Comparative Analyses of Stock Returns Properties and Predictability

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Abstract

This study investigates the stock returns of the Dow Jones Industrial Average (DJIA), Standard and Poor’s (S&P) 500, and the National Association of Securities Dealers Automated Quotations (NASDAQ) to analyze and compare their properties and to determine their relative predictability. While it is commonly accepted that price per earnings ratio and corporate earnings are the main determinants of stock market returns, this assertion may not hold equally for monthly and daily stock returns. In addition, does this assertion hold equally for stock returns regardless of the stock market index? This work uses nonparametric spectral estimation to study the underlying properties of stock returns and uses monthly corporate earnings (corporate profits after tax) and 3-month Treasury bill interest rate (proxy for price per earnings ratio) to forecast the monthly stock returns of S&P 500, DJIA, and NASDAQ indices. Since corporate earnings are issued quarterly, this data set had to be interpolated to produce the monthly corporate earnings. Overall, the analyses and forecasting were facilitated with both statistical and digital signal processing techniques. Some examples of the techniques used include Hurst exponent to determine predictability, nonparametric spectral estimation to determine the underlying properties of the stock returns, and correlation and root mean square error to determine the forecasting accuracy. The results of this study provide evidence to support that economic and financial time series such as interest rates, corporate earnings, and stock market returns are time varying and non-Gaussian with smooth compactly supported and essentially bandlimited power spectral density estimates. It further shows that the forecasts of the different stock market returns align well with the desired values and the S&P 500 forecasted stock returns were the best.

Keywords: Stock Returns; Forecasting; Power Spectral Density Estimate; Spectrogram; Stock Price Indices; Interest Rate; Corporate Earnings

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1. Introduction

This paper investigates stock returns’ properties and predictability relative to each other. Price per earnings (P/E) ratios and real corporate earnings (corporate profits after tax) have been found to influence stock market returns through stock prices\(^1, 2, 3\). Since interest rates and P/E ratios are inversely related to each other and interest rates are used “as a proxy for the P/E ratio”\(^4\), inverted interest rates and corporate earnings also strongly impact stock market prices and returns. These two determinants of stock prices are directly affected by the changes in real money, changes in the price level, and changes in total spending, which are ultimately affected by the corporate tax rate, potential output, changes in nominal money, and changes in government spending as described in the Keran stock price determination model\(^1\). Lorie and Hamilton\(^2\) and AL-Shubiri\(^4\) added other influencing factors of stock returns in dividend yield and the general economy’s evolution. Major stock market indices such as the Dow Jones Industrial Average (DJIA), Standard and Poor’s (S&P) 500, and the National Association of Securities Dealers Automated Quotations (NASDAQ) are descriptively very similar to each other – their fluctuations typically coincide\(^1, 2\), suggesting moderately high to very high correlations between pairs of them. In fact, for stock price samples under study, the Pearson correlations between S&P 500 and DJIA, S&P 500 and NASDAQ, and DJIA and NASDAQ were found to be 0.952, 0.824, and 0.729 respectively over the period where their durations coincide: January 1986 to May 2015. Moreover, according to Levy and Sarnat\(^5\) and Wong et al\(^6\) stock returns in different industries within the typical economy tend to move together with sufficiently high positive correlations.

Stock market returns reflect a relatively cheap alternative to loans for obtaining capital to finance a business\(^4\). Under the efficient market hypothesis, stock returns are an example of a random walk process.\(^2\) However, some researchers have produced evidence to the contrary. For example, Lo and MacKinlay\(^7\) used a volatility-based specification test to reject the notion that stock returns follow the random walk model at least for weekly stock returns. Their conclusion was supported by the evidence of amply high “positive serial correlation” in weekly and monthly stock market data. Furthermore, Officer\(^8\) found that stock market returns are described by ‘fat tailed’ distributions with some level of stability that appears more consistently in monthly returns. In addition, Biswal\(^9\) provided an overview of the applications of spectral analyses and wavelets used to uncover the inherent hidden periodicities, nonstationarities, nonlinearity, and complex dynamics and relationships in the behavior of stock price changes (returns). Spectral analysis was also used to argue against the randomness of stock returns by demonstrating that their spectra vary in important ways and were therefore not flat\(^10\). Moreover, stock markets in developing countries may produce stock returns with attributes different from stock market returns in developed countries. For example, Bhandari and Kamaiah\(^11\) used cross-spectral analysis to study the relationships between returns yielded on India’s BSE 30 and those generated on well-established stock markets in the United States of America (DJIA and NASDAQ), United Kingdom (FTSE 100), and Japan (Nikkei 225) and discovered “some evidence” showing that the relationships are nonlinear with short-term synchrony, and that the BSE 30 was lagging in the long-run. Finally, to capture the spectra of business cycles as they evolve through time, Turhan-Sayan and Sayan\(^12\) used both linear (Gabor Transform and short-time Fourier Transform) and nonlinear (Page distribution and Wigner distribution) time-frequency methods to compare their relative effectiveness in detecting cycles in stock market data, and found the Page distribution to be the most effective on both synthetic and real stock market data.

Before applying a forecasting regime to time series data such as stock market returns, a desirable precursor is to determine the predictability of the time series\(^13\). An effective way to accomplish this task is to estimate the Hurst exponent\(^13, 14, 15, 16, 17\) of the time series data. The Hurst exponent provides a reasonable estimate of the relative persistence inherent in a time series\(^14, 15, 16, 17\). A persistent time series has Hurst exponent in the half-open interval of (0.5, 1) and consequently has long-term memory (correlations)\(^16, 17\), and is therefore predictable over the discernable trends, at least, in the short-run\(^15\). These time series data that include stock market returns have a fractal dimension (2 – Hurst exponent) in the half-open interval of [1, 1.5), implying that the fractal dimension of a time series decreases as its Hurst exponent increases suggesting less jaggedness in the time series. Mandelbrot and Hudson\(^16\) reported an average Hurst exponent of about 0.635 for S&P 500 stock returns from their examination of several studies. More specifically, Lobato and Savin\(^18\) reported that their findings support the existence of meaningful long-term memory in stock returns.

Stock market returns are inherently noisy, volatile, multivariate, complex time series with nonlinear, nonstationary, and non-Gaussian features\(^14, 16, 19\). They also possess uncertain and intricate relationships\(^16, 20\). Of the
many different linear and nonlinear models used to forecast financial time series including stock returns, neural
network related models seem to be the favorite among researchers. Neural network related models tend to
outperform other types of models in the forecasting of financial time series.

This study explores the underlying properties of stock returns using descriptive and inferential statistics as well as
nonparametric spectral estimation in spectral and time frequency analyses through the Welch power spectral density
and the short-time Fourier Transform based spectrogram respectively. In addition, the stock returns, which are
generated from S&P 500, DJIA, and NASDAQ composite monthly stock closing prices, are forecasted using
predictor variables that included their past values and inverted 3-month Treasury bill interest rate and/or corporate
earnings in the framework of focused gamma neural network models. The forecasts are compared relative to each
other using visual examination as well as root mean square error and correlation. On these bases, the overall best
forecasts are associated with the S&P 500 stock returns. The forecasted stock returns for the S&P 500 are slightly
better than those for the NASDAQ composite, which are slightly better than those for the DJIA.

2. Materials and Methods

The materials for this study include predictor variables that are obtained from quarterly corporate profits after tax
with inventory valuation adjustment and capital consumption adjustment (corporate earnings) and monthly 3-month
Treasury bill interest rate, and response variables that are obtained from adjusted monthly stock closing prices of
S&P 500, DJIA, and NASDAQ composite indices. The corporate earnings and the 3-month Treasury bill (T-bill)
interest rate were obtained from the Federal Reserve Bank of St. Louis Research & Data and the stock market data
were obtained from Yahoo Finance Historical Data. Each of these economic and financial time series data sets had
different time durations. Corporate earnings had a duration spanning from January 1947 to January 2015; 3-month
T-bill interest rate had a duration spanning from January 1934 to May 2015; S&P 500 stock prices had a duration
spanning from January 1950 to June 2015; DJIA stock prices had a duration spanning January 1985 to June 2015;
and NASDAQ Composite stock prices had a duration spanning from February 1971 to June 2015. The tools used to
render these data sets in the appropriate form for preprocessing as well as processing to produce the desired results
include Microsoft Excel 2010, Mathworks Matlab R2015a, and NeuroDimension NeuroSolutions.

The preprocessing of the economic and financial time series data first involved the interpolation of corporate
earnings and the inversion of the 3-month T-bill interest rate. The interpolation was done to convert quarterly
corporate earnings time series data into monthly corporate earnings data by filling in the missing months. This
interpolation was executed with the Matlab interp function. The inverted 3-month T-bill interest rate time series data
produced from multiplying the actual 3-month T-bill interest rate data by -1 was further subjected to a 36-month backward first difference and scaled to within ±1 by its maximum absolute value. Corporate earnings and S&P 500, DJIA, NASDAQ composite stock prices were subjected to 12-month relative changes. The changes in stock prices are stock returns. Further preprocessing of five time series data sets included nonlinear filtering using the 3rd order median filter and mean removal. The resulting time series data sets now have durations of 773 months (inverted T-bill interest rate), 771 months (corporate earnings), 773 months (S&P 500 stock returns), 353 months (DJIA stock returns), and 520 months (NASDAQ composite stock returns) with starting dates of January 1951 for inverted T-bill interest rate, corporate earnings, and S&P 500 stock returns, January 1986 for DJIA stock returns, and February 1972 for NASDAQ composite stock returns.

Prior to the nonparametric spectral estimation, the following descriptive and inferential statistics were generated:
standard deviation, skewness, kurtosis, Hurst exponent, correlation, and Jarque-Bera test; and subsequent to the
forecasting of the stock returns, root mean square error (RMSE) and correlation were performed to determine the
goodness of the forecasted stock return values relative to the desired ones. The nonparametric spectral estimation
was performed in the Matlab environment using the pwelch function for the power spectral density and spectrogram
function for the time-frequency analysis. In each of these cases, the Chebyshev window was used. The forecasting of
the stock returns was performed in the NeuroSolutions neural network simulation environment. Nine focused gamma
neural network models were developed to forecast each type of stock returns (S&P 500, DJIA, and NASDAQ composite) in sets of three. Some models were similar and some training and testing sets were essentially the same. For each desired stock return output, the inputs were either inverted T-bill interest rate (or corporate earnings) and the past stock return values or inverted T-bill interest rate, corporate earnings, and the past stock return values. In
fact, models 1, 4, and 7 used inverted interest rate and the past stock return values as inputs; models 2, 5, and 8 used corporate earnings and the past stock return values as inputs; and models 3, 6, and 9 used inverted interest rate, corporate earnings, and the stock return values as inputs. The design parameters for each model are generally described in Table 1. The trajectory length is a factor of the training set. Hence, among the considerations in selecting the training set for a model is its relative extent in the number of samples and the number of trajectory lengths derivable from it since simply changing the trajectory length can affect a model’s output. To satisfy these considerations, the training set for each model was 59% of the total number of samples provided to the model and the testing set used for the out-of-sample forecasts was 41% (see Table 1). The total number of samples provided to each model was conditioned on the prediction mode horizon of 1-month ahead, the time duration variations in the data sets, and the nonzero lead months at maximum correlation minus one taken into account for the model design.

Table 1. Design parameters for focused gamma neural network models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Network Models</th>
<th>Inputs</th>
<th>Hidden Layer</th>
<th>Output Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topology</td>
<td>TLRN: Focused Gamma</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Prediction Mode Horizon</td>
<td>1 Step-ahead</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Taps</td>
<td>1; 2; 3; 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tap Delay</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of Samples</td>
<td>2; 4; 5; 7;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trajectory Length</td>
<td>151; 147; 29; 91; 62; 48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training Data Subsets</td>
<td>59%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Testing Data Subsets</td>
<td>41%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total No. of Samples</td>
<td>770; 736; 734; 511; 497; 332; 325</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Processing Elements</td>
<td>2; 4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Weights + Biases</td>
<td>2; 3</td>
<td>6; 26; 14; 20; 28</td>
<td>3; 5</td>
<td></td>
</tr>
<tr>
<td>Activation Function</td>
<td>Tanh</td>
<td>Tanh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight Update Mode</td>
<td>Batch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Learning Algorithm</td>
<td>Rprop</td>
<td>Rprop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Step Size Initial Value</td>
<td>1.00</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of Learning</td>
<td>Supervised</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights on Testing</td>
<td>Best Weights on Training</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error Termination</td>
<td>MSE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epochs/Run</td>
<td>11; 28; 30; 31; 32; 36; 42; 54</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: TLRN and Rprop represent time-lagged recurrent network and resilient backpropagation respectively. MSE termination threshold = 0.0001 minimum on training.

For example, in the design of models 1, 2, 3 for forecasting S&P 500 stock returns, the total number of samples provided to each of these models was 736 samples for model 1, 770 samples for model 2, and 734 samples for model 3 with the respective training sets being 435, 455, and 434 samples. Since the S&P 500 stock returns, interest, and corporate earnings data set lengths were 773, 773, 771 samples after the preprocessing, to obtain the actual number of samples provided to the models, 37 samples were discarded from 773 for model 1 design – one sample for the prediction mode horizon and 36 samples for the nonzero lead months for maximum correlation minus 1 (see Table 2); one sample was discarded for model 2 design – one sample for the prediction mode horizon; and 39 samples were discarded for model 3 design – one sample for the prediction mode horizon, 2 samples for time duration variations, 36 samples for the nonzero lead months for maximum correlation minus 1 (see Tables 1 and 2). Overall, the forecasting horizon was one month. The trajectory lengths for models 1, 2, and 3 were 29, 91, and 62 respectively (see Table 1). The models 4, 5, and 6 associated with forecasting DJIA stock returns and models 7, 8, and 9 associated with forecasting NASDAQ composite stock returns were designed in a manner similar to models 1, 2, and 3. Because the neural network models are of the focused gamma type with inherent input short-term memory, the learning paradigm used was backpropagation through time with the Resilient Backpropagation algorithm to facilitate faster learning. This learning paradigm trains the gamma neural network models using trajectory learning.
The focused gamma neural network model is a form focused time-lagged feedforward network (or time-lagged recurrent network) with the input short-term memory unit made from the infinite impulse response gamma filter\textsuperscript{22}. Moreover, the input short-term gamma memory has memory depth that is decoupled from the number of taps, and it is stable if its resolution is within the open interval (0, 2).

3. Results and Discussion

The descriptive and inferential statistics showed that the filtered economic time series inverted 3-month T-bill interest rate (interest rate henceforth) and corporate earnings and the financial time series S&P 500, DJIA, and NASDAQ composite stock returns are non-Gaussian and predictable at least in the short run. These are evident in Table 2.

Table 2. Attributes of filtered preprocessed data

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Data</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera Test (p-value)</th>
<th>Hurst Exponent</th>
<th>Max Correlation/Lead Months for Interest Rate (Corporate Earnings) Lead on Stock Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent Variables</td>
<td>Interest Rate*</td>
<td>0.3062</td>
<td>-0.1822</td>
<td>3.5904</td>
<td>0.0025</td>
<td>0.7628</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Corporate Earnings</td>
<td>0.1386</td>
<td>0.2283</td>
<td>3.0003</td>
<td>0.0355</td>
<td>0.6168</td>
<td></td>
</tr>
<tr>
<td>Dependent Variables</td>
<td>S&amp;P 500</td>
<td>0.1562</td>
<td>-0.3548</td>
<td>3.1705</td>
<td>0.0017</td>
<td>0.6869</td>
<td>0.134 (0.215)/(37 (0))</td>
</tr>
<tr>
<td></td>
<td>DJIA</td>
<td>0.1535</td>
<td>-0.5028</td>
<td>3.6894</td>
<td>0.0013</td>
<td>0.7565</td>
<td>0.255 (0.474)/(28 (21))</td>
</tr>
<tr>
<td></td>
<td>NASDAQ</td>
<td>0.2403</td>
<td>-0.1249</td>
<td>3.6285</td>
<td>0.0125</td>
<td>0.7161</td>
<td>0.054 (0.226)/(9 (23))</td>
</tr>
</tbody>
</table>

Notes: *36 month backward first difference. It is the additive inverse of 3-month Treasury bills (T-bills). The five datasets are of zero mean and are filtered with a third order nonlinear median filter. The stock returns and corporate earnings (profits after tax) are 12-month relative changes.

Table 2 from the skewness and kurtosis values and substantiated by the Jarque-Bera test for non-Gaussianity, and the Hurst exponent values exceed 0.5 for each of the time series data, suggesting predictability, which is especially important for the response variables in the stock returns. For example, Jarque-Bera test rejects at the 0.05 significance level the null hypothesis that any of these time series come from a normal distribution and each of the time series is skewed. Even though corporate earnings have a kurtosis value of 3.00, it is skewed right (0.2283) and has a probability value of 0.0355 on the Jarque-Bera test. Interest rate has the largest spread about the mean with a standard deviation value of 0.3062 followed by the NASDAQ composite with a standard deviation value of 0.2403. In terms of lead/lag relationships, interest rate leads stock returns by at least nine months (NASDAQ composite) and corporate earnings lead stock returns by at least zero months (S&P 500).

In the nonparametric spectral estimation using the Welch power spectral density estimate of the preprocessed predictor and response variables, the power spectral density estimate of the predictor variables interest rate and corporate earnings time series data of length 773 and 771 samples respectively are found to be both similar to and different from the power spectral density estimate of the response variables S&P 500, DJIA, and NASDAQ composite stock returns time series data of length of 773, 353, and 520 samples respectively (see Fig. 1). They are both low frequency with most of the estimated power bounded within the normalized frequency interval of (0, 0.0840]. While interest rate and corporate earnings peak at about the normalized frequency averaged to be 0.0132 from 0.0117 and 0.0147 respectively and the magnitude of interest rate is about 11 times that of corporate earnings, the peak of DJIA stock returns occurs at a distinctively different normalized frequency than those of S&P 500 and NASDAQ composite stock returns, which occur at essentially the same normalized frequency (see Fig. 1). The DJIA stock returns power spectral density peaked at the normalized frequency of 0.001. Moreover, the fluctuations in the three stock market returns lasted much longer than those evident in interest rate and corporate earnings. In all cases, the predictor and response variables’ power spectral densities are quite smooth and bandlimited.

The spectrograms of the predictor and response variables complement the power spectral density estimates of these variables by providing additional details on how the power is distributed in frequency as time evolves. Over time, the power levels fluctuate with frequency with the highest concentration of the power being in the lower frequencies, typically not exceeding 60nHz. The variation in the power intensity as time and frequency changes is clear evidence supporting the time varying property of the predictor (interest rate and corporate earnings) and response (S&P 500, DJIA, NASDAQ composite stock returns) variables. For example, interest rate has consistently
high power per frequency variation at 9.042 nHz starting at about the year 17.92 to the end of the time series with the power ranging from 64.16 dB/Hz to 65.52 dB/Hz, and reaching its highest point at the year 29.42 when it was 75.58 dB/Hz. In this same year 29.42, relatively higher amounts of power existed throughout the frequency band: at S&P 500 frequency 192.9 nHz it decreased to 20.05 dB/Hz. The spectrogram of the stock returns shows high power levels spread across different frequencies over time (see Fig. 2). Some of these high power levels occur at frequency 6.028 nHz in time year 2.63 and year 55.23 with values of 66.09 dB/Hz and 66.62 dB/Hz respectively. Other instances of high power levels include frequency 3.014 nHz in year 44.71 and frequency 15.07 nHz in years 7.89 and 34.19. So, while the power spectral density of the economic and financial time series established that most of their energies are concentrated in the lowest frequencies, the spectrogram provides the time frequency locations of those energies with
their relative strengths. The spectrogram also shows that the energy in an economic or financial time series is not necessarily uniform, but is punctuated by varying levels across different time frequency cells, sometimes extending throughout the frequency band over a limited time slot (see Fig. 2).

From the Hurst exponent, it is established that stock market returns produced from indices of S&P 500, DJIA, NASDAQ can be predicted. However, the evidence that stock market returns can be predicted does not necessarily mean that they can be arbitrarily predicted and under any conditions. Therefore, this study employs interest rate, corporate earnings, and past values of the stock returns as the predictor variables. For each set of three models for forecasting stock returns of S&P 500, DJIA, and NASDAQ composite indices, the design of the models follows the sequence where the predictor variables are applied with first interest rate and the past stock return values, followed by corporate earnings and the past stock return values, and then finally interest rate, corporate earnings, and the past stock return values. From Table 3, it is obvious that the forecasted values of stock returns are relatively close to the desired stock return values both within and across stock return types. The RMSE values are relatively small (less than 0.2) and the correlation values are relatively high (over 0.9). For example (see Table 3), the lowest RMSE values and the highest correlation value are associated with forecasts of S&P 500 stock return values. The very lowest RMSE value of 0.1030 and the highest correlation value of 0.9584 are produced by model 3 whose inputted predictor variables are the past values of S&P 500 stock return values, interest rate, and corporate earnings. The other RMSE and correlation values associated with the forecasted S&P 500 stock returns are such that for model 1, with inputted predictor variables of the past stock returns and interest rate, they are 0.1109 and 0.9481 respectively and for model 2, with inputted predictor variables of the past stock returns and corporate earnings, they are 0.1166 and 0.9517 respectively. In every instance, these RMSE and correlation values are better than those associated with the corresponding forecasts of DJIA stock returns, which are produced by models 4, 5, and 6. However, while the RMSE values associated with S&P 500 stock returns forecasts are lower than the corresponding ones associated with NASDAQ composite stock returns forecasts by 0.0295, 0.0187, and 0.0315 respectively, the correlation values associated with NASDAQ composite stock returns forecasts produced by models 7 and 8, which correspond to models 1 and 2, are greater by 0.0013. In spite of these facts and despite the slight differences in the forecasts, S&P 500 stock returns forecasted values more closely aligned with the desired values while the worst forecasts belonged to DJIA stock returns. For S&P 500 and NASDAQ composite stock returns, the best returns were yielded in the last models (3 and 9) in their sets when the predictor variables were interest rate, corporate earnings, and the past stock return values.

Table 3. Forecasting models and out-of-sample forecasts

<table>
<thead>
<tr>
<th>Response Variables</th>
<th>Models</th>
<th>Time Span</th>
<th>Samples</th>
<th>RMSE</th>
<th>Correlation</th>
<th>Time Span</th>
<th>Samples</th>
<th>RMSE</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>Model 1</td>
<td>Feb. 1954 – Apr. 1990</td>
<td>435</td>
<td>0.1000</td>
<td>0.9552</td>
<td>May 1990 – May 2015</td>
<td>301</td>
<td>0.1109</td>
<td>0.9481</td>
</tr>
<tr>
<td></td>
<td>Model 2</td>
<td>Feb. 1951 – Dec. 1988</td>
<td>455</td>
<td>0.1010</td>
<td>0.9533</td>
<td>Jan. 1989 – Mar. 2015</td>
<td>315</td>
<td>0.1166</td>
<td>0.9517</td>
</tr>
<tr>
<td></td>
<td>Model 3</td>
<td>Feb. 1954 – Mar. 1990</td>
<td>434</td>
<td>0.1005</td>
<td>0.9548</td>
<td>Apr. 1990 – Mar. 2015</td>
<td>300</td>
<td>0.1030</td>
<td>0.9584</td>
</tr>
<tr>
<td>DJIA</td>
<td>Model 4</td>
<td>May 1988 – Apr. 2004</td>
<td>192</td>
<td>0.1334</td>
<td>0.9529</td>
<td>May 2004 – May 2015</td>
<td>133</td>
<td>0.1634</td>
<td>0.9477</td>
</tr>
<tr>
<td></td>
<td>Model 6</td>
<td>May 1988 – Apr. 2004</td>
<td>192</td>
<td>0.1308</td>
<td>0.9528</td>
<td>May 2004 – May 2015</td>
<td>133</td>
<td>0.1889</td>
<td>0.9328</td>
</tr>
<tr>
<td>NASDAQ</td>
<td>Model 7</td>
<td>Nov. 1972 – Dec. 1997</td>
<td>302</td>
<td>0.1077</td>
<td>0.9379</td>
<td>Jan. 1998 – May 2015</td>
<td>209</td>
<td>0.1404</td>
<td>0.9494</td>
</tr>
</tbody>
</table>

Note: The predictor variables for each response variable of S&P 500, DJIA, and NASDAQ stock returns include the past stock return values and interest rate (or corporate earnings) or the past stock return values, interest rate, and corporate earnings.

The plots seem to confirm the forecasted results in Table 3 and only show the out-of-sample forecasts over the testing period. In general, the plots for DJIA desired and forecasted stock returns are not as closely aligned as the desired and forecasted stock returns for S&P 500 and NASDAQ, and the forecasted values of S&P 500 stock returns seem to align with the desired values more snugly than is observed with the fit between NASDAQ composite stock returns desired and forecasted values. Fig. 3 provides an example of the stock returns forecasts in showing the S&P
500 forecasts over the testing period associated with models 1, 2, and 3 (see Table 3). None of the forecasts captured the intensity of the great recession, but each of the forecasts captured the trajectory of the recession reasonably well.

The great recession period of December 2007 (month 228) through June 2009 (month 246) occurred from month 222 through month 242.

Since stock prices are a leading economic indicator and moreover the S&P 500 provides a good measurement of the general stock market behavior in the USA, one would expect the great recession to show-up in stock returns of such indices as S&P 500 before becoming evident in the general economy, which was indeed the case. In Fig. 3, the great recession period started in month 222 (June 2007) and ended in month 242 (February 2009) whereas the actual recession occurred from month 228 (December 2007) to month 246 (June 2009). Furthermore, the starting and ending time of the great recession are the same for S&P 500 and DJIA stock returns, but is different for NASDAQ stock returns. The NASDAQ stock returns showed that the great recession started in November 2007 and ended in November 2008. In addition to the great recession, S&P 500 and NASDAQ stock returns include other recessions: the 8-month recessions of 2001 (S&P 500 and NASDAQ) and 1990-1991 (S&P 500). In both cases, the start (month 141 or September 2000 and month 13 or January 1990) of the S&P 500 stock returns downturn preceded the official start of the recessions by six months, which is similar 6-month presage as evidenced with the great recession. The relative difference in the precise time of capturing the 2001 and the 2007-2009 recessions by S&P 500 and NASDAQ stock returns is consistent with the fact that the behavior of these two indices sometimes diverge. The fact that DJIA stock returns forecasts was not as good as those for S&P 500 and NASDAQ composite might be a consequence of the relative shortness of DJIA time series.

Overall, the models seem to perform generally well in forecasting the desired stock market returns. Given the inherent complexity of stock returns data in terms of nonlinearity, nonstationarity, and nonGaussianity as well as the irregular changes in the data over time it is unlikely that a regression model would have performed at a comparable level. As stated in Saboo et al., complex time series like stock prices (and returns) are “difficult for econometric modeling, non-stationary, very noisy and badly fitted by linear models.”

4. Conclusion

This study confirms that stock returns, interest rate, and corporate earnings were time varying nonGaussian time series that can be predicted over an appropriate time horizon with desirable predictor (or explanatory) variables. It was also observed that most of the energy in these types of financial and economic time series data is concentrated
in the very low frequencies. While most of the energy is indeed in the lowest frequencies and is smooth and essentially bandlimited, the actual distribution of the energy in a financial (e.g. S&P 500 stock returns) or economic (e.g., interest rate) time series is spread across the time frequency lattice with instances of high energy over different time frequency cells. Accounting for the complexity and some underlying properties of stock returns and their relationship to interest rate and corporate earnings, relatively and reasonably very good forecasts were produced for S&P 500, DJIA, and NASDAQ composite stock returns. These forecasts captured behavior of the great recession, but missed its intensity. Nonetheless, overall the forecasts relating to S&P 500 stock returns were slightly the best. One reason for this might be its much longer length. A future study of stock market returns will involve repeating these experiments using daily or higher frequency time series values.

References