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Condition for superradiance in higher-dimensional rotating black holes

Eylee Jung, Sung Hoon Kim, D.K. Park

Department of Physics, Kyungnam University, Masan 631-701, South Korea

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Abstract

It is shown that the superradiance modes always exist in the radiation by the $(4+n)$-dimensional rotating black holes. Using a Bekenstein argument the condition for the superradiance modes is shown to be $0 < \omega < m\Omega$ for the scalar, electromagnetic and gravitational waves when the spacetime background has a single angular momentum parameter about an axis on the brane, where Ω is a rotational frequency of the black hole and m is an azimuthal quantum number of the radiated wave.

Recent brane-world scenarios such as large extra dimensions [1,2] or compactified extra dimensions with warped factor [3] predict a TeV-scale gravity. The emergence for the TeV-scale gravity in the higher-dimensional theories opens the possibility to make the black hole factories in the future high-energy colliders [4–7]. In this context it is of interest to examine the various properties of the higher-dimensional black holes.

The absorption and emission for the different particles by the $(4+n)$-dimensional Schwarzschild background have been studied analytically [8] and numerically [9]. It was shown that the presence of the extra dimensions in general decreases the absorptivity and increases the emission rate on the brane. The decrease of the absorptivity may be due to the decrease of the effective radius [10] $r_c \equiv \sqrt{\sigma_\infty/\pi}$, where σ_∞ is a high-energy limit of the total absorption cross section. Although it may explain why the absorptivity is suppressed in the high-energy regime, it does not seem to provide a satisfactory physical reason for the suppression of the absorptivity in the full range of the particle energy. The enhancement of the emission rate may be caused by the increase of the Hawking temperature in the presence of the extra dimensions. This means that the Planck factor is more crucial than the greybody factor in the Hawking radiation. For the case of

E-mail addresses: eylee@kyungnam.ac.kr (E. Jung), shoon@kyungnam.ac.kr (S.H. Kim), dkpark@hep.kyungnam.ac.kr (D.K. Park).

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the minimally coupled massless scalar the low-energy absorption cross section (LACS) always equals to the horizon area [11]. Thus, for the brane-localized scalar the LACS is always $4\pi r_H^2$ while for the bulk scalar it is equal to $\Omega_{n+2} r_H^{n+2}$, where

$$\Omega_{n+2} = \frac{2\pi^{(n+3)/2}}{\Gamma[(n+3)/2]}$$

is the area of a unit $(n+2)$-sphere. The ratio of the LACS for the Dirac field to that for the scalar was shown to be $2^{(n-3)/(n+1)}$ for the brane-localized case [8] and $2^{-(n+3)/(n+1)}$ for the bulk case [12]. Therefore, the ratio factor $1/8$, which was obtained by Unruh long ago [13], was recovered when $n = 0$. The dependence on the dimensionality in these ratio factors may be used to prove the existence of the extra dimensions in the future black hole experiments. The relative bulk-to-brane energy emissivity was also calculated in Ref. [9] numerically, which confirmed the main result of Ref. [10], i.e., *black holes radiate mainly on the brane*.

For the higher-dimensional charged black holes the full absorption and emission spectra have been computed numerically in Ref. [14]. It has been shown that contrary to the effect of the extra dimension the presence of the nonzero inner horizon parameter r_- generally enhances the absorptivity and suppresses the emission rate. It has been shown also that the relative bulk-to-brane emissivity decreases with increasing the inner horizon parameter r_-. The LACS for the minimally coupled massless scalar always equals to the horizon area. For the Dirac fermion the LACS becomes [12]

$$\sigma_F^{\rm BL} = 2^{-\frac{n+3}{n+1}} \left[1 - \left(\frac{r_-}{r_+}\right)^{n+1}\right]^{\frac{n+2}{n+1}} \sigma_S^{\rm BL} \tag{1}$$

for the bulk case and

$$\sigma_F^{\rm BR} = 2^{\frac{n-3}{n+1}} \left[1 - \left(\frac{r_-}{r_+}\right)^{n+1}\right]^{\frac{2}{n+1}} \sigma_S^{\rm BR} \tag{2}$$

for the brane-localized case. In Eqs. (1) and (2) $\sigma_S^{\rm BL}$ and $\sigma_S^{\rm BR}$ are the LACSs for the bulk and brane-localized scalars, respectively.

The absorption and emission problems in the higher-dimensional rotating black holes were recently discussed in Refs. [15–17], where the existence of the superradiance modes [18,19] is predicted analytically and numerically in the presence of the extra dimensions. The existence of the superradiance is very important for the experimental signature in the future colliders because it may change [15,20,21] the standard claim that *black holes radiate mainly on the brane*. In this context it is important to derive a criterion for the existence of the superradiance. In this short note we will derive this criterion using a Bekenstein's argument [22].

The gravitational background around a $(4+n)$-dimensional, rotating, uncharged black hole having single angular momentum parameter about an axis in the brane is given by [23]

$$\begin{aligned} ds^2 = {} & -\left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 - \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}}\, dt\, d\phi \\ & + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2 \\ & + \left(r^2 + a^2 + \frac{a^2 \mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\phi^2 \\ & + r^2 \cos^2\theta\, d\Omega_n, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \Delta &= r^2 + a^2 - \frac{\mu}{r^{n-1}}, \\ \Sigma &= r^2 + a^2 \cos^2\theta, \end{aligned} \tag{4}$$

and $d\Omega_n$ is a line-element on a unit n-sphere.

It is worthwhile noting that the $(4+n)$-dimensional rotating black holes can have $1 + n/2$ angular momentum parameters for even n and $(3+n)/2$ parameters for odd n maximally [23]. Although our following argument can be applicable to this general case, it seems to be complicated in the calculation. Thus, we would like to consider the simpler case by reducing the angular momentum parameters. That is why we choose a single angular momentum parameter in Eq. (3). The detailed calculation for the spacetime background having multiple angular momentum parameters will be reported elsewhere.

The horizon radius r_H is determined from $\Delta = 0$, i.e.,

$$r_H^2 + a^2 - \frac{\mu}{r_H^{n-1}} = 0. \tag{5}$$

The horizon area $\tilde{A}$, mass M, angular momentum J and Hawking temperature T_H are given by

$$\tilde{A} = \frac{\Omega_n r_H^n}{n+1} A, \qquad M = \frac{(n+2)\Omega_{n+2}}{16\pi}\mu,$$
$$J = \frac{2}{n+2} M a,$$
$$T_H = \frac{2}{A}\left[r_H + \frac{8\pi(n-1)M}{(n+2)\Omega_{n+2} r_H^n}\right], \tag{6}$$

where $\Omega_N = 2\pi^{(N+1)/2}/\Gamma[(N+1)/2]$ is an area of unit N-sphere and $A = 4\pi(r_H^2 + a^2)$. It is easy to show that the various quantities in Eq. (6) are related to each other in the form

$$A T_H = 2r_H + (n-1)\frac{\mu}{r_H^n} = 2r_H + \frac{(n-1)A}{4\pi r_H}. \tag{7}$$

Now we assume M and J are independent variables. Then elementary mathematics gives

$$d\tilde{A} = \frac{\partial \tilde{A}}{\partial M} dM + \frac{\partial \tilde{A}}{\partial J} dJ. \tag{8}$$

Firstly, let us calculate $\partial A/\partial M$, which is given by

$$\frac{\partial A}{\partial M} = 8\pi\left(r_H \frac{\partial r_H}{\partial M} + a\frac{\partial a}{\partial M}\right). \tag{9}$$

Differentiating Eq. (5) with respect to M and using Eq. (7), it is easy to show

$$\frac{\partial r_H}{\partial M} = \frac{1}{A T_H}\left[\frac{16\pi}{(n+2)\Omega_{n+2} r_H^{n-1}} + \frac{2a^2}{M}\right],$$
$$\frac{\partial a}{\partial M} = -\frac{n+2}{2M^2} J = -\frac{a}{M}, \tag{10}$$

which results in

$$\frac{\partial A}{\partial M} = \frac{2}{M T_H r_H}\left[\left(r_H^2 + a^2\right) - n a^2\right]. \tag{11}$$

Combining Eqs. (10) and (11), one can show

$$\frac{\partial \tilde{A}}{\partial M} = \frac{(n+2)\Omega_n r_H^{n-1}}{(n+1) M T_H}\left(r_H^2 + a^2\right). \tag{12}$$

Differentiating Eq. (5) with respect to J and following the previous procedure, one also can show

$$\frac{\partial A}{\partial J} = \frac{(n-1)(n+2)a}{M T_H r_H},$$
$$\frac{\partial \tilde{A}}{\partial J} = -\frac{(n+2)\Omega_n r_H^{n-1} a}{(n+1) M T_H}. \tag{13}$$

Inserting Eqs. (12) and (13) into Eq. (8), Eq. (8) becomes in the following

$$d\tilde{A} = \left[1 - \Omega \frac{dJ}{dM}\right] \frac{\partial \tilde{A}}{\partial M} dM, \tag{14}$$

where

$$\Omega = \frac{a}{r_H^2 + a^2} \tag{15}$$

is a rotational frequency of the black hole. Bekenstein showed in Ref. [22] that for scalar, electromagnetic and gravitational waves dJ/dM becomes

$$\frac{dJ}{dM} = -\frac{T_\phi^r}{T_t^r} = \frac{m}{\omega}, \tag{16}$$

where m and ω are azimuthal quantum number and energy of the incident wave, respectively, and $T_{\mu\nu}$ is a stress-energy tensor. Thus, Eq. (14) becomes

$$d\tilde{A} = \left[1 - \frac{m}{\omega}\Omega\right] \frac{\partial \tilde{A}}{\partial M} dM. \tag{17}$$

Since $\partial \tilde{A}/\partial M$ is always positive from Eq. (12) and $d\tilde{A} > 0$ because $\tilde{A}/4$ is a black hole entropy, Eq. (17) gives a condition

$$0 < \omega < m\Omega \tag{18}$$

if $dM < 0$, which is a condition for the existence of the superradiance.

One may apply the same procedure to the brane-localized fields to derive a criterion for the existence of the superradiance modes. In this case we should use the induced metric

$$ds_{\mathrm{BR}}^2 = -\left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^2 - \frac{2a\mu \sin^2\theta}{\Sigma r^{n-1}} dt\, d\phi$$
$$+ \frac{\Sigma}{\Delta} dr^2 + \Sigma\, d\theta^2$$
$$+ \left(r^2 + a^2 + \frac{a^2\mu \sin^2\theta}{\Sigma r^{n-1}}\right) \sin^2\theta\, d\phi^2. \tag{19}$$

However, the metric (19) is not exact black hole solution of the Einstein field equation. Thus it is obscure whether we can identify the quarter of the horizon area with a black hole entropy. Since, furthermore, the metric (19) is not a vacuum solution unlike Kerr black hole, it generates its own stress-energy tensor and hence total energy–momentum tensor should be $T_{\mu\nu}^{\mathrm{tot}} = T_{\mu\nu}^f + T_{\mu\nu}^m$, where $T_{\mu\nu}^f$ and $T_{\mu\nu}^m$ are the stress-energy tensors contributed from field and metric, respectively. Thus, it is not evident for the brane-localized waves whether we can use $-T_\phi^{r,\mathrm{tot}}/T_t^{r,\mathrm{tot}} = m/\omega$ or not.

As commented earlier, our procedure is not restricted to the rotating black hole with single angular parameter. In $4+n$ dimensions the rotating black holes can have $(n+3)/2$ angular momentum parameters for odd n and $(n+2)/2$ parameters for even n. So it is interesting to apply our method to this black holes to derive a general condition for the existence of the superradiance.

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References

[1] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 429 (1998) 263, hep-ph/9803315.
[2] L. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 436 (1998) 257, hep-ph/9804398.
[3] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370, hep-ph/9905221.
[4] S.B. Giddings, T. Thomas, Phys. Rev. D 65 (2002) 056010, hep-ph/0106219.
[5] S. Dimopoulos, G. Landsberg, Phys. Rev. Lett. 87 (2001) 161602, hep-ph/0106295.
[6] D.M. Eardley, S.B. Giddings, Phys. Rev. D 66 (2002) 044011, gr-qc/0201034.
[7] D. Stojkovic, Phys. Rev. Lett. 94 (2005) 011603, hep-ph/0409124.
[8] E. Jung, S.H. Kim, D.K. Park, JHEP 0409 (2004) 005, hep-th/0406117.
[9] C.M. Harris, P. Kanti, JHEP 0310 (2003) 014, hep-ph/0309054.
[10] R. Emparan, G.T. Horowitz, R.C. Myers, Phys. Rev. Lett. 85 (2000) 499, hep-th/0003118.
[11] S.R. Das, G. Gibbons, S.D. Mathur, Phys. Rev. Lett. 78 (1997) 417, hep-th/9609052.
[12] E. Jung, S.H. Kim, D.K. Park, hep-th/0503027.
[13] W.G. Unruh, Phys. Rev. D 14 (1976) 3251.
[14] E. Jung, D.K. Park, hep-th/0502002.
[15] V. Frolov, D. Stojković, Phys. Rev. D 67 (2003) 084004, gr-qc/0211055.
[16] D. Ida, K. Oda, S.C. Park, hep-ph/0501210.
[17] C.M. Harris, P. Kanti, hep-th/0503010.
[18] Y.B. Zel'dovich, JETP Lett. 14 (1971) 180.
[19] W.H. Press, S.A. Teukolsky, Nature 238 (1972) 211.
[20] V. Frolov, D. Stojković, Phys. Rev. D 66 (2002) 084002, hep-th/0206046.
[21] V. Frolov, D. Stojković, Phys. Rev. Lett. 89 (2002) 151302, hep-th/0208102.
[22] J.D. Bekenstein, Phys. Rev. D 7 (1973) 949.
[23] R.C. Myers, M.J. Perry, Ann. Phys. 172 (1986) 304.