Global Colloquium in Recent Advancement and Effectual Researches in Engineering, Science and Technology (RAEREST 2016)

Longitudinal Guidance of Unmanned Aerial Vehicle Using Integral Sliding Mode Control

Varsha Haridas a*, Vivek A a

*Department of Electrical and Electronics Engineering, Amrita Vishwa Vidyapeetham, Kollam-691001, Kerala, India

Abstract

This paper proposes a nonlinear scheme for guidance and longitudinal control of Unmanned Aerial Vehicle (UAV). The main objective of the guidance algorithm is to minimize the errors in altitude and flight path angle of the vehicle during flight. The guidance scheme must perform well in the case of small as well as large longitudinal errors, without saturating the pitch angle of the vehicle, which act as the control input. Integral Sliding Mode Control (ISMC) is used in longitudinal control of UAV. It is an improved sliding control method. Initially a linear sliding surface is employed for longitudinal guidance but it cannot provide satisfactory performance for both large and small errors in altitude and flight path angle and hence a nonlinear sliding surface is proposed. The simulations are carried out in MATLAB®/SIMULINK®. The simulation results show the effectiveness and robustness of the proposed control scheme.

1. Introduction

The term UAV is an abbreviation of Unmanned Aerial Vehicle, meaning aerial vehicles which operate without a human pilot. UAVs are commonly used in both the military and police forces in situations where the risk of sending a human piloted aircraft is unacceptable, or the situation makes using a manned aircraft impractical. More advanced UAVs used radio technology for guidance, allowing them to fly missions and return. They were constantly controlled by a human pilot, and were not capable of flying themselves. Modern UAVs are controlled with both autopilots, and human controllers in ground stations. This allows them to fly long, uneventfully flights under their own control, and fly under the command of a human pilot during complicated phases of the mission.

UAVs must have advanced path planning algorithms which is combined with effective and robust guidance. These
effective techniques ensure satisfactory operation in the case of system uncertainties and environmental disturbances. The important criterion of the guidance and control system is its ability of precise longitudinal control in the presence of disturbing forces. Longitudinal control of UAVs is mainly done by two approaches. In the first approach, the guidance and control design problems are separated into an outer guidance loop and an inner control loop while in the second approach, the guidance and control loops are unified to a single framework [1],[2]. The second approach is more complicated due to coupling of different guidance and control variables. Thus in most applications the first approach is employed.

There has been a lot of interest in guidance laws based on Model Predictive Control (MPC) and NMPC methods in recent years. The MPC-based techniques are quite challenging due to computational and implementation complexity. A high-level controller for small fixed-wing UAVs using NMPC, and minimization of the proposed cost function that results in minimization of the cross-track error is presented in [3]. A nonlinear guidance scheme for ground track control of aerial vehicles is discussed in [4]. The main objective of the guidance algorithm is to control the longitudinal errors such as error in altitude and error in flight path angle and try to keep it as small as possible. An autopilot controllers test platform for UAV using MATLAB®/SIMULINK® or X-plane is presented in [6]. The proposed scheme is implemented in the control computer. Mainly classical control theory such as root locus and frequency domain analysis methods are used in outer guidance loop design [5],[7],[8].

In this paper, an integral sliding mode control is used for the longitudinal guidance of UAV. The main aim is to minimize the errors in altitude and flight path angle. First, a sliding mode control with nonlinear sliding surface is proposed. But the disadvantage of SMC is high frequency switching (chattering) in the control signal. Chattering can be eliminated by using an integral sliding mode control. Simulation results show the robustness of the proposed guidance scheme.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>pitch angle</td>
</tr>
<tr>
<td>γ</td>
<td>flight path angle</td>
</tr>
<tr>
<td>α</td>
<td>angle of attack</td>
</tr>
<tr>
<td>V_g</td>
<td>ground velocity</td>
</tr>
<tr>
<td>q</td>
<td>pitch rate</td>
</tr>
<tr>
<td>m</td>
<td>mass of UAV</td>
</tr>
<tr>
<td>a_e</td>
<td>error in altitude</td>
</tr>
<tr>
<td>γ_e</td>
<td>error in flight path angle</td>
</tr>
<tr>
<td>F_L</td>
<td>lift force</td>
</tr>
</tbody>
</table>

2. Problem Formulation

The problem discussed in this paper is the guidance of an air vehicle from waypoint WP-1 to waypoint WP-2 (as shown in Fig.1.(a)) with minimum error in altitude and flight path angle in the presence of disturbances i.e. to keep a_e as small as possible and to make γ_e ≈ 0 as a_e ≈ 0. The UAV control system consists of outer guidance loop and inner control loop.
Fig. 1. (a) Guidance problem definition; (b) Longitudinal Guidance and Control System

Fig. 1(b) shows the structure of the overall longitudinal guidance and control system. The outer guidance block gets the current position and ground velocity vector inputs from sensor and the waypoint information from the mission plan. Based on these information, the guidance block will generate a reference pitch angle ($\theta_r$) for the inner control loop to follow. The inner control loop gets current pitch angle and pitch rate information from the sensor and generates commands which is used to actuate the elevators. The design of outer guidance logic will be done by using integral sliding mode approach.

2.1. Assumptions

For an outer guidance logic design, the following assumptions are used. They are:

(a) Pitch angle ($\theta$) is measurable.
(b) Inner loop dynamics are faster than the outer loop dynamics.
(c) Control law for the inner loop is available[10].

2.2. System Dynamics

Many flight mechanics books gives equations of motion for guidance and control of aerospace vehicles[9],[12]. The forces acting on aerospace vehicles during accelerating climb/decent is given in Fig. 2.

Adding up all the forces, then the lift force is given by

$$F_L = mg \cos \gamma + mV_g \dot{\gamma}$$

where $F_L$ is the lift force, $m$ is the mass, $g$ is the gravitational acceleration and $V_g$ is the velocity of the vehicle relative to ground. Equation (1) can be written in the form

$$\dot{\gamma} = \frac{g}{V_g} \left( \frac{F_L}{mg} - \cos \gamma \right)$$

where $\frac{F_L}{mg}$ is known as the load factor which is a dimensionless term.
From the assumption that an inner loop dynamics are faster than outer loop dynamics, then $F_{\text{req}} \approx F_L$ and thus equation (2) can be written as

$$\dot{\gamma} = \frac{g}{V_g} \left( \frac{F_{\text{req}}}{mg} - \cos \gamma \right)$$  \hspace{1cm} (3)

The ground velocity $V_g$ mainly has two components in the vertical plane

$$\dot{x} = V_g \cos \gamma \quad \dot{a} = V_g \sin \gamma$$  \hspace{1cm} (4)

From equations (3) and (4), the state equations for outer loop guidance problem can be written as

$$\dot{a} = V_g \sin \gamma$$  \hspace{1cm} (5)

$$\dot{\gamma} = \frac{g}{V_g} \left( \frac{F_{\text{req}}}{mg} - \cos \gamma \right)$$  \hspace{1cm} (6)

where the state variables are altitude and flight path angle. Let error in altitude be $a_e = a - a_{ref}$ and $\gamma_e = \gamma - \gamma_{ref}$ be the error in flight path angle. Now the state equations can be written as

$$\dot{a}_e = V_g \cos \gamma_{ref} \sin \gamma_e$$  \hspace{1cm} (7)

$$\dot{\gamma}_e = \frac{g}{V_g} \left( \frac{F_{\text{req}}}{mg} - \cos \gamma \right) - \gamma_{ref}$$  \hspace{1cm} (8)

3. Sliding Mode Control (SMC)

Sliding Mode Control (SMC) is a nonlinear control technique having remarkable properties such as accuracy, robustness, and easy tuning and implementation. SMC systems are designed to drive the system states onto a particular surface in the state space, called sliding surface. Once the sliding surface is reached, sliding mode control keeps the states on the neighborhood of the sliding surface. Thus the sliding mode control is a two part controller design in which the first part involves the design of a sliding surface so that the sliding motion satisfies design specifications and the second is concerned with the selection of a control law that will make the switching surface attractive to the system state[13]. In SMC, linear sliding surface cannot provide satisfactory performance and hence a nonlinear sliding surface is taken[4].
3.1. Guidance law based on a nonlinear sliding surface using SMC

The proposed nonlinear sliding surface is

\[ s = \gamma_e + B_1 \arctan (B_2 a_e) = 0 \]  \hspace{1cm} (9)

where the constants \( B_1 \) and \( B_2 \) are real positive numbers and \( B_1 \leq 1 \).

Motion on the sliding surface is obtained by putting \( s=0 \), thus implying \( \gamma_e = -B_1 \arctan (B_2 a_e) \).

Then the motion on sliding surface is given by:

\[ a_e = -V \sin [B_1 \arctan (B_2 a_e)] \]  \hspace{1cm} (10)

In the case of disturbances \( F_{\text{req}} \) is obtained from the equation \( \dot{s} = -k \text{sgn}(s) \).

\[ \dot{g} \left( \frac{F_{\text{req}}}{mg} - \cos \gamma \right) + \left[ (B_1B_2) / (1+B_2^2 a_e^2) \right] \bar{V} \cos \gamma_{\text{ref}} \sin \gamma_e = -k \text{sgn}(s) \]  \hspace{1cm} (11)

where \( \bar{g}, \bar{m}, \bar{V} \) are the estimated values of gravitational acceleration \( g \), mass \( m \) and ground velocity \( V \).

\[ F_{\text{req}} = \bar{m} \bar{V} \left[ \frac{\dot{g}}{mg} \cos \gamma + \left[ (B_1B_2) / (1+B_2^2 a_e^2) \right] \bar{V} \cos \gamma_{\text{ref}} \sin \gamma_e \right] \]  \hspace{1cm} (12)

The required or reference angle of attack \( \alpha_{\text{ref}} \) is given as

\[ F_{\text{req}} = \frac{1}{2} \rho V^2 S_{\text{ref}} (C_L_0 + C_L \alpha_{\text{ref}}) \]  \hspace{1cm} (13)

Feedback gain \( k \) can be taken such that \( \dot{W} = s \dot{s} < 0 \) in the domain of attraction.

\[ s \dot{g} \left( \frac{F_{\text{req}}}{mg} - \cos \gamma \right) + \left[ (B_1B_2) / (1+B_2^2 a_e^2) \right] \bar{V} \cos \gamma_{\text{ref}} \sin \gamma_e = 0 \]  \hspace{1cm} (14)

The value of \( k \) can be obtained by simplifying the above reachability condition.

4. Integral Sliding Mode Control (ISM C)

A new sliding mode design concept, namely Integral Sliding Mode Control (ISM C) is employed in guidance control. The order of the motion equation in ISMC is equal to the order of the original system, rather than reduced by the number of dimension of the control input. As a result, robustness of the system can be guaranteed throughout an entire response of the system starting from the initial time instance[11].

4.1. Guidance law based on linear sliding surface

Let us choose a linear sliding surface \( s = \gamma_e + \lambda a_e + \int a_e = 0 \) where \( \lambda \) is a positive scalar. Motion on the sliding surface is given by \( s=0 \) and thus gives \( \gamma_e = -\lambda a_e - \int a_e \). In the case of disturbances \( F_{\text{req}} \) is derived from \( \dot{s} = -k \text{sgn}(s) \).

\[ \dot{g} \left( \frac{F_{\text{req}}}{mg} - \cos \gamma \right) + \lambda \bar{V} \cos \gamma_{\text{ref}} \sin \gamma_e = -k \text{sgn}(s) \]  \hspace{1cm} (15)

where \( \bar{g}, \bar{m}, \bar{V} \) are the estimated values of gravitational acceleration \( g \), mass \( m \) and ground velocity \( V \).
Using the relation $\theta = \gamma + \alpha$, the reference pitch angle $\theta_r$ command can be generated for the inner loop. Simulations are done for an initial error of 600 m. The error in altitude versus time for different values of $\lambda$ is as shown in Fig. 3(a). The error decreases to zero in 155 s (approximately) in the case of $\lambda=0.004$ and 180 s (approximately) in the case of $\lambda=0.0025$. Thus it is seen that larger $\lambda$ gives better performance i.e. fast convergence to zero.

The maximum value of pitch angle is chosen as 35 degrees. So the reference pitch angle should be less than or equal to the maximum value of pitch angle i.e. $|\theta_r| \leq |\theta_{max}|$. But for higher values of $\lambda$ i.e. at better performance the pitch angle exceeds the maximum limit of 35 degrees (as shown in Fig. 3(b)). Thus linear sliding surface cannot provide better performance in the case of errors and hence a nonlinear sliding surface is designed.

![Fig.3.(a) Simulation results for error in altitude versus time for different values of $\lambda$.](image-a)

![Fig.3.(b) Simulation results for reference pitch angle with $\lambda=0.004$.](image-b)

### 4.2. Guidance law based on a nonlinear sliding surface using ISMC

The proposed nonlinear sliding surface is given by

$$s = \gamma_e + B_1 \arctan(B_2a_e) - \int a_e$$

where the constants $B_1$ and $B_2$ are real positive numbers and $B_1 \leq 1$.

Motion on the sliding surface is given by putting $s=0$ and hence $\gamma_e = -B_1 \arctan(B_2a_e) - \int a_e$.

In the case of any disturbances $F_{req}$ is derived from $\dot{s} = -k \text{sgn}(s)$.

$$\frac{\dot{\gamma}}{V} (F_{req} - \cos \gamma) + \left( \frac{B_1B_2}{(1+B_2^2a_e^2)} \right) \cos \gamma_r \sin \gamma_r a_e - k \text{sgn}(s) = 0$$

Feedback gain $k$ can be taken such that $\dot{W} = s \dot{s} < 0$ in the domain of attraction.

$$s \left( \frac{\dot{\gamma}}{V} (F_{req} - \cos \gamma) + \left( \frac{B_1B_2}{(1+B_2^2a_e^2)} \right) \cos \gamma_r \sin \gamma_r a_e - k \text{sgn}(s) \right) = 0$$
The value of \( k \) can be obtained by simplifying the reachability condition (21). The simulation result for error in altitude versus time using SMC and ISMC is given in Fig.4.(a).

\[ \text{Error in altitude (m)} \]

\[ \text{Time(s)} \]

\[ \text{SMC} \]

\[ \text{ISMC} \]

Fig.4. (a) Simulation plot of error in altitude versus time using ISMC and SMC; (b) Simulation plot of error in flight path angle versus time using SMC and ISMC

It can be seen that the error in altitude converges to zero at a faster rate by using ISMC compared to SMC. Simulation result for error in flight path angle using SMC and ISMC is shown in Fig.4.(b). It can be seen that the error in flight path angle converges to zero at \( t=90 \text{s} \) (approximately) by using SMC while the flight path angle error converges to zero at a faster rate while using ISMC.

Simulation results of reference pitch angle using SMC and ISMC are shown in Fig.5. The disadvantage of SMC is high frequency switching called chattering which can be eliminated by using ISMC. While using SMC there is high irregularities in the control signal (as shown in Fig.5.(a)). But these irregularities are absent while using an ISMC (as given in Fig.5.(b)).

Thus by using ISMC, the error in altitude and error in flight path angle converges to zero at a faster rate compared to SMC and also eliminate chattering.

\[ \text{Reference pitch angle (degrees)} \]

\[ \text{Time(s)} \]

\[ \text{SMC} \]

\[ \text{ISMC} \]

Fig.5.(a) Simulation result of reference pitch angle versus time using SMC; (b) Simulation result of reference pitch angle versus time using ISMC
5. Conclusion

An integral sliding mode control with nonlinear sliding surface is proposed for longitudinal guidance control of aerial vehicles which incorporates parameters which can be chosen to satisfy given performance conditions. A nonlinear guidance law is then derived. The stability of the proposed sliding surface is proved by using an appropriate Lyapunov function. Control saturation is avoided. Also desired performance is achieved without saturating the pitch angle command to the control system of the vehicle.

Simulations were performed in the SIMULINK® and simulation results obtained using ISMC were compared with the simulation results obtained by using SMC. The maximum control efforts were then successfully obtained. The results demonstrated the effectiveness of the proposed control schemes.

References