A new Multiuser Detection Algorithm Based on Robust Kalman Filtering

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Abstract

Inspired from $H^\infty$ robust control technique, this research applies $H^\infty$ filtering to Multi-user Detection (MUD), and proposes a new MUD algorithm based on robust Kalman filtering. Because it can rapidly converge when the statistical property of interferences or noises are unknown, this algorithm has better performance than existing blind adapting MUD based Kalman filtering.

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I. Introduction

Normal Kalman Algorithm is originated from $H^2$ estimation criterion which needs a accurate system model and the statistical property of external interference signal. However, the actual system always has a unknown error. The state equation of $w_i(n+1) \approx w_i(n)$ in the multiuser detection model is a kind of approximate model, and the statistical property of the noise factor $e_i(n)$ in the observation equation is approximate. If a burst noise factor occurs, the normal Kalman may have a slow convergence, low accuracy, even a filter failure. The Robust control is mainly for the control of non-sure external inference feature. As the existence of the non-determinacy of filtering system, Robust control appears in out eyesight. For instance the $H^\infty$ filter is a unique and universal example.

$H^\infty$ Filter [1] draws into the Hardy normal number to construct a typical filter that can minimize the Hardy normal number $H^\infty$. No hypothesis is addressed to the spectrum property of the interference signal of $H^\infty$ Filter (opposite to the Normal Kalman[2]), and the minimum estimated error can be achieved even in the worst cases.
2. Kalman Filtering Algorithm

Kalman Filter is proposed by R. E. Kalman in 1960. It is an filtering algorithm that can estimate signals via the observed values of the abstracted signals. The state space was introduced into the theory of stochastic estimation, and the signal process was deemed as an output of linear systems in course of the white noise. The typical I/O relationship is described with state equations. The filtering algorithm is formed with statistical features of system state equations, observation equations, and white noise excitation (system and observation noise). As all of the variables are time-domain, we can estimate not only the one dimension stationary random process but also the non stationary with multi-dimensions. This feature can completely break the restrictions which encountered by Wiener filtering design in frequency domain. A broader sphere of application can be reached as a result.

For any random discrete linear systems, the mathematical model can be shown as the state and observation equation as below.

\[
X_k = \Phi_{k,k-1}X_{k-1} + \Gamma_{k,k-1}W_{k-1}
\]

\[
Z_k = H_kX_k + V_k
\]

In this paper, \(X_k\) with \(n\) dimensions means the state vector at the moment \(k\). It is unobservable; Matrix \(\Phi_{k,k-1}\) with \(n \times n\) dimensions is called state one step transformation matrix. It tells you how the transformation implements from state of moment \(k-1\) to \(k\) in the dynamic system. It is known; Vector \(W_{k-1}\) is process noise vector which can describe the additive noise and difference. \(\Gamma_{k,k-1}\) is a noise input matrix with \(n \times p\) dimensions. \(Z_k\) with \(m\) dimensions observation vector at the moment \(k\). \(H_k\) observation matrix with \(m \times n\) dimensions which can effect the relationship between the state and observation vector. It is known; \(V_k\) with \(m\) dimensions is the random system noise sequence.

For the random system, it must meet the following assumptions and conditions:

1) Process noise vector \(W_k\) of the System and observation noise vector \(V_k\) are the white noise random process vector, it can be written as:

\[
E\left[ W_kW^T_j \right] = Q_k \delta_{kj}
\]

\[
E\left[ V_kV^T_j \right] = R_k \delta_{kj}
\]

\(Q_k\) is covariance matrix of process noise vector \(W_k\) of the system, which is the symmetric nonnegative definite matrix; \(R_k\) is the variance matrix of observation noise vector, which is the symmetric positive definite matrix; \(\delta_{kj}\) is Kronecker \(\delta\) function.

2) Process noise vector \(W_k\) of the System and observation noise vector \(V_k\) are not related or \(\delta\) associated, it can be written as:

\[
E\left[ W_kV^T_j \right] = 0
\]

or

\[
E\left[ W_kV^T_j \right] = S_k \delta_{kj}
\]

3) The system initial state \(X_0\) is a known distribution or normal distribution of random vector, and its mean and variance matrix, respectively are written as:

\[
E\left[ X_0 \right] = \mu_X
\]

\[
E\left[ X_0^T X_0^T \right] = P_0
\]
4) The System process noise vector \(W_k\) and observation noise vectors \(V_k\) are not related to the initial state \(X_0\), can be given by:

\[
\begin{align*}
E[X_0W_k^T] &= 0 \\
E[X_0V_0^T] &= 0
\end{align*}
\]

Meeting the above four assumptions and conditions, you can get the equation of Kalman filter in the random linear discrete system, so the estimating \(\hat{X}_k\) of \(X_k\) can be solved by the following equation:

One step state prediction:

\[
\hat{X}_{k,k-1} = \Phi_{k,k-1}\hat{X}_{k-1}
\]

State Estimation:

\[
\hat{X}_k = \hat{X}_{k,k-1} + K_k [Z_k - H_k\hat{X}_{k,k-1}]
\]

Filter gain matrix:

\[
K_k = P_{k,k-1}H_k^T [H_k P_{k,k-1}H_k^T + R_k]^{-1}
\]

One step prediction error matrix:

\[
P_{k,k-1} = \Phi_{k,k-1}P_{k,k-1}\Phi_{k,k-1}^T + \Gamma_{k,k-1}\Omega_{k,k-1}\Gamma_{k,k-1}^T
\]

Estimation error variance matrix:

\[
P_k = [I - K_k H_k]P_{k,k-1}[I - K_k H_k]^T + K_k R_k K_k^T
\]

As long as the initial value \(X_0\) and \(P_0\) are given, according to the \(k\) time of observation \(Z_k\), we can recursively calculate the \(k\) time state estimation \(\hat{X}_k\).

3. Robust Kalman Filtering Algorithm

1) The description of the \(H^\infty\) Filter problem

Considering the described stochastic linear discrete systems, the initial state \(X_0\) based the system; assuming that \(\hat{X}_0\) is the estimates for the initial state, the initial estimation error variance matrix can be written as:

\[
P_0 = E\{[X_0 - \hat{X}_0][X_0 - \hat{X}_0]^T\}
\]

Here, we do not do any assumptions of the statistical property of the system process noise \(W_k\) and observation noise \(V_k\), but the initial error \(X_0 - \hat{X}_0\) of the system, the system noise \(W_k\) and \(V_k\) are the unknown disturbance input.

In general, we hope to use observation \(Z_k\) to estimate a linear combination of the system state.

\[
z_k = L_k X_k
\]

In the formula, \(L_k \in \mathbb{R}^{n \times n}\) is for a given matrix., under the given observation \(\{Z_k\}\), So that \(\hat{z}_k = F_z(z_0, Z_1, \cdots, Z_k)\) is the estimate of \(z_k\), the definition of the filtering error can be written as:
\[ e_k = \hat{z}_k - L_k X_k \]  

Figure 1, set \( T_k(F_f) \) is the transfer function of the unknown disturbance map to the filtering error \( \{e_k\} \), then the \( H^\infty \) filter can be described as:

1) The optimal \( H^\infty \) filtering problem: to find the optimal \( H^\infty \) estimate \( \hat{z}_k = F_f(Z_0, Z_1, \cdots, Z_k) \) to make \( \|T_k(F_f)\|_\infty \) reach the minimum. It can be given by:

\[
g^2 = \inf_{F_f} \|T_k(F_f)\|_\infty = \inf_{F_f} \sup_{X_k, W_{ch}, V_{ch}} \|X_k - \hat{X}_k\|_{\infty}^2 + \|W_k\|_{\infty}^2 + \|V_k\|_{\infty}^2 \]  

The definition above demonstrate that the optimal \( H^\infty \) filtering ensures all the possible interference input with the defined energy to minimize the estimated error energy gain.

2) Inferior \( H^\infty \) filtering problem: Given a positive number \( \gamma > 0 \), look for sub-optimal \( H^\infty \) estimate \( \hat{z}_k = F_f(Z_0, Z_1, \cdots, Z_k) \), so \( \|T_k(F_f)\|_\infty < \gamma \), and it can be written as:

\[
\inf_{F_f} \sup_{X_k, W_{ch}, V_{ch}} \|X_k - \hat{X}_k\|_{\infty}^2 + \|W_k\|_{\infty}^2 + \|V_k\|_{\infty}^2 < \gamma^2
\]

Based on the definition above, inferior \( H^\infty \) Filtering ensures the possible interference input with a kind of defined energy. The estimated error energy gain is less than an assured value \( \gamma^2 \). It worth noting that the solution of optimal \( H^\infty \) Filtering can be obtained by an expected precision of iterating the \( \gamma \) of \( H^\infty \) Filtering problem. As a result, we only discuss the solution of inferior \( H^\infty \) filtering here.
In the formula, \( \mathbf{R}_{\gamma,k} = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{k} & \mathbf{L}_{k} \end{bmatrix} \mathbf{p}_{k,k-1} \begin{bmatrix} \mathbf{H}_{k}^T & \mathbf{L}_{k}^T \end{bmatrix} \)

That is, if type (16) is set up, a \( H^\infty \) filter is given as follows:

\[
\hat{z}_k = \mathbf{L}_k \hat{x}_k
\]  

(18)

Here \( \hat{x}_k \) recurrence formula can be given by:

\[
\dot{\hat{x}}_{k+1} = \Phi_{k+1} \hat{x}_k + \mathbf{p}_{k+1} \hat{h}_{k+1} \left[ \mathbf{I} + \mathbf{H}_k \mathbf{p}_{k+1} \hat{h}_{k+1} \right]^{-1} \left[ \mathbf{z}_k - \mathbf{H}_k \Phi_{k+1} \hat{x}_k \right]
\]  

(19)

4. A New MUD Algorithm Based On Robust Kalman Filtering

Considering a synchronous DS-CDMA system with \( K \) active users, the wireless channel to be slowly varying Rayleigh fading channel, that channel in a symbol cycle remains unchanged, then the received signal can be expressed as [3,4,5]:

\[
y(n) = \sum_{k=1}^{K} h_k(n) b_k(n) s_k(n) + \sigma \nu(n)
\]  

(20)

In the formula, \( h_k \) stands for the channel coefficient of the user \( k \) to the receiver. \( b_k(n) \) and \( s_k(n) \) respectively express the user’s transmitted symbol and the spreading sequence, spread spectrum gain is \( N \), the additive Gaussian white noise is \( \nu(n) \).

Without loss of generality, assuming that user 1 is the desired user, and setting \( c_1 \) as the user a linear multi-user detector weight vector, similar the approach to decomposition of the constraint component and non-bound components [5]:

\[
c_1(n) = s_1 - \mathbf{C}_{1,\text{null}} \mathbf{w}_1(n)
\]  

(21)

In the formula, the \( (N-1) \) dimension vector \( \mathbf{w}_1(n) \) is the adaptive part of \( c_1(n) \), \( s_1 \) is the non-adaptive part of \( c_1(n) \). The column of \( N \times (N-1) \) dimension matrix \( \mathbf{C}_{1,\text{null}} \) generate the zero space of \( s_1 \), it can be written as: \( \langle \mathbf{C}_{1,\text{null}}, s_1 \rangle = 0 \).

As we all know, the general non-stationary CDMA system has the following state equation:

\[
\mathbf{w}_1(n+1) = \mathbf{F}(n+1,n) \mathbf{w}_1(n) + \mathbf{W}(n)
\]  

(22)

In the formula, \( \mathbf{F}(n+1,n) \) is the state transition matrix, \( \mathbf{W}(n+1) \) is that the other unknown disturbances or the sudden noise impact on the system state.

In the channel to be slowly varying circumstances, the state equation to meet:

\[
\mathbf{w}_1(n+1) = \mathbf{w}_1(n) + \delta \mathbf{w}_1(n)
\]  

(23)

That is, \( \mathbf{F}(n+1,n) = I \), \( \mathbf{W}(n) = \delta \mathbf{w}_1(n) \)

For the observation equation, we obtained the following method:

\[
e(n) = \mathbf{e}_1, \mathbf{y} = \mathbf{e}_1^T(n) \mathbf{y}(n)
\]  

(24)

Bundles (21) into equation (24), we can get the formula:
Using \( y(n) = s_i^\top y(n) \), \( d^H(n) = y(n)^H C_{i,\text{null}} \), the equation (25) can be rewritten as follows:

\[
\tilde{y}(n) = d^H(n)w_i(n) + e(n)
\]  

The formula (26) can be used as the required observation equation of the Kalman filter algorithm.

Being different from the standard Kalman filtering algorithm, the state equation \( \sigma w_i(n) \) and observation equation \( e(n) \) whose property of without any assumption, and they are treated as unknown interference, which is consistent with the actual system.

Using \( L(n) = I_{(N-1)\times(N-1)} \) as the direct estimation \( w_i(n) \), rather than its linear combination. By the \( H^\infty \) filtering theorem, we can get multi-user detection algorithm based the robust \( H^\infty \) filter, as follows:

\[
\hat{w}_i(n) = \hat{w}_i(n-1) + P(n)\hat{d}(n)[I + d^H(n)P(n)d(n)]^{-1}[\hat{y}(n) - d^H(n)\hat{w}(n-1)] 
\]  

In the formula, the recursive equation of \( P(n) \) is can be written as:

\[
P(n+1) = P(n) - P(n)d(n)L^T(n)R_{e,n}^{-1}\begin{bmatrix} d^H(n) \\ L(n) \end{bmatrix}P(n) \tag{28}
\]

\[
R_{e,n} = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} d^H(n) \\ L(n) \end{bmatrix}P(n)[d(n) L^T(n)] \tag{29}
\]

By the \( H^\infty \) filtering theorem, when \( \gamma^2 \) of (29) needs to satisfy the \( H^\infty \) filter convergence condition, it can be given by:

\[
P^{-1}(n) + d(n)d^H(n) - \gamma^2 L^T(n)L(n) > 0 \tag{30}
\]

The initial value is written as:

\[
\hat{w}_i(0) = E[\hat{w}_i(0)] = 0 \tag{31}
\]

\[
P(0) = E[([w_i(0) - \hat{w}_i(0)][w_i(0) - \hat{w}_i(0)]^H)] = I \tag{32}
\]

5. Simulation Results

The counter-compared curved shaped in Fig 2 is shown that the comparison of time- average SNR and iteration times of the two algorithm in the stationary Gaussian Channel. With these parameters below: Bit energy 1 of the expected user, 20 dB of SNR, 9 interference users, 5 users of 30dB , 4 users of 40dB SNR. we can draw a conclusion that the two algorithm get nearly the same performance, Robust Kalman filter algorithm better than Normal Kalman filter algorithm. Because the hypothesis feature of normal Kalman algorithm is more accurate than Robust algorithm in this condition.
The counter-compared curved shaped in Fig 3 is shown as error performance parameter in non stationary Gaussian Channel that shows a comparison of time- average SNR and iteration times. With these parameters below: $N = 31$ of Spreading Gain, bit energy $1$ of the expected user, $20$ dB of SNR, $2$ users of $50$dB added when reached $500^{th}$ iteration, $2$ users of $60$dB added when $1500^{th}$, we can draw a conclusion that the convergence speed of Robust Kalman algorithm is obviously faster than the normal Kalman algorithm. It is a clear evidence that the new algorithm has the ability to provide a stronger adaption and trace feature, but the complexity and time delay appear.

Fig. 2 Time Average SIR（Stationary Channel）

Fig. 3 Time Average SIR（Nonstationary Channel）

Fig. 4 BER of Robost Kalman and Standard Kalman（Stationary Channel）
The counter-compared curved shaped in Fig 4 is shown as error performance parameter in non stationary Gaussian Channel with the same simulation parameter of Simulation 1. The Error Rate of Robust Kalman Algorithm can get a better performance than normal Kalman.

![Fig. 5 The BER comparison of semi-blind multiuser detectors](image)

The counter-compared curved shaped in Fig 5 is shown as error performance parameter in non stationary Gaussian Channel implemented in two different algorithms with which 20000 iteration calculations of SNR. At the beginning of iteration there are 4 interference user of 30dB; 3 users of 40dB added at the 5000th iteration; finally 2 users of 50dB at 10000th iteration. As we see, in the non stationary Gaussian Channel, the normal Kalman filtering algorithm achieved a poor potential performance of error rate in that only $10^{-3}$ order of magnitude can be reached; but Robust Kalman filtering algorithm can get $10^{-4}$ order of magnitude in cost of a test delay time in order to search for a $\gamma$ value compiled with the $H^\infty$ Filter convergence.

6. Conclusions

In this paper, a new robust Kalman filtering multiuser detection algorithm is proposed for the CDMA system. Because it can rapidly converge when the statistical property of interferences or noises are unknown, this algorithm has better performance than existing blind adapting MUD based Kalman filtering. The simulation results show that the proposed new algorithm performs better in terms of SINR and BER, and has low computational complexity.

References


