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Trimaximal mixing with predicted θ_{13} from a new type of constrained sequential dominance

Stefan Antusch [a,b], Stephen F. King [c], Christoph Luhn [c], Martin Spinrath [d,*]

[a] *Department of Physics, University of Basel, Klingelbergstr. 82, CH-4056 Basel, Switzerland*
[b] *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, D-80805 München, Germany*
[c] *School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, United Kingdom*
[d] *SISSA/ISAS and INFN, Via Bonomea 265, I-34136 Trieste, Italy*

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Abstract

Following the recent T2K indication of a sizeable reactor angle, we present a class of models which fix θ_{13} while preserving trimaximal solar mixing. The models are based on a new type of constrained sequential dominance involving new vacuum alignments, along the $(1,2,0)^T$ or $(1,0,2)^T$ directions in flavour space. We show that such alignments are easily achieved using orthogonality, and may replace the role of the subdominant flavon alignment $(1,1,1)^T$ in constrained sequential dominance. In such models, with a normal hierarchical spectrum, the reactor angle is related to a ratio of neutrino masses by $\theta_{13} = \frac{\sqrt{2}}{3}\frac{m_2^\nu}{m_3^\nu}$, leading to $\theta_{13} \sim 5°–6°$, while the atmospheric angle is given by the sum rule $\theta_{23} \approx 45° + \sqrt{2}\theta_{13}\cos\delta$. We find that leptogenesis is unsuppressed due to the violation of form dominance and that the CP violating phase responsible for leptogenesis is precisely equal to the Dirac CP phase δ, providing a direct link between leptogenesis and neutrino mixing in this class of models.

* Corresponding author.
E-mail addresses: stefan.antusch@unibas.ch (S. Antusch), king@soton.ac.uk (S.F. King), christoph.luhn@soton.ac.uk (C. Luhn), spinrath@sissa.it (M. Spinrath).

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1. Introduction

Recently T2K have published evidence for a large non-zero reactor angle [1] which, when combined with data from MINOS and other experiments in a global fit, yields [2,3]

$$6^\circ \lesssim \theta_{13} \lesssim 9^\circ, \tag{1.1}$$

with a statistical significance of a non-zero reactor angle of about 3σ. If confirmed this would rule out the hypothesis of exact tri–bimaximal (TB) mixing [4], and a flurry of alternative proposals have recently been put forward [5].

For example, an attractive scheme based on trimaximal (TM) mixing remains viable [6]. TM mixing is defined to maintain the second column of the TB mixing matrix and hence preserves the solar mixing angle prediction $\sin\theta_{12} \approx 1/\sqrt{3}$. However there is another variation of TM mixing which also preserves this good solar mixing angle prediction by maintaining the first column of the TB matrix, namely TM_1 mixing [7]:

$$U_{\mathrm{TM}_1} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\vartheta & \frac{1}{\sqrt{3}}\sin\vartheta e^{i\rho} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\vartheta - \frac{1}{\sqrt{2}}\sin\vartheta e^{-i\rho} & \frac{1}{\sqrt{2}}\cos\vartheta + \frac{1}{\sqrt{3}}\sin\vartheta e^{i\rho} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\cos\vartheta + \frac{1}{\sqrt{2}}\sin\vartheta e^{-i\rho} & -\frac{1}{\sqrt{2}}\cos\vartheta + \frac{1}{\sqrt{3}}\sin\vartheta e^{i\rho} \end{pmatrix} P, \tag{1.2}$$

where $\frac{1}{\sqrt{3}}\sin\vartheta = \sin\theta_{13}$, P' is a diagonal phase matrix required to put U_{TM_1} into the PDG convention [8], and $P = \mathrm{diag}(1, e^{i\frac{\alpha_2}{2}}, e^{i\frac{\alpha_3}{2}})$ contains the usual Majorana phases. In particular TM_1 mixing approximately preserves the successful TB mixing for the solar mixing angle $\theta_{12} \approx 35^\circ$ as the correction due to a non-zero but relatively small reactor angle is of second order. Although TM_1 mixing reduces to TB mixing in the limit that $\vartheta \to 0$, it is worth emphasising that in general TM_1 mixing involves an undetermined reactor angle θ_{13} which could in principle be large or even maximal (e.g. 45°). The observed smallness of the reactor angle θ_{13} compared to the atmospheric angle $\theta_{23} \approx 45^\circ$ and the solar angle $\theta_{12} \approx 34^\circ$ [2] is therefore not explained by the TM_1 hypothesis alone. Clearly the relative smallness of the reactor angle can only be explained with additional model dependent input. Although there are models of TM mixing which can account for the smallness of the reactor angle [9] so far there is no model in the literature for TM_1 mixing, let alone one which fixes the reactor angle.

In this paper we propose a model of TM_1 mixing where the magnitude of the reactor angle is fixed. The model we discuss is actually representative of a general strategy for obtaining TM_1 mixing using sequential dominance (SD) [10] and vacuum alignment. The strategy of combining SD with vacuum alignment is familiar from the constrained sequential dominance (CSD) approach to TB mixing [11] where a neutrino mass hierarchy is assumed and the dominant and subdominant flavons responsible for the atmospheric and solar neutrino masses are aligned in the directions of the third and second columns of the TB mixing matrix, namely $\langle\phi_1^\nu\rangle \propto (0,1,-1)^T$ and $\langle\phi_2^\nu\rangle \propto (1,1,1)^T$. The new idea here is to maintain the usual vacuum alignment for the dominant flavon, $\langle\phi_1^\nu\rangle \propto (0,1,-1)^T$ as in CSD, but to replace the effect of the subdominant flavon vacuum alignment by a different one, namely either $\langle\phi_{120}\rangle \propto (1,2,0)^T$ or $\langle\phi_{102}\rangle \propto (1,0,2)^T$, where such alignments may be naturally achieved from the standard ones using orthogonality arguments. We shall refer to this new approach as CSD2. We shall show that CSD2 leads to TM_1 mixing and a reactor angle which, at leading order, is predicted to be proportional to the ratio of the solar to the atmospheric neutrino masses, $\theta_{13} = \frac{\sqrt{2}}{3}\frac{m_2^\nu}{m_3^\nu}$.

It is interesting to compare the predictions of CSD2 to another alternative to CSD that has been proposed to account for a reactor angle called partially constrained sequential dominance (PCSD) [12]. PCSD involves a vacuum misalignment of the dominant flavon alignment to $(\varepsilon, 1, -1)^T$, with a subdominant flavon alignment $(1, 1, 1)^T$, leading to tri–bimaximal-reactor (TBR) mixing [12] in which only the reactor angle is switched on, while the atmospheric and solar angles retain their TB values. However, in the case of PCSD, the value of the reactor angle is not predicted whereas CSD2 leads to the above relation.

The layout of the rest of the paper is as follows. In Section 2 we describe CSD2 including a discussion of vacuum alignment and an example of a model based on CSD2. In Section 3 we discuss the phenomenology of CSD2, first showing that it reproduces TM_1 mixing exactly, then comparing the second order analytic results to a numerical treatment. In Section 4 we show that leptogenesis is unsuppressed and moreover CSD2 leads to a link between the CP phase for leptogenesis and the Dirac CP phase δ. We conclude in Section 5.

2. A new type of constrained sequential dominance

2.1. CSD and TB mixing

Assuming the type I see-saw mechanism, in the diagonal right-handed (RH) neutrino mass basis we may write $M_R = \mathrm{diag}(M_A, M_B, M_C)$ and the neutrino Yukawa matrix as $Y_\nu = (A, B, C)$ where A, B, C are three column vectors. Then the type I see-saw formula $M_\nu = Y_\nu M_R^{-1} Y_\nu^T$ gives

$$M_\nu = \frac{v^2 AA^T}{M_A} + \frac{v^2 BB^T}{M_B} + \frac{v^2 CC^T}{M_C}. \tag{2.1}$$

SD corresponds to a hierarchy of contributions $\frac{AA^T}{M_A} \gg \frac{BB^T}{M_B} \gg \frac{CC^T}{M_C}$ corresponding to the physical neutrino mass hierarchy $m_3^\nu \gg m_2^\nu \gg m_1^\nu$ [10]. The Yukawa couplings in A determine the atmospheric angle, while those in B determine the solar angle, with the reactor angle dependent on both [10]. For a strong hierarchy one can ignore the Yukawa couplings in C and the effect of the third right-handed neutrino.

TB mixing naturally emerges from CSD [11] where the neutrino Yukawa matrix Y_ν and the right-handed neutrino mass matrix M_R take the constrained form

$$Y_\nu = \begin{pmatrix} 0 & b \\ a & b \\ -a & b \end{pmatrix}, \qquad M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}, \tag{2.2}$$

assuming $m_1^\nu = 0$. In models with non-Abelian family symmetries this structure can be explained by two flavons pointing in the directions in flavour space defined by the columns of Y_ν:

$$\langle \phi_1^\nu \rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \qquad \langle \phi_2^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{2.3}$$

2.2. CSD2 and vacuum alignment

As our starting point, we consider the vacuum alignment sector of any SD flavour model where the flavons, which furnish triplet representations under a family symmetry G_F, are typically aligned in the three orthogonal directions

$$\langle\phi_1^\nu\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \qquad \langle\phi_2^\nu\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad \langle\phi_3^\nu\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}. \tag{2.4}$$

Applying these flavons to build up the neutrino Yukawa matrix, one would obtain TB neutrino mixing and a form diagonalisable neutrino mass matrix.

In many models three additional orthogonal flavons are present,

$$\langle\phi_1^e\rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \langle\phi_2^e\rangle \propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \langle\phi_3^e\rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{2.5}$$

The third flavon ϕ_3^e is generically introduced to generate hierarchical quark and charged lepton mass matrices, and typically governs the masses of the third generation of charged particles.

In this paper we augment this set of CSD flavons by another flavon pointing in the direction

$$\langle\phi_{120}\rangle \propto \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad \text{or} \quad \langle\phi_{102}\rangle \propto \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}. \tag{2.6}$$

These alignments can be obtained by requiring the orthogonality conditions $\langle\phi_{120}\rangle \cdot \langle\phi_3^\nu\rangle = \langle\phi_{120}\rangle \cdot \langle\phi_3^e\rangle = 0$ or alternatively $\langle\phi_{102}\rangle \cdot \langle\phi_3^\nu\rangle = \langle\phi_{102}\rangle \cdot \langle\phi_2^e\rangle = 0$, where the "·" denotes the usual $SO(3)$ inner product. Implicitly we assume real triplet representations, but it is straightforward to extend this mechanism to complex representations. The orthogonality conditions can be realised easily with "Lagrange multiplier" superfields D_1 and D_2, which are singlets under the family symmetry G_F. They give the following terms in the superpotential

$$D_1\big(\phi_{120} \cdot \phi_3^\nu\big) + D_2\big(\phi_{120} \cdot \phi_3^e\big) \quad \text{or} \quad D_1\big(\phi_{102} \cdot \phi_3^\nu\big) + D_2\big(\phi_{102} \cdot \phi_2^e\big). \tag{2.7}$$

The F-term conditions $|F_{D_1}| = |F_{D_2}| = 0$ are equivalent to the orthogonality conditions and therefore yield the desired alignments.

The shaping symmetries of a specific model select the flavons which couple to the RH neutrinos and thus determine the structure of the neutrino Yukawa matrix. CSD2 corresponds to the case where the dominant flavon is ϕ_1^ν as in CSD, and the subdominant flavon is taken to be either ϕ_{120} or ϕ_{102}. In the following subsection, we sketch a model in which this is achieved.

2.3. An example of a model with CSD2

In the following we briefly outline how to implement the discussed alignment into a concrete model of lepton masses and mixing angles. As the model is mainly for the purpose of illustration, we do not discuss the quark sector and the charged lepton mass matrix is diagonal by construction.

We start with a short discussion on how to achieve the alignments in Eqs. (2.4) and (2.5). This is rather standard and we follow closely the alignment as discussed in [13]. Although we do not implement spontaneous CP violation we stick to Z_4 symmetries to make the analogy more obvious.

The vacuum alignment of $\phi_{1,2,3}^e$ is generated by the renormalisable superpotential

$$\mathcal{W}_e \sim A_i^e\big(\phi_i^e \star \phi_i^e\big) + O_{ij}^e\big(\phi_i^e \cdot \phi_j^e\big), \tag{2.8}$$

where "⋆" denotes the symmetric cross product in A_4, which is defined as $(x \star y)_i = s_{ijk} x^j y^k$, with $s_{ijk} = +1$ on all permutations $\{i, j, k\} \in \{1, 2, 3\}$ and zero otherwise, see also [16]. Here

Table 1
The symmetries of the example model of lepton masses and mixings. Only the matter, the Higgs and the flavon fields are shown. The charges of the driving fields can be easily inferred from the corresponding superpotential terms.

	$SU(2)\times U(1)$	A_4	$Z_4^{(1)}$	$Z_4^{(2)}$	$Z_4^{(3)}$	$Z_4^{(4)}$	$Z_4^{(5)}$	$Z_4^{(6)}$	$Z_4^{(7)}$
L	$\mathbf{2}_{-1}$	$\mathbf{3}$							
e^c	$\mathbf{1}_2$	$\mathbf{1}$	3						
μ^c	$\mathbf{1}_2$	$\mathbf{1}$		3					
τ^c	$\mathbf{1}_2$	$\mathbf{1}$			3				
N_1	$\mathbf{1}_0$	$\mathbf{1}$				3			
N_2	$\mathbf{1}_0$	$\mathbf{1}$					3		
H_1	$\mathbf{2}_{-1}$	$\mathbf{1}$							
H_2	$\mathbf{2}_1$	$\mathbf{1}$							
ϕ_1^e	$\mathbf{1}_0$	$\mathbf{3}$	1						
ϕ_2^e	$\mathbf{1}_0$	$\mathbf{3}$		1					
ϕ_3^e	$\mathbf{1}_0$	$\mathbf{3}$			1				
ϕ_1^ν	$\mathbf{1}_0$	$\mathbf{3}$				1			
ϕ_2^ν	$\mathbf{1}_0$	$\mathbf{3}$						1	
ξ	$\mathbf{1}_0$	$\mathbf{1}$						1	
ϕ_3^ν	$\mathbf{1}_0$	$\mathbf{3}$							1
ϕ_{120}	$\mathbf{1}_0$	$\mathbf{3}$					1		
ϕ_{102}	$\mathbf{1}_0$	$\mathbf{3}$					1		

and in the rest of this section we drop coupling constants in the superpotential to increase the readability. The F-term conditions of the triplet driving fields A_i^e align the flavon vevs in the desired directions[1] and the singlet driving fields O_{ij}^e enforce the flavons to point in three different directions. Note that A_i^e is doubly charged under $Z_4^{(i)}$ while O_{ij}^e carries a charge of three under both $Z_4^{(i)}$ and $Z_4^{(j)}$. We also assume a $U(1)_R$ symmetry under which the matter fields carry a charge of one, the flavons are neutral and the driving fields are doubly charged.

For the alignment of the ϕ^ν flavons we employ the superpotential

$$\mathcal{W}_\nu \sim A_2^\nu\big(\xi\phi_2^\nu + \phi_2^\nu \star \phi_2^\nu\big) + O_{ij}^\nu\big(\phi_i^\nu \cdot \phi_j^\nu\big) + O_{11}^{\nu e}\big(\phi_1^\nu \cdot \phi_1^e\big), \tag{2.9}$$

where we use a similar notation as before and the charges under the A_4 and Z_4 symmetries are distributed in the same way. Note that we have introduced an additional auxiliary singlet flavon field ξ with a non-zero vev to align ϕ_2^ν. Having obtained the alignment $\langle\phi_2^\nu\rangle$ from the A_2^ν driving field, the remaining neutrino flavons ϕ_1^ν and ϕ_3^ν are aligned by the driving fields $O_{11}^{\nu e}$ and O_{ij}^ν.

Eventually, the new flavon alignments are achieved by adding the two singlet driving fields D_1 and D_2 of Eq. (2.7) as discussed previously. D_1 is charged under $Z_4^{(5)}$ and $Z_4^{(7)}$ while D_2 is charged under $Z_4^{(5)}$ and $Z_4^{(2/3)}$ for $\phi_{102/120}$.

With the symmetries and the field content as given in Table 1 we end up with the following Yukawa superpotential

$$\mathcal{W}_{\text{Yuk}} \sim \frac{1}{\Lambda}\big(\phi_1^e \cdot L e^c H_1 + \phi_2^e \cdot L\mu^c H_1 + \phi_3^e \cdot L\tau^c H_1 + \phi_1^\nu \cdot L N_1 H_2 + \phi_{120/102} \cdot L N_2 H_2\big), \tag{2.10}$$

[1] At this stage the flavon vevs could also vanish. We assume that they are driven to non-zero values by either soft SUSY breaking mass terms or higher-dimensional terms in the superpotential.

which give the Yukawa couplings after the flavons develop their vevs. Λ is a generic messenger mass scale. The charged lepton Yukawa matrix is diagonal due to the alignment of the ϕ^e flavons:

$$Y_e = \mathrm{diag}(y_e, y_\mu, y_\tau), \tag{2.11}$$

and the neutrino Yukawa couplings $Y_\nu = (A, B)$ read

$$Y_\nu^{(120)} = \begin{pmatrix} 0 & b \\ a & 2b \\ -a & 0 \end{pmatrix} \quad \text{or} \quad Y_\nu^{(102)} = \begin{pmatrix} 0 & b \\ a & 0 \\ -a & 2b \end{pmatrix}, \tag{2.12}$$

depending on the choice of the subdominant flavon, either ϕ_{120} or ϕ_{102}. The parameters a and b can be determined from the parameters in the superpotential. Later on we will see, that a relative phase difference $\arg(a/b) = 45°, 135°, 225°$ or $315°$, which translates into a Dirac CP phase $\delta = 90°$ or $270°$, is preferred by experimental data and that this would also maximise the generated baryon asymmetry. Such a phase difference can be easily obtained in the context of spontaneous CP violation from discrete symmetries as discussed in [13], which could be applied here straightforwardly.

Due to the Z_4 symmetries the RH neutrinos have no mass terms at the renormalisable level, but they become massive after the flavons develop their vevs due to the following terms in the superpotential

$$\mathcal{W}_R \sim \frac{1}{\Lambda}(\phi_1^\nu)^2 N_1^2 + \frac{1}{\Lambda}\phi_{120/102}^2 N_2^2. \tag{2.13}$$

From the symmetries alone also terms like $\phi_1^\nu \cdot \phi_{120/102} N_1 N_2$ would be allowed, but we assume, that the messenger fields mediating such operators are absent. Under this common assumption the RH neutrino mass matrix is diagonal

$$M_R = \begin{pmatrix} M_A & 0 \\ 0 & M_B \end{pmatrix}. \tag{2.14}$$

3. The phenomenology of CSD2

3.1. Predictive trimaximal mixing from CSD2

With the charged lepton mass matrix being diagonal, the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing originates solely from the neutrino sector. As discussed in the previous section we introduce two RH neutrinos N_i $(i = 1, 2)$ which entails one massless light neutrino. The RH neutrino mass matrix M_R is assumed to be diagonal and each N_i couples to its own flavon. Adopting ϕ_1^ν for the dominant and ϕ_{120} for the subdominant term, the resulting neutrino Yukawa matrix is $Y_\nu^{(120)}$, see Eq. (2.12).[2] Then the (type-I) see-saw formula leads to a simple effective light neutrino mass matrix, given by

[2] We comment on the case where the subdominant flavon is taken to be ϕ_{102} below Eqs. (3.6)–(3.13). Note that this resembles the case with two-texture zeros in the neutrino Yukawa matrix, whose phenomenology was extensively discussed in [14,15]. Here we go beyond those papers by giving an explicit vacuum alignment mechanism for such a texture, where all mixing angles and phases depend on a single complex parameter.

$$
\begin{aligned}
M_\nu &= \frac{v^2 AA^T}{M_A} + \frac{v^2 BB^T}{M_B} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + m_b \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= m_a \begin{pmatrix} \frac{m_b}{m_a} & 2\frac{m_b}{m_a} & 0 \\ 2\frac{m_b}{m_a} & 1+4\frac{m_b}{m_a} & -1 \\ 0 & -1 & 1 \end{pmatrix},
\end{aligned} \tag{3.1}
$$

where $m_a = \frac{v^2 a^2}{M_A}$ and $m_b = \frac{v^2 b^2}{M_B}$ can in general be complex. We assume $|m_b| \ll |m_a|$, which can originate from a hierarchy in M_A and M_B, in the parameters a and b, or a combination of both. This is nothing but sequential dominance.

The scale of family symmetry breaking, the messenger scale Λ and the scale of the right-handed neutrino masses is undetermined and can be chosen appropriately. This is due to the fact, that

$$
M_\nu \sim v^2 \frac{\langle \phi \rangle^2}{M_R \Lambda^2}, \tag{3.2}
$$

from which this freedom of choice is obvious.

Clearly, the unitary matrix that diagonalises M_ν depends on only one complex parameter

$$
\frac{m_b}{m_a} = \epsilon e^{i\alpha}, \quad \epsilon, \alpha \in \mathbb{R}. \tag{3.3}
$$

An overall Majorana phase related to the parameter m_a can be set to zero without loss of generality. With $|m_b| \ll |m_a|$ we can use ϵ as an expansion parameter for the resulting neutrino masses and mixing angles. We have computed our results to second order in ϵ. General formulas can be found in [10].

Before determining the PMNS matrix explicitly, it is worth to show why the neutrino mass matrix M_ν in Eq. (3.1) implies the $\mathrm{TM_1}$ mixing form in Eq. (1.2) where the first column is proportional to $(-2, 1, 1)^T$. The reason is simply that $\langle \phi_3^\nu \rangle \propto (-2, 1, 1)^T$ is an eigenvector of M_ν in Eq. (3.1) with a zero eigenvalue corresponding to the first neutrino mass being zero. The reason for this is that M_ν in Eq. (3.1) is a sum of two terms, the first being proportional to $\langle \phi_1^\nu \rangle \langle \phi_1^\nu \rangle^T$ and the second being proportional to $\langle \phi_{120} \rangle \langle \phi_{120}^T \rangle$. Since $\langle \phi_3^\nu \rangle \propto (-2, 1, 1)^T$ is orthogonal to both $\langle \phi_1^\nu \rangle$ and $\langle \phi_{120} \rangle$ it is then clearly annihilated by the neutrino mass matrix, i.e. it is an eigenvector with zero eigenvalue. Therefore we immediately expect M_ν in Eq. (3.1) to be diagonalised by the $\mathrm{TM_1}$ mixing matrix where the first column is proportional to $\langle \phi_3^\nu \rangle \propto (-2, 1, 1)^T$. Although the remainder of this subsection gives a perturbative diagonalisation of M_ν in Eq. (3.1), we already know it must lead to $\mathrm{TM_1}$ mixing exactly to all orders according to this general argument.

Mindful of the PDG phase conventions for the PMNS mixing matrix we write

$$
U_{\mathrm{PMNS}}^T P' M_\nu P' U_{\mathrm{PMNS}} = M_\nu^{\mathrm{diag}} = \mathrm{diag}\big(0, m_2^\nu, m_3^\nu\big), \tag{3.4}
$$

with

$$
U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P, \tag{3.5}
$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. Furthermore, $P = \mathrm{diag}(1, e^{i\frac{\alpha_2}{2}}, e^{i\frac{\alpha_3}{2}})$ is the Majorana phase matrix and $P' = \mathrm{diag}(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau})$ is an unphysical phase matrix which is required to bring the PMNS matrix into PDG form.

In order to determine U_{PMNS} we first calculate the eigenvalues of $(M_\nu^\dagger M_\nu)/m_a^2$ which will be functions of ϵ and α. Requiring orthogonality, the corresponding eigenvectors can be obtained analytically. These eigenvectors, normalised to the unit length, comprise the unitary matrix which diagonalises $M_\nu^\dagger M_\nu$ and thus also M_ν. It can therefore be identified with $P' U_{\text{PMNS}}$ once the Majorana phases have been adjusted to give real masses in Eq. (3.4). Using ϵ as our expansion parameter we obtain the second order result,[3]

$$m_2^\nu = \left[3\epsilon - 3\epsilon^2 \cos\alpha\right] m_a, \tag{3.6}$$

$$m_3^\nu = \left[2 + 2\epsilon\cos\alpha + \frac{\epsilon^2}{2}(7 - \cos 2\alpha)\right] m_a, \tag{3.7}$$

$$\theta_{23} = \frac{\pi}{4} + \epsilon\cos\alpha + \epsilon^2\left(\frac{3}{2} - \cos 2\alpha\right), \tag{3.8}$$

$$\theta_{12} = \arcsin\frac{1}{\sqrt{3}} - \frac{\epsilon^2}{2\sqrt{2}}, \tag{3.9}$$

$$\theta_{13} = \frac{\epsilon}{\sqrt{2}} + \frac{\epsilon^2}{2\sqrt{2}}\cos\alpha, \tag{3.10}$$

$$\delta = \alpha - \epsilon\frac{5}{2}\sin\alpha \quad \text{(only up to order } \epsilon), \tag{3.11}$$

$$\alpha_2 = -\alpha + 2\epsilon\sin\alpha - 3\epsilon^2\sin 2\alpha, \tag{3.12}$$

$$\alpha_3 = 0. \tag{3.13}$$

Note that the PMNS matrix has only one non-trivial Majorana phase as one of the neutrinos is exactly massless. These results are only slightly modified if we choose the $(1, 0, 2)^T$ alignment for the subdominant neutrino term: $\theta_{23} \to \frac{\pi}{2} - \theta_{23}$, $\delta \to \pi + \delta$, $\delta_e \to \pi + \delta_e$, and $\delta_\mu \leftrightarrow \delta_\tau$. All observables in the neutrino sector can be expressed in terms of m_a, ϵ and α. Excluding Majorana phases (and the mass of the massless neutrino), this means that the model class makes three predictions which should be testable in future oscillation experiments since θ_{13} is comparatively large.

It is useful to compare the above predictions to a general leading order parametrisation of the PMNS mixing matrix in the PDG convention in terms of deviations from TB mixing [17],

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{2}{\sqrt{6}}(1 - \frac{1}{2}s) & \frac{1}{\sqrt{3}}(1 + s) & \frac{1}{\sqrt{2}} r e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 + s - a + r e^{i\delta}) & \frac{1}{\sqrt{3}}(1 - \frac{1}{2}s - a - \frac{1}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}}(1 + a) \\ \frac{1}{\sqrt{6}}(1 + s + a - r e^{i\delta}) & -\frac{1}{\sqrt{3}}(1 - \frac{1}{2}s + a + \frac{1}{2} r e^{i\delta}) & \frac{1}{\sqrt{2}}(1 - a) \end{pmatrix} P, \tag{3.14}$$

where the deviation parameters s, a, r are defined as [17],

$$\sin\theta_{12} = \frac{1}{\sqrt{3}}(1 + s), \qquad \sin\theta_{23} = \frac{1}{\sqrt{2}}(1 + a), \qquad \sin\theta_{13} = \frac{r}{\sqrt{2}}. \tag{3.15}$$

[3] We remark that the CP violating Dirac phase δ is only determined to order ϵ as it always appears together with $\sin\theta_{13}$ which in turn is already of order ϵ. For completeness we also list the results for the unphysical phases $\delta_e = -\frac{\epsilon}{2}\sin\alpha(1 - 5\epsilon\cos\alpha)$, $\delta_\mu = -\frac{\epsilon}{2}\sin\alpha(3 - 7\epsilon\cos\alpha)$, and $\delta_\tau = \pi + \frac{\epsilon}{2}\sin\alpha(1 - \epsilon\cos\alpha)$.

At leading order the above predictions can be expressed by

$$a = r\cos\delta, \qquad s = 0, \tag{3.16}$$

where

$$r = \frac{2}{3}\frac{m_2^\nu}{m_3^\nu} \sim \frac{2}{15} \to \theta_{13} \sim 5°\text{–}6°, \tag{3.17}$$

where the predicted reactor angle may be compared to Eq. (1.1).[4] We emphasise that these predictions hold true for both the $(1,2,0)^T$ as well as the $(1,0,2)^T$ alignment. In both cases, with a suitable choice of phase convention, the leading order mixing matrix can be written in the form,

$$U_{\mathrm{TM}_1} = P' \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} re^{-i\delta} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1 - \frac{3}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1 + re^{-i\delta}) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}}(1 + \frac{3}{2}re^{i\delta}) & -\frac{1}{\sqrt{2}}(1 - re^{-i\delta}) \end{pmatrix} P, \tag{3.18}$$

where Eq. (3.18) corresponds to a small angle expansion of TM_1 mixing in Eq. (1.2). However, from the general argument given earlier in this subsection, we expect TM_1 mixing in Eq. (1.2) to be valid to all orders beyond the small angle approximation.

3.2. Numerical results

In this section we present the numerical results for the two CSD2 cases defined by Eqs. (2.12) and (2.14) in Figs. 1 and 2. We used random values for m_a, m_b and α as input and calculated with the Mixing Parameter Tools provided with the REAP package [19] the resulting neutrino masses, mixing angles and CP violating phases. We used then the recent global fit results from [2] for the solar and atmospheric neutrino mass squared differences and mixing angles deduced from the new reactor fluxes as a constraint. We have also checked that the numerical results agree well with the second order analytic results in Eqs. (3.6)–(3.13).

There are some interesting features of the plots. First of all, note that θ_{13} can go up to more than 7° in the 3σ interval. It is, however, more interesting to look at the 1σ regions. The atmospheric mixing angle θ_{23} has an upper 1σ bound of 45°, which is very restrictive for the $(1,2,0)^T$ alignment. Indeed, by this bound, $\theta_{13} > 5°$ is disfavoured in the $(1,2,0)^T$ case, while for the $(1,0,2)^T$ alignment values up to 6.4° are still allowed, see upper panels of Fig. 1. This is due to the fact that the deviations from $\theta_{23} = 45°$ have opposite signs for both cases. Turning to the solar mixing angle, the 1σ region for θ_{12} induces a lower bound on θ_{13} of approximately 4.5°, which is identical in both cases, see lower panels of Fig. 1.

It is also interesting to look at the phases in Fig. 2. In the $(1,2,0)^T$ alignment case a phase difference α, cf. Eq. (3.3), of approximately 90°–100° or 260°–270° is preferred as can be seen in the upper left panel of Fig. 2.[5] For the $(1,0,2)^T$ alignment, the preferred values of the Dirac CP phase span bigger regions, but still the CP conserving case is not preferred, see upper right panel of Fig. 2. Actually, the maximally CP violating cases $\delta = \pm 90°$ are in both cases at the edge

[4] Note that in a model where the charged lepton mass matrix is not diagonal, one must combine the charged lepton corrections with the underlying TB neutrino mixing deviations to formulate the total observed deviation from TB mixing as discussed in [18].

[5] Keep in mind that the Dirac CP phase is almost identical to the phase difference α in this case.

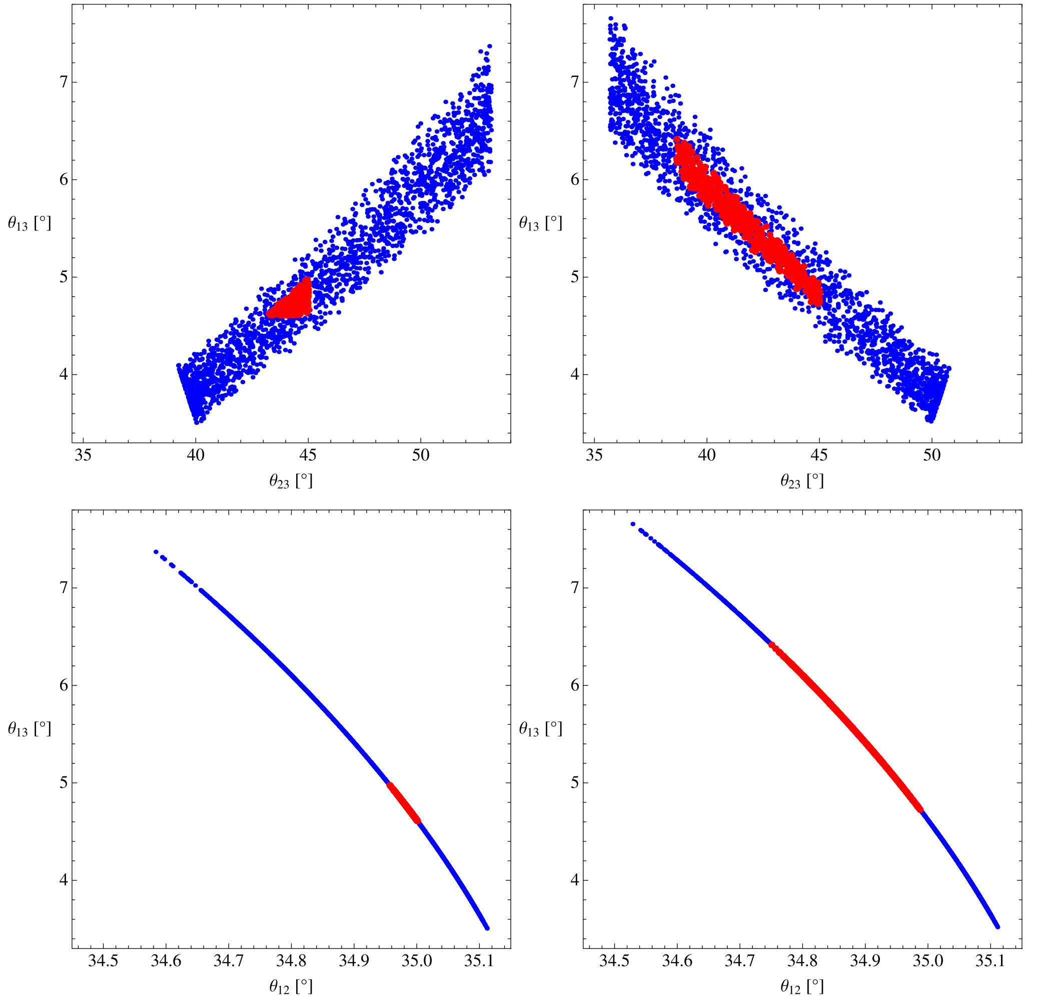

Fig. 1. The correlations between θ_{13} and the other two mixing angles in CSD2. The panels on the left/right show the results for the $(1,2,0)^T/(1,0,2)^T$ alignment. The regions compatible with the 1σ (3σ) ranges of the atmospheric and solar neutrino mass squared differences and mixing angles, taken from [2], are depicted by the red (blue) points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

of the preferred regions. This is due to the fact that the corrections to a maximal atmospheric mixing are very small for $\alpha \approx \delta = \pm 90°$, see Eq. (3.8). Such a phase can emerge naturally in models with spontaneous CP violation from discrete symmetries [13].

4. PMNS-leptogenesis link

As has been noticed in [20–22], in models where TB mixing is realised via flavons which are orthogonal to each other, as in CSD or more generally in scenarios which satisfy the conditions of form dominance [22,23], the CP asymmetries for leptogenesis vanish.

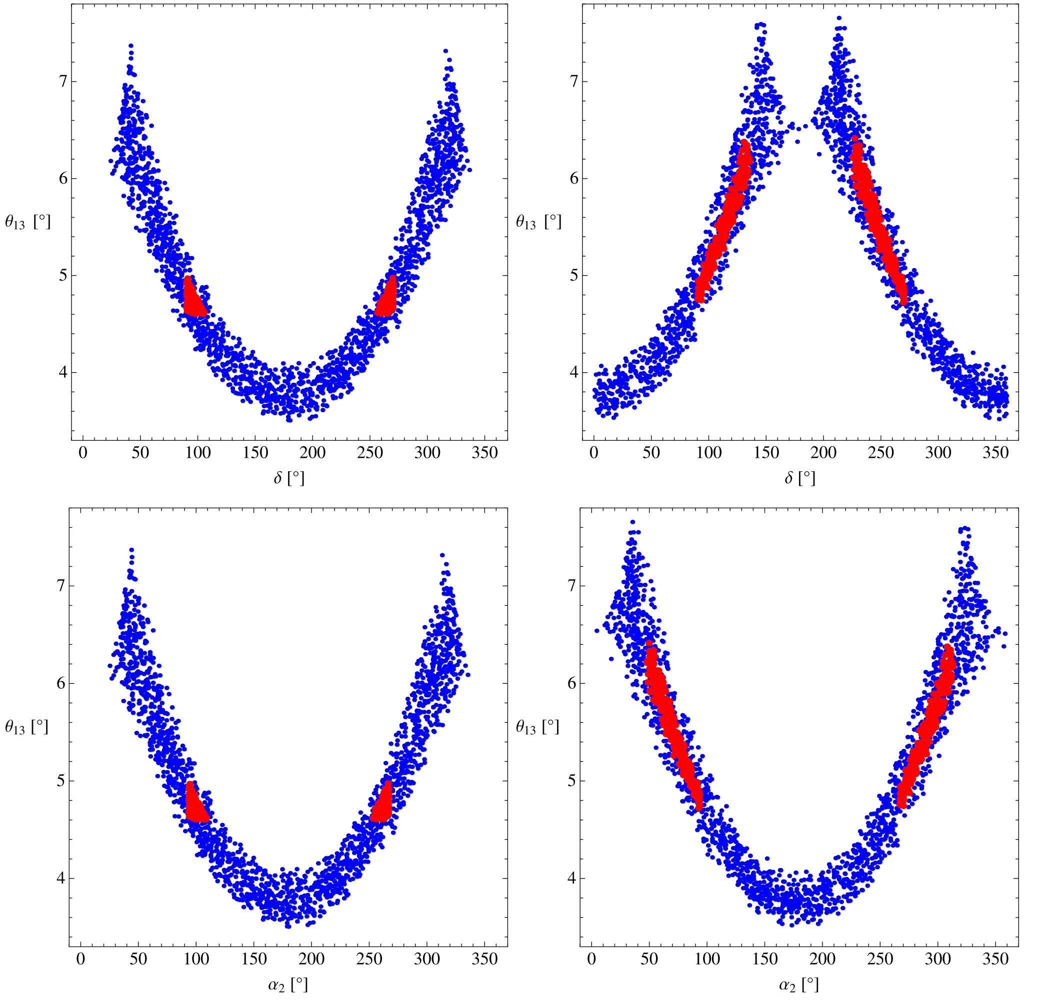

Fig. 2. The correlations between θ_{13} and the two physical phases in CSD2. The panels on the left/right show the results for the $(1, 2, 0)^T/(1, 0, 2)^T$ alignment. The regions compatible with the 1σ (3σ) ranges of the atmospheric and solar neutrino mass squared differences and mixing angles, taken from [2], are depicted by the red (blue) points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

On the contrary, in models with $(1, 2, 0)^T$ or $(1, 0, 2)^T$ vacuum alignment – since the flavon vevs of the model are now no longer orthogonal – the asymmetry does not vanish, rendering models of this type attractive for cosmology.

Furthermore, the two zero textures in Y_ν imply a direct link between the CP violation for leptogenesis and the Dirac CP phase δ, as has been discussed for models with sequential dominance and hierarchical RH neutrinos in [15,20,24]. The produced baryon asymmetry Y_B from leptogenesis in models with $(1, 2, 0)^T$ or $(1, 0, 2)^T$ vacuum alignment satisfies

$$Y_B \propto \pm \sin\delta, \tag{4.1}$$

meaning that a measurement of δ in future neutrino oscillation experiments allows to draw conclusions about the prospects for leptogenesis.

The sign in Eq. (4.1) depends on the choice of the new flavon alignment, either $(1,2,0)^T$ or $(1,0,2)^T$, as well as on which of the RH neutrinos is the lightest, N_1 with mass M_A or N_2 with mass M_B (cf. [20]). Explicitly, the "+" sign applies to the $(1,0,2)^T$ alignment with $M_A \ll M_B$ and to the $(1,2,0)^T$ alignment with $M_B \ll M_A$. The "−" sign holds for the other two cases, the $(1,0,2)^T$ alignment with $M_B \ll M_A$ and the $(1,2,0)^T$ alignment with $M_A \ll M_B$.

Since the baryon asymmetry Y_B is positive, it follows that, in models with a fixed alignment and RH neutrino masses, leptogenesis requires δ in a specific range. In models where the "+" sign applies in Eq. (4.1) only the region around $\delta = 90°$ generates the correct positive Y_B, while in models where the "−" sign holds only the region around $\delta = 270°$ is valid.

Current global fits [3] favour $\sin\delta$ being negative and this suggests that the negative sign is favoured in Eq. (4.1). According to the above discussion this suggests either the $(1,0,2)^T$ alignment with $M_B \ll M_A$ or the $(1,2,0)^T$ alignment with $M_A \ll M_B$. The latter possibility corresponds to so called "light sequential dominance" which plays a special role in leptogenesis within the framework of two right-handed neutrino models as recently discussed in [25].

5. Summary and conclusions

Recently T2K have published evidence for a large non-zero reactor angle which, if confirmed, would exclude the tri–bimaximal mixing pattern. In this paper we have presented a model which fixes the reactor angle while preserving trimaximal solar mixing. In particular we have shown how a variant of trimaximal mixing, called TM_1 mixing in Eq. (1.2) with the solar angle given by $\sin\theta_{12} \approx 1/\sqrt{3}$, results from an extension of constrained sequential dominance involving new vacuum alignments along the $(1,2,0)^T$ or $(1,0,2)^T$ directions in flavour space. We have shown that such alignments are naturally achieved using orthogonality, and may replace the role of the subdominant flavon alignment $(1,1,1)^T$ in constrained sequential dominance. We have proposed the first model in the literature of this kind leading to TM_1 mixing where the reactor angle is related to the ratio of the solar to the atmospheric neutrino masses, $\theta_{13} = \frac{\sqrt{2}}{3}\frac{m_2^\nu}{m_3^\nu}$. We emphasise that the considered model is merely representative of a general strategy based on CSD2 for obtaining TM_1 mixing together with the above prediction for the reactor angle.

We have studied the phenomenological consequences of CSD2 both analytically and numerically. The analytic treatment confirms that TM_1 mixing results, at leading order, in a reactor angle which is predicted to be proportional to the ratio of the solar to the atmospheric neutrino masses, yielding $\theta_{13} \sim 5°$–$6°$, while the atmospheric angle is given by the sum rule $\theta_{23} \approx 45° + \sqrt{2}\,\theta_{13}\cos\delta$, where the leptonic Dirac CP phase δ is undetermined by CSD2, but experimentally preferred to lie in a range of 90°–130° or 230°–270°. The numerical results agree well with the second order analytic results, and demonstrate the full range of neutrino mixing parameters possible with CSD2, although this range could be extended in models which contain additional contributions from charged lepton mixing.

Finally we have seen that in CSD2 leptogenesis is unsuppressed due to the violation of form dominance and that the decay asymmetries feature a direct link between the CP phase for leptogenesis and the Dirac CP phase δ, with the produced baryon asymmetry $Y_B \propto \pm\sin\delta$.

In conclusion, CSD2 leads to a highly predictive form of leptonic mixing, with the solar angle tightly constrained to its trimaximal value, the reactor angle predicted to be within the range of recent global fits, and the atmospheric angle correlated with the Dirac CP phase δ which is precisely equal to the leptogenesis phase. The large reactor angle indicated by T2K therefore opens up the exciting possibility of an early measurement of the low energy CP violating phase δ

which is also responsible for the matter–antimatter asymmetry of the Universe within this class of models.

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