31]). Here $\hat{p}_{i\mu}$ are the partonic momenta ($q^\mu = \hat{p}_{1\mu} + \hat{p}_{2\mu}$). With the help of (35) and (36), the lepton momenta can be written as (this is the lepton c.m. system)

$$l_{1\mu} = \frac{1}{2}q_\mu + \frac{Q}{2}f_\mu(\theta,\varphi;\hat{X},\hat{Y},\hat{Z}),$$
$$l_{2\mu} = \frac{1}{2}q_\mu - \frac{Q}{2}f_\mu(\theta,\varphi;\hat{X},\hat{Y},\hat{Z}), \tag{37}$$

where $\hat{A} = A/\sqrt{-A^2}$ and

$$f_\mu(\theta,\varphi;\hat{X},\hat{Y},\hat{Z}) = \hat{X}_\mu\cos\varphi\sin\theta + \hat{Y}_\mu\sin\varphi\sin\theta + \hat{Z}_\mu\cos\theta. \tag{38}$$

Within this frame, the contraction of the lepton tensor with the gauge invariant hadron tensor (34) reads

$$\mathcal{L}_{\mu\nu}\overline{\mathcal{W}}^{\mathrm{GI}}_{\mu\nu} = -2\cos\theta\,\varepsilon_{\nu S^T p_1 p_2}\bar{q}(y_B)T(x_B,x_B). \tag{39}$$

We want to emphasize that this differs by the factor of 2 in comparison with the case where only one diagram, presented in Fig. 1(a), has been included in the (gauge non-invariant) hadron tensor, i.e.

$$\mathcal{L}_{\mu\nu}\overline{\mathcal{W}}^{(1)}_{\mu\nu} = \frac{1}{2}\mathcal{L}_{\mu\nu}\overline{\mathcal{W}}^{\mathrm{GI}}_{\mu\nu}. \tag{40}$$

Therefore, from the practical point of view, the neglecting of the diagram in Fig. 1(b) or, in other words, the use of the QED gauge non-invariant hadron tensor yields the error of the factor of two.

Indeed, taking the contribution of the diagram in Fig. 1(a) corresponds to keeping of only the term proportional to $\hat{p}_{1\mu}$ in (35). The contraction with (gauge invariant) leptonic tensor is equivalent to making it gauge invariant by substitution

$$\hat{p}_{1\mu} \quad\Rightarrow\quad \hat{p}_{1\mu} - q_\mu\frac{\hat{p}\cdot q}{Q^2} = \frac{p_{1\mu} - p_{2\mu}}{2}. \tag{41}$$

It is this factor of 2 which makes the difference with the correct gauge invariant expression.

4. Conclusions and discussions

The essence of this Letter consists in the exploration of the electromagnetic gauge invariance of the transverse polarized DY hadron tensor. We showed that it is mandatory to include a contribution of the extra diagram which naively does not have an imaginary part. The account for this extra contribution leads to the amplification of SSA by the factor of 2.

This additional contribution emanates from the complex gluonic pole prescription in the representation of the twist 3 correlator $B^V(x_1,x_2)$ which, in its turn, is directly related to the complex pole prescription in the quark propagator forming the hard part of the corresponding hadron tensor.

We stress that in the previous considerations (see, for example, [26]), the B^V-function was always assumed to be purely real one, while the needed imaginary part was ensured by means of the specially introduced "propagator"[1] in the hard part of the hadron tensor.

In the present Letter, the causal prescription in the quark propagator, involved in the hard part of the diagram in Fig. 1(a), selects from the physical axial gauges the contour gauge defined by Eq. (11). At the same time, the contour gauge predestines Eq. (12) and, therefore, the representation of $B^V(x_1,x_2)$ in the form of the gluonic pole with the complex prescription, see (15). Since both diagrams in Fig. 1(a) and (b) should be considered within the same (contour) gauge, the representation (15), which we advocate, has to be applied for the diagram depicted in Fig. 1(b). As a result of this, the diagram in Fig. 1(b), in contrast to naive assumptions, has the imaginary part. In some sense, the diagram in Fig. 1(b) feels the complex prescription in the hard part of the diagram in Fig. 1(a) by means of the contour gauge which we make used. Note that, from the physical point of view, the consideration of each of the diagrams in Fig. 1 individually makes no sense.

This is completely similar to the case of exclusive dijet production [38] when the pole prescription in (twist two) matrix element of gluonic fields is controlled by the corresponding hard subprocess.

We have argued that, in addition to the electromagnetic gauge invariance, the inclusion of new-found contributions corrects by the factor of 2 the expression for SSA in the transverse polarized Drell–Yan process.

Finally, we proved that the complex prescription in the quark propagator forming the hard part of the hadron tensor, the starting point in the contour gauge, the representation of $B^V(x_1,x_2)$ like (15) and the electromagnetic gauge invariance of the hadron tensor must be considered together as the deeply related items.

Acknowledgements

We would like to thank A.B. Arbuzov, D. Boer, A.V. Efremov, D. Ivanov, N. Kivel, B. Pire, M.V. Polyakov, J. Qiu, P.G. Ratcliffe and N.G. Stefanis for useful discussions and correspondence. This work is partly supported by the DAAD program and the RFBR (grants 09-02-01149 and 09-02-00732).

References

[1] P.A.M. Guichon, M. Vanderhaegen, Prog. Part. Phys. 41 (1998) 125; X. Ji, J. Phys. G 24 (1998) 1181.
[2] B. Pire, O.V. Teryaev, Proceedings of the 13th International Symposium on High Energy Spin Physics, 8–12 September 1998, Protvino, Russia, arXiv:hep-ph/9904375.
[3] I.V. Anikin, B. Pire, O.V. Teryaev, Phys. Rev. D 62 (2000) 071501, arXiv:hep-ph/0003203.
[4] A.V. Belitsky, D. Mueller, L. Niedermeier, A. Schafer, Nucl. Phys. B 593 (2001) 289, arXiv:hep-ph/0004059.
[5] M. Penttinen, M.V. Polyakov, A.G. Shuvaev, M. Strikman, Phys. Lett. B 491 (2000) 96, arXiv:hep-ph/0006321.
[6] A.V. Efremov, O.V. Teryaev, Sov. J. Nucl. Phys. 39 (1984) 962, Yad. Fiz. 39 (1984) 1517.
[7] A. Efremov, V. Korotkiian, O. Teryaev, Phys. Lett. B 348 (1995) 577.
[8] A.V. Efremov, O.V. Teryaev, Phys. Lett. B 150 (1985) 383.
[9] J.w. Qiu, G. Sterman, Phys. Rev. Lett. 67 (1991) 2264.
[10] D. Boer, P.J. Mulders, F. Pijlman, Nucl. Phys. B 667 (2003) 201, arXiv:hep-ph/0303034.
[11] J.C. Collins, Phys. Lett. B 536 (2002) 43, arXiv:hep-ph/0204004.
[12] A.V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B 656 (2003) 165, arXiv:hep-ph/0208038.
[13] J.C. Collins, A. Metz, Phys. Rev. Lett. 93 (2004) 252001, arXiv:hep-ph/0408249.
[14] I.O. Cherednikov, N.G. Stefanis, Nucl. Phys. B 802 (2008) 146, arXiv:0802.2821 [hep-ph].
[15] B. Pire, J.P. Ralston, Phys. Rev. D 28 (1983) 260.

[1] This is the so-called special propagator originally suggested by J.w. Qiu.

[16] R.D. Carlitz, R.S. Willey, Phys. Rev. D 45 (1992) 2323.
[17] A. Brandenburg, D. Mueller, O.V. Teryaev, Phys. Rev. D 53 (1996) 6180, arXiv:hep-ph/9511356.
[18] A.P. Bakulev, N.G. Stefanis, O.V. Teryaev, Phys. Rev. D 76 (2007) 074032, arXiv:0706.4222 [hep-ph].
[19] A.V. Radyushkin, Phys. Rev. D 80 (2009) 094009, arXiv:0906.0323 [hep-ph].
[20] M.V. Polyakov, JETP Lett. 90 (2009) 228, arXiv:0906.0538 [hep-ph].
[21] S.V. Mikhailov, N.G. Stefanis, Mod. Phys. Lett. A 24 (2009) 2858, arXiv:0910.3498 [hep-ph].
[22] A. Brandenburg, S.J. Brodsky, V.V. Khoze, D. Mueller, Phys. Rev. Lett. 73 (1994) 939, arXiv:hep-ph/9403361.
[23] N. Hammon, O. Teryaev, A. Schafer, Phys. Lett. B 390 (1997) 409, arXiv:hep-ph/9611359.
[24] D. Boer, P.J. Mulders, O.V. Teryaev, arXiv:hep-ph/9710525;
D. Boer, P.J. Mulders, O.V. Teryaev, Phys. Rev. D 57 (1998) 3057, arXiv:hep-ph/9710223.
[25] O.V. Teryaev, RIKEN Rev. 28 (2000) 101.
[26] D. Boer, J.w. Qiu, Phys. Rev. D 65 (2002) 034008, arXiv:hep-ph/0108179.
[27] P.G. Ratcliffe, O. Teryaev, Mod. Phys. Lett. A 24 (2009) 2984, arXiv:0910.5348 [hep-ph].
[28] H.G. Cao, J.P. Ma, H.Z. Sang, Commun. Theor. Phys. 53 (2010) 313, arXiv:0901.2966 [hep-ph].
[29] J. Zhou, F. Yuan, Z.T. Liang, Phys. Rev. D 81 (2010) 054008, arXiv:0909.2238 [hep-ph].
[30] J.P. Ma, Q. Wang, Eur. Phys. J. C 37 (2004) 293, arXiv:hep-ph/0310245.
[31] V. Barone, A. Drago, P.G. Ratcliffe, Phys. Rep. 359 (2002) 1, arXiv:hep-ph/0104283.
[32] I.V. Anikin, O.V. Teryaev, Phys. Part. Nucl. Lett. 6 (2009) 3, arXiv:hep-ph/0608230.
[33] N.N. Bogolyubov, D.V. Shirkov, Introduction to the Theory of Quantized Fields, Intersci. Monogr. Phys. Astron., vol. 3, Interscience Publishers, New York, 1959, p. 1.
[34] A.V. Efremov, A.V. Radyushkin, Theor. Math. Phys. 44 (1981) 774, Teor. Mat. Fiz. 44 (1980) 327.
[35] S.V. Ivanov, G.P. Korchemsky, A.V. Radyushkin, Yad. Fiz. 44 (1986) 230, Sov. J. Nucl. Phys. 44 (1986) 145;
S.V. Ivanov, G.P. Korchemsky, Phys. Lett. B 154 (1985) 197;
S.V. Ivanov, Fiz. Elem. Chast. Atom. Yadra 21 (1990) 75.
[36] I.V. Anikin, D.Y. Ivanov, B. Pire, L. Szymanowski, S. Wallon, Nucl. Phys. B 828 (2010) 1, arXiv:0909.4090 [hep-ph].
[37] A.V. Radyushkin, Phys. Rev. D 56 (1997) 5524, arXiv:hep-ph/9704207.
[38] V.M. Braun, D.Y. Ivanov, A. Schafer, L. Szymanowski, Nucl. Phys. B 638 (2002) 111, arXiv:hep-ph/0204191.
[39] Y. Koike, K. Tanaka, arXiv:0907.2797 [hep-ph].