Physics Letters B 675 (2009) 305–307

Contents lists available at ScienceDirect

Physics Letters B

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Δ contribution in $e^+ + e^- \to p + \bar{p}$ at small s

Hai Qing Zhou [a,*], Dian Yong Chen [b], Yu Bing Dong [b,c]

[a] Department of Physics, Southeast University, Nanjing 211189, PR China
[b] Institute of High Energy Physics, Chinese Academy of Science, Beijing 100049, PR China
[c] Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, PR China

ARTICLE INFO

Article history:
Received 2 March 2009
Accepted 13 April 2009
Available online 17 April 2009
Editor: W. Haxton

PACS:
13.40.Gp
13.60.-r
25.30.-c

Keywords:
Two-photon exchange
Delta
Form factor

ABSTRACT

Two-photon annihilate contributions in the process $e^+ + e^- \to p + \bar{p}$ including N and Δ intermediate are discussed in a simple hadronic model. The corrections to the unpolarized cross section and polarized observables P_x, P_z are presented. The results show the two-photon annihilate correction to unpolarized cross section is small and its angle dependence becomes weak at small s after considering the N and $\Delta(1232)$ contributions simultaneously, while the correction to P_z is enhanced.

1. Introduction

The Two-Photon-Exchange (TPE) effect has attracted many interests after its success in explaining the un-consistent measurements of $R = \mu_p G_E/G_M$ from $ep \to ep$ by Rosenbluth technique and polarized methods [1–4]. It is found that the TPE corrections play an important role in extracting the proton's form factors due to its explicit angle dependence. Later some other processes [5–7] are suggested to measure the TPE like effects. The $e^+ + e^- \to p + \bar{p}$ is one of such processes and the two-photon annihilate corrections in this process have been discussed by [8] where only the N intermediate was included. The estimate by [8] showed the two-photon annihilate corrections are about a few percent in the magnitude but strongly depend on the hadron production angle. On another hand, the calculation in [9] showed the $\Delta(1232)$ intermediates also innegligible in the TPE corrections in the simple hadronic model [2,4,9]. These researches prompt us to extent the estimate of the two-photon annihilate corrections in [8] to include Δ intermediate state. In this work, we present such results.

2. Two-photon annihilate corrections including N and $\Delta(1232)$ as intermediate state

Considering the process $e^+(k_2) + e^-(k_1) \to p(p_2) + \bar{p}(p_1)$, the Born diagram is showed as Fig. 1. The differential cross section for this process at the tree level can be written as [10]

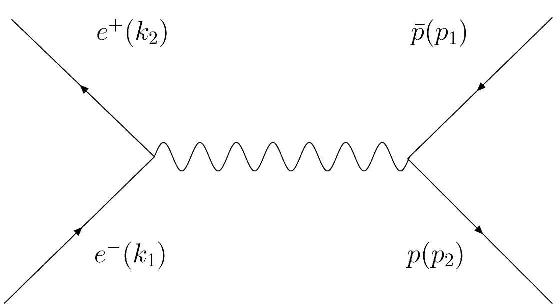

Fig. 1. One photon annihilating diagram for $e^+ + e^- \to p + \bar{p}$.

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\alpha^2\sqrt{1-4M_N^2/q^2}}{4q^2} \times \left(|G_M|^2\left(1+\cos^2\theta\right) + \frac{1}{\tau}|G_E|^2\sin^2\theta\right) \qquad (1)$$

where $q = k_1 + k_2$, $\tau = q^2/4M_N^2 > 1$ and θ is the angle between the momentum of finial antiproton and initial electron in the center of mass frame. The Sachs form factors have been used as

$$G_M(q^2) = F_1(q^2) + F_2(q^2), \qquad G_E(q^2) = F_1(q^2) + \tau F_2(q^2). \qquad (2)$$

In principle, the form factors at certain $s = q^2$ can be extracted from the measurement of the unpolarized differential cross section

* Corresponding author.
E-mail address: zhouhq@mail.ihep.ac.cn (H.Q. Zhou).

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doi:10.1016/j.physletb.2009.04.029

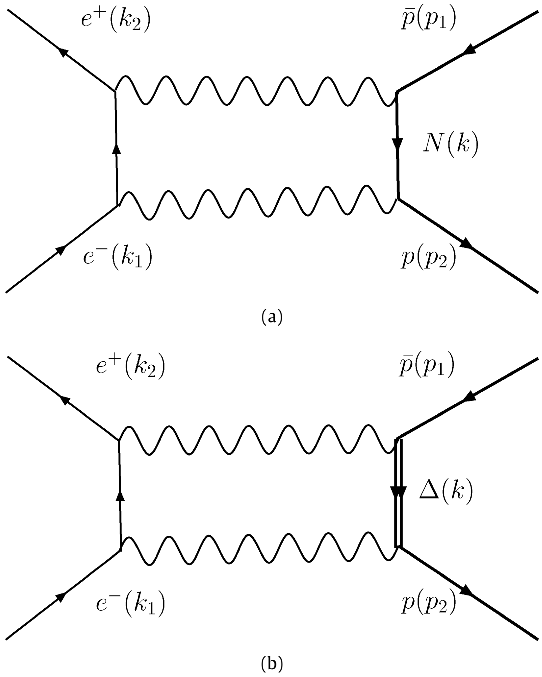

Fig. 2. Two-photon annihilating diagrams (a) with N as intermediate state, (b) with $\Delta(1232)$ as intermediate state. Corresponding cross-box diagrams are implied.

at different angle. To extract the form factors more precisely, the radiative corrections should be considered. Among the one loop radiative corrections, the box and crossed box diagrams play special role due to their strong angle dependence. This leads us restrict our discussions on the two-photon annihilate correction firstly.

Using the simple hadronic model developed in [2,4,9] and including N and Δ as the intermediate state like Fig. 2, the unpolarized cross section can be written as

$$d\sigma = d\sigma_0\left(1+\delta_N^{2\gamma}+\delta_\Delta^{2\gamma}\right) \propto \sum_{\text{helicity}} \left|\mathcal{M}_0+\mathcal{M}_N^{2\gamma}+\mathcal{M}_\Delta^{2\gamma}\right|^2, \tag{3}$$

where $\mathcal{M}_0$ is the contribution of one-photon annihilate diagram and $\mathcal{M}_{N,\Delta}^{2\gamma}$ denote the contribution from two-photon annihilate diagrams with N and Δ as intermediate state. The corrections to the unpolarized cross section can defined as

$$\delta_{N,\Delta}^{2\gamma} = \frac{\sum_{\text{helicity}} 2\,Re\{\mathcal{M}_{N,\Delta}^{2\gamma}\mathcal{M}_0^\dagger\}}{\sum_{\text{helicity}} |\mathcal{M}_0|^2}. \tag{4}$$

The corrections from N have been discussed in [8]. To discuss the correction from Δ, we take the following matrix elements as [9,11]

$$\begin{aligned}
&\left\langle N(p_2)\middle|J_\mu^{em}\middle|\Delta(k)\right\rangle \\
&\quad= \frac{-F_\Delta(q_1^2)}{M_N^2}\bar{u}(p_2)\left[g_1\left(g_\mu^\alpha \not{k}\not{q}_1 - k_\mu\gamma^\alpha\not{q}_1 - \gamma_\mu\gamma^\alpha k\cdot q_1 + \gamma_\mu\not{k}q_1^\alpha\right)\right. \\
&\qquad + g_2\left(k_\mu q_1^\alpha - k\cdot q_1 g_\mu^\alpha\right) + g_3/M_N\left(q_1^2\left(k_\mu\gamma^\alpha - g_\mu^\alpha\not{k}\right)\right. \\
&\qquad \left.\left. + q_{1\mu}\left(q_1^\alpha\not{k} - \gamma^\alpha k\cdot q_1\right)\right)\right]\gamma_5 T_3 u_\alpha^\Delta(k), \\
&\left\langle\Delta(k)\bar{N}(p_1)\middle|J_\nu^{em}\middle|0\right\rangle \\
&\quad= \frac{-F_\Delta(q_2^2)}{M_N^2}\bar{u}_\beta^\Delta(k)T_3^+\gamma_5\left[g_1\left(g_\nu^\beta\not{q}_2\not{k} - k_\nu\not{q}_2\gamma^\beta\right.\right. \\
&\qquad \left. - \gamma^\beta\gamma_\nu k\cdot q_2 + \not{k}\gamma_\nu q_2^\beta\right) \\
&\qquad + g_2\left(k_\nu q_2^\beta - k\cdot q_2 g_\nu^\beta\right) - g_3/M_N\left(q_2^2\left(k_\nu\gamma^\beta - g_\nu^\beta\not{k}\right)\right. \\
&\qquad \left.\left. + q_{2\nu}\left(q_2^\beta\not{k} - \gamma^\beta k\cdot q_2\right)\right)\right]v(p_1),
\end{aligned} \tag{5}$$

where $q_1 = p_2 - k$, $q_2 = k + p_1$ and T_3 is the third component of the $N \to \Delta$ isospin transition operator and is $\sqrt{2/3}$ here. The effective vertexes of $\gamma N\Delta$ are defined as $\bar{u}(p_2)\Gamma_\mu^\alpha(\gamma\Delta \to N) \times u_\alpha^\Delta(k) = -ie\langle N(p_2)|J_\mu^{em}|\Delta(k)\rangle$, $\bar{u}_\beta^\Delta(k)\Gamma_\nu^\beta(\gamma \to \bar{N}\Delta)v(p_1) = -ie \times \langle\Delta(k)\bar{N}(p_1)|J_\nu^{em}|0\rangle$. Both the two vertexes satisfy the conditions $q_{1,2}^\mu\Gamma_\mu = 0$ and $k_\alpha\Gamma^\alpha = 0$, the first condition ensure the gauge invariance of the result and the second condition ensure to select only the physical spin 3/2 component [9].

For the propagator of Δ, the same form is employed as [9]

$$\begin{aligned}
&S_{\alpha\beta}^\Delta(k) = \frac{-i(\not{k}+M_\Delta)}{k^2-M_\Delta^2+i\epsilon}P_{\alpha\beta}^{3/2}(k), \\
&P_{\alpha\beta}^{3/2}(k) = g_{\alpha\beta} - \gamma_\alpha\gamma_\beta/3 - (\not{k}\gamma_\alpha k_\beta + k_\alpha\gamma_\beta\not{k})/3k^2.
\end{aligned} \tag{6}$$

Such propagator is different with the usual RS one which read as

$$\begin{aligned}
S_{\alpha\beta}^{\text{RS}}(k) = \frac{\not{k}+M_\Delta}{k^2-M_\Delta^2+i\epsilon}\Bigg[&-g_{\alpha\beta} + \frac{1}{3}\gamma_\alpha\gamma_\beta + \frac{1}{3M_\Delta}(\gamma_\alpha k_\beta - \gamma_\beta k_\alpha) \\
&+ \frac{2}{3M_\Delta^2}k_\alpha k_\beta\Bigg].
\end{aligned} \tag{7}$$

After using the properties of the vertexes, these two forms result in the same amplitude.

By this effective interaction, the amplitude of box diagram Fig. 2(b) can be written as

$$\begin{aligned}
M^{(2b)} = -i\int &\frac{d^4k}{(2\pi)^4}\bar{u}(k_2)(-ie\gamma_\mu)\frac{i(\not{p}_1+\not{k}-\not{k}_2+m_e)}{(p_1+k-k_2)^2-m_e^2+i\varepsilon} \\
&\times(-ie\gamma_\nu)v(k_1)\frac{-i}{(p_1+k)^2+i\varepsilon}\frac{-i}{(p_2-k)^2+i\varepsilon} \\
&\times\bar{u}(p_2)\Gamma_{\gamma\Delta\to N}^{\mu\alpha}\frac{-i(\not{k}+M_\Delta)}{k^2-M_\Delta^2+i\varepsilon}P_{\alpha\beta}^{3/2}(k)\Gamma_{\gamma\to\bar{N}\Delta}^{\beta\nu}v(p_1),
\end{aligned} \tag{8}$$

where Feynman gauge invariance has been used. Similarly one can get the amplitude of crossed box diagram with Δ intermediate state.

In the practical calculation, we take the form factor F_Δ in the monopole form as G_E in N case [8]

$$F_\Delta(q^2) = G_E(q^2) = G_M/\mu_p(q^2) = \frac{-\Lambda_1^2}{q^2-\Lambda_1^2}, \tag{9}$$

the coupling parameters and cut-offs are the same as [8,11]

$$g_1 = 1.91, \qquad g_2 = 2.63, \qquad g_3 = 1.58, \qquad \Lambda_1 = 0.84\ \text{GeV}. \tag{10}$$

3. Numerical results and discussion

Using the above as input, the two-photon annihilate corrections can be calculated directly. We use the package FeynCalc [12] and LoopTools [13] to carry out the calculation. The IR divergence in the N intermediate case is treated as [8] and there is no divergence in the $\Delta(1232)$ case. The numerical results for $\delta_{N,\Delta}^{2\gamma}$ are showed in Fig. 3. The similar calculation can be applied to the polarized quantities P_x and P_z as [8,14] with the definitions

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}[1 + P_y\xi_y + \lambda_e P_x\xi_x + \lambda_e P_z\xi_z]. \tag{11}$$

The results of the corrections to P_x and P_z are presented in Fig. 4. In our previous results [8], when discussing the TPE corrections to polarized observables, only the contributions in term $\frac{d\sigma}{d\Omega}$ are considered, while the corrections in $\frac{d\sigma_{un}}{d\Omega}$ are neglected. Here the calculations are improved to include both corrections.

As showed in Fig. 3, the correction $\delta_\Delta^{2\gamma}$ is found to be always opposite to the corrections $\delta_N^{2\gamma}$ in all the angle region. This behavior is similar to the ep scattering case [9]. Detailedly, at $s = 4\ \text{GeV}^2$

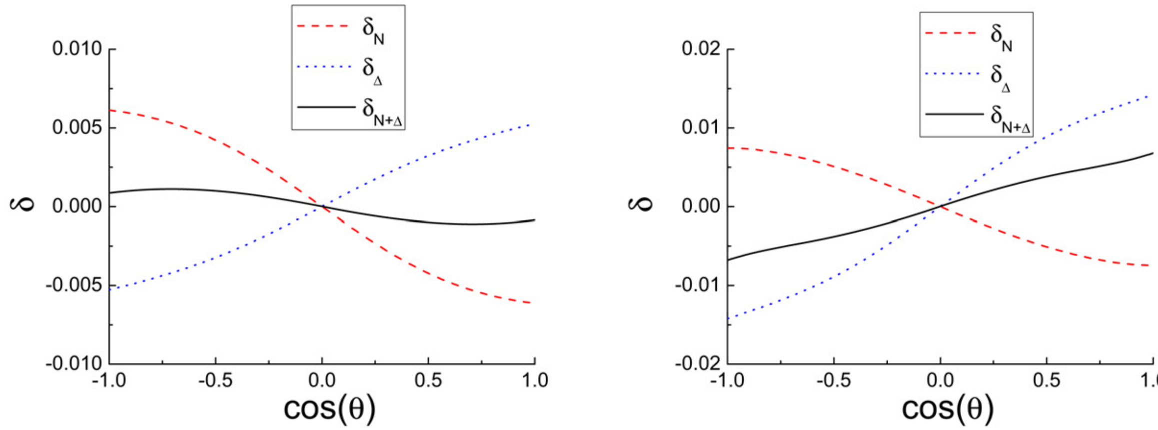

Fig. 3. Cosine θ dependence of two-photon-annihilating corrections to unpolarized cross section. The dashed and dotted lines denote to the correction from N and δ_Δ, respectively, and their sum is given by the solid lines. The left result is for $s = 4\ \text{GeV}^2$ and the right one for $s = 5\ \text{GeV}^2$.

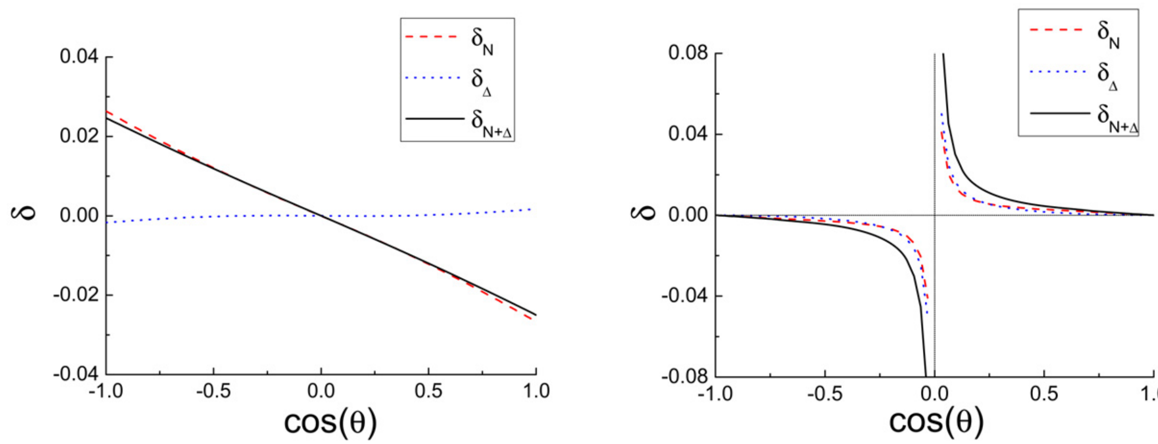

Fig. 4. Cosine θ dependence of two-photon-annihilating corrections to P_x and P_z. The dashed and dotted lines denote to the correction from N and δ_Δ, respectively, and their sum is given by the solid lines. The left result is for P_x and the right one for P_z, both with $s = 4\ \text{GeV}^2$.

the absolute magnitude of $\delta_\Delta^{2\gamma}$ is so close to $\delta_N^{2\gamma}$ which results in the large cancelation and small total correction to unpolarized cross section. The small $\delta_{N+\Delta}^{2\gamma}$ and its weak angle dependence suggest the Rosenbluth method will work well in this region. This conclusion is some different with the *ep* scattering case where the cancelation is much smaller and the total correction still strongly depend on the scattering angle. At $s = 5\ \text{GeV}^2$, the absolute magnitude of $\delta_\Delta^{2\gamma}$ becomes larger than $\delta_N^{2\gamma}$ which suggests the important roles played by $\Delta(1232)$ intermediate state in the process of $e^+ + e^- \to p + \bar{p}$.

For the polarized observables, Fig. 4 shows the correction to P_x from Δ is much smaller than N and the correction to P_z from Δ is close to N. The former property suggests $\Delta(1232)$ gives no new correction than [8] while the latter property increases the two-photon annihilate corrections to P_z which enhances our previous suggestion that the nonzero P_z at $\theta = \pi/2$ may be a good place to measure the two-photon exchange like effects directly.

Acknowledgements

This work is supported by the National Sciences Foundations of China under Grant Nos. 10747118, 10805009, 10475088, and by CAS Knowledge Innovation Project No. KC2-SW-N02.

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