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Letter

Structural optimum design of bistable cylindrical shell for broadband energy harvesting application

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ABSTRACT

The shallow cylindrical structure is suitable to develop broadband vibration energy harvesters due to the property of the inherent mechanical bistability. In this letter, the optimum design of the bistable cylindrical shell for broadband energy harvesting application is investigated from the structural point of view. The output power is evaluated by the concept of the harvestable power, which balances the frequency of snap through and the referred output energy associated with each snap through. The non-dimensional harvestable power is analytically expressed as the function of the non-dimensional curvature parameter and one constructed parameter. The universal dependence of the optimal curvature parameter and the associated optimal harvestable power on the constructed parameter is derived, which can be well approximated by the linear relation in double logarithmic coordinate.

Vibration energy harvesting technique has received extensive attention due to its important significance in different fields of technology such as wireless sensors, data transmitters and medical implants [1–3]. Conventional linear harvesters utilize the principle of linear resonance and thus operate well only when the external excitation frequency matches to the fundamental frequency of the device. The time-varying, multi-frequency and random characteristics of the ambient vibration, however, render the typical linear harvesters unsuitable for most practical applications [4,5]. To address this issue, energy harvesting technique exploiting stiffness nonlinearity has been proposed for broadband transduction [6,7]. Compared to the nonlinear monostable harvesters, the bistable harvesters exhibit broader effective frequency bandwidth and larger output power relying on the fantastic dynamic phenomena [7,8]. It is even more important that the bistable harvesters exhibit the highest robustness to the changing excitation environment and the uncertainty of design parameters compared to the linear and nonlinear monostable harvesters [8,9]. The advantages mentioned above verify the applicability of the bistable harvesters on the broadband energy harvesting.

According to the aufbau principle of bistable potential shape, the bistable harvesters can be classified as three categories, i.e., the magnetic attraction, magnetic repulsion and mechanical bistability [8]. The bistable harvesters with magnetic components require the obtrusive arrangement of magnets and inevitably generate unwanted electromagnetic field, which dramatically limits the miniaturization and degrades the performance. The typical bistable harvesters with mechanical bistability are achieved through buckled mechanism, such as the clamped or hinged beam buckled by an axial force beyond the critical buckling force [10–12] and the inverted clamped beam buckled by the gravity of an elaborately selected tip mass [13]. Recently, a novel bistable harvester utilizing composite laminates with an asymmetric lay-up has been suggested [14–16]. The inherent mechanical bistability means asymmetric composite laminates occupy smaller space and induces that this type bistable harvester is potentially more suitable for miniaturization than the bistable harvesters with magnet-induced bistability. Furthermore, the structures with inherent mechanical bistability can be easily fabricated through strain mismatch, which is a mature technique in the micro electronics industry [17].

The broadband response of the bistable harvester comes down to the solution of a set of essentially electromechanical coupling equations. The broadband response and parameter optimization have been investigated through some established techniques, such as Monte Carlo simulation, moment method, Galerkin procedure, finite element method and equivalent linearization technique [6,8,18–21]. All above mentioned are numerical or semi-analytical techniques, and so far not any analytical technique has been established unless confining the large ratio between the period of the mechanical subsystem and the time constant of the harvesting circuit [7]. Besides, most works contribute to the optimum design

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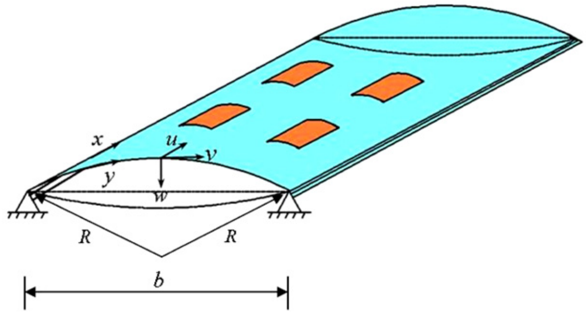

Fig. 1. Shallow cylindrical structure integrated with piezoelectric patches.

of the mathematical system, not the practical physical system. As an exclusive work toward the optimum design of the bistable harvester with inherent mechanical bistability, the authors discover the optimal configuration based on the statics of the device, not the practical broadband response [16].

The isotropic cylindrical structure, as a common structure with inherent mechanical bistability, is more easily fabricated through microelectronic process than the asymmetric composite laminates. Similar to the bistable harvester with asymmetric composite laminates, the isotropic cylindrical shell integrated with a group of piezoelectric patches constitutes a simple and reliable bistable harvester, as shown in Fig. 1. The host structure vibrates under the stimulation of the external excitation, and the piezoelectric patches deform and generate electric output through piezoelectric mechanism. This letter concentrates on the optimum design of the cylindrical shell-type bistable harvester, and tries to analytically establish the universal design curves. Due to the randomness of broadband excitation and the complexity of the electromechanical coupling, it is almost impossible to analytically optimize the actual output power. Based on the above consideration, we neglect the influence of the piezoelectric components and the harvesting circuit on structural responses and establish the optimum design only from the structural point of view.

Consider a shallow cylindrical shell of thickness h, curvature radius R, and span b with two opposite edges hinged support. The uniformly distributed pressure $\xi(t)$ acting radially inwards is broadband excitation and approximately described by Gaussian white noise with the intensity $2D$. With the assumption that the shell is sufficiently flat, the transverse deflection is represented by the fundamental mode, i.e., $w(y,t) = hq(t)\sin(\pi y/b)$, in which $q(t)$ denotes the non-dimensional amplitude of the transverse deflection [22]. The in-plane displacement $\upsilon(y,t)$ can be expressed through the amplitude $q(t)$ by integrating the in-plane equilibrium equation and applying the boundary conditions $\upsilon(0,t) = \upsilon(b,t) = 0$, i.e., $\upsilon = hq\{-\pi hq\sin(2\pi y/b)/(8b) + b[1-\cos(\pi y/b)]/(\pi R) - 2y/(\pi R)\}$, and then the strain energy per unit length is calculated by $U = E'h^5\left[\left(2k^2/\pi^2 + \pi^4/48\right)q^2 - \pi k q^3/2 + \pi^4 q^4/32\right]/b^3$, in which $E' = E/\left(1-\upsilon^2\right)$, E and υ denote the plane-strain modulus, Young's modulus and Poisson's ratio, respectively. $k = b^2/(Rh)$ is a non-dimensional curvature parameter which can measure the value of curvature radius. The kinetic energy per unit length is $T = bmh^2\dot{q}^2/4$ and the dissipation function is $D_f = bh^2\varepsilon\dot{q}^2/2$, in which, m denotes the mass per unit mid-surface area and ε represents the coefficient of viscous damping. The generalized force associated with the time-dependent random excitation is expressed as, $Q = 2bh\xi(t)/\pi$. Then, the nonlinear stochastic differential equation which describes the random responses of the shallow cylindrical structure is derived through the Lagrange procedure [23,24]

$$\ddot{q} + \frac{2\varepsilon}{m}\dot{q} + \frac{2E'h^3}{b^4 m}\left[\left(\frac{4k^2}{\pi^2} + \frac{\pi^4}{24}\right)q - \frac{3\pi}{2}kq^2 + \frac{\pi^4}{8}q^3\right] = \frac{4}{\pi m h}\xi(t). \quad (1)$$

The equilibrium configuration of the shallow cylindrical structure free from external excitation should be first investigated. Removing the acceleration, velocity and the external excitation terms from Eq. (1) yields the following algebraic equation,

$$\left(\frac{4k^2}{\pi^2} + \frac{\pi^4}{24}\right)q - \frac{3\pi}{2}kq^2 + \frac{\pi^4}{8}q^3 = 0. \quad (2)$$

The solutions of the above equation $q_1 = 0$, $q_{2,3} = \left(6k \pm \sqrt{4k^2 - \pi^6/3}\right)/\pi^3$ represent the possible equilibrium positions of the shallow cylindrical structure. In the case of $k < \pi^3\sqrt{3}/6$, $q_{2,3}$ are the imaginary roots and those imply that the cylindrical shell have only one equilibrium position $q_1 = 0$. In the case of $k > \pi^3\sqrt{3}/6$, however, $q_{2,3}$ are the positive real roots. The cylindrical structure possesses three equilibrium positions, in which q_1, q_3 are stable equilibrium positions while q_2 is the unstable equilibrium position. The critical value $k = \pi^3\sqrt{3}/6$ corresponds to the transition from mono-stable case to bistable case. The strain energy per unit length associated with the mono-stable, bi-stable and the switch status are shown in Fig. 2. It is worth pointing out that the potential energy function is asymmetrical, and the bistable harvesters with asymmetrical potential are relatively complex compared to those with symmetrical potential [18–21,25]. The investigation in this letter is concentrated on the case of $k > \pi^3\sqrt{3}/6$, i.e., the bi-stable configuration of the shallow cylindrical structure.

Some representative response samples of the shallow cylindrical shell are calculated through the Monte Carlo simulation and shown in Fig. 3. It is obvious that the random responses of the cylindrical shell represent the bistable property. The random response consists of the intra-well micro-vibration around one of the stable equilibrium positions and the snap through behavior crossing the unstable equilibrium position [7,8]. Compared to the intra-well micro-vibration, each snap through corresponds to a larger strain variation, which means the much higher output energy of piezoelectric patches attached. To assess the output power of the cylindrical shell-type bistable harvester, it is reasonable to ignore the contribution of the intra-well micro-vibration and only calculate the output power associated with the snap-through process. Consequently, the frequency of snap through and the output energy associated with each snap through are two crucial parameters. The large values of these two parameters correspond to the high efficiency of the bistable harvester. The non-dimensional curvature parameter k is inversely proportional to the curvature radius R. With the increase of curvature parameter k, the potential barrier ascends and the frequency of snap through decreases, vice versa, as shown in Fig. 3. On the contrary, the structure strain and then the output energy associated with each snap through increase with the curvature parameter k. Thus, *the frequency of snap through competes with the output energy associated with each snap through with the variation of curvature parameter k, and the optimal configuration can be derived by balancing these two crucial parameters.*

Essentially, the frequency of snap through, i.e., the rate of crossing, is a random process. From the view of statistics, the stationary rate of expectation crossing, i.e., the expectation of the rate of crossing for the stationary stage is appropriate to assess the frequency of the snap through [26,27]. The stationary joint probability density function of mechanical states can be analytically expressed as

$$p_s(q,\dot{q}) = C\cdot\exp\left\{-\frac{\pi^2\varepsilon m h^2}{16D}\left\{\dot{q}^2 + \frac{4E'h^3}{b^4 m}\left[\left(\frac{2k^2}{\pi^2} + \frac{\pi^4}{48}\right)q^2 - \frac{\pi}{2}kq^3 + \frac{\pi^4}{32}q^4\right]\right\}\right\}, \quad (3)$$

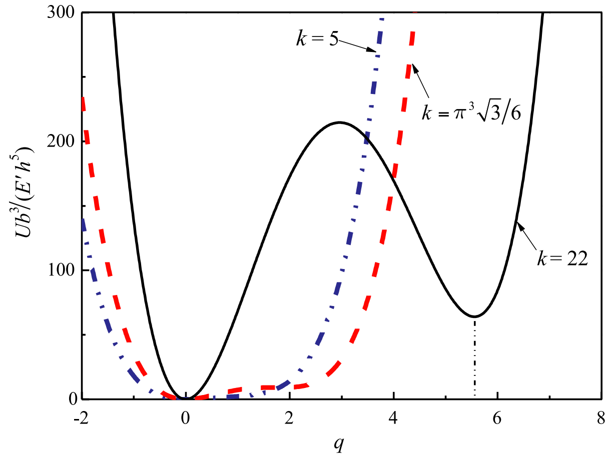

Fig. 2. Asymmetrical potential shapes for representative values of curvature parameter k.

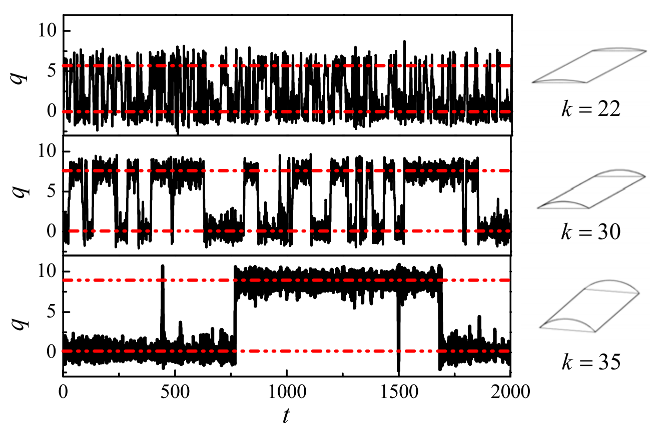

Fig. 3. Response samples of the shallow cylindrical structure excited by the Gaussian white noise. System parameters are set as $\varepsilon/m = 0.15$, $D = 0.05$, $mh = 1/3$, $2E'h^3/(b^4m) = 0.01$, and $k = 22, 30, 35$. Dashed lines represent the stable equilibrium positions.

in which C is a positive constant and can be determined by the normalization condition $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_s(q, \dot{q})\, dq d\dot{q} = 1$. Suppose that once the shallow cylindrical shell reaches the plane defined by two straight edges the snap through behavior occurs. In other words, the snap through threshold is set as $q_0 = \left(R - \sqrt{R^2 - b^2/4}\right)/h \approx k/8$. Through the formula $\nu_a = \int_{-\infty}^{\infty} |\dot{q}|\, p_s(q_0, \dot{q})\, d\dot{q}$, the stationary ratio of expectation crossing are derived as [26,27]

$$\nu_a = \sqrt{\frac{16D}{\pi^3\varepsilon mh^2}} \cdot \frac{\exp\left\{-A\left[\left(2k^2/\pi^2 + \pi^4/48\right)k^2/64 - \pi k^4/1024 + (\pi k/8)^4/32\right]\right\}}{\int_{-\infty}^{\infty}\exp\left\{-A\left[\left(2k^2/\pi^2 + \pi^4/48\right)q^2 - (\pi/2)kq^3 + (\pi^4/32)q^4\right]\right\}dq}, \quad (4)$$

in which $A = \varepsilon\pi^2E'h^5/\left(4Db^4\right)$ is a non-dimensional constructed parameter. For a given constructed parameter value $A = 10^{-4}$, the dependence of the stationary ratio of expectation crossing on the curvature parameter k is shown in Fig. 4. The stationary ratio of expectation crossing decreases with the increase of the curvature parameter. Particularly, as the curvature parameter is too large, the stationary ratio of expectation crossing approaches zero, which means that snap through behavior almost does not happen. The consistency of the analytical results and the results from Monte Carlo simulation verifies the precision of the analytical expression in Eq. (4).

The output energy associated with each snap through can be evaluated by a referred deformation energy. The referred deformation energy is defined by the strain energy as the shallow cylindrical shell located in the plane specified by two straight edges, i.e.,

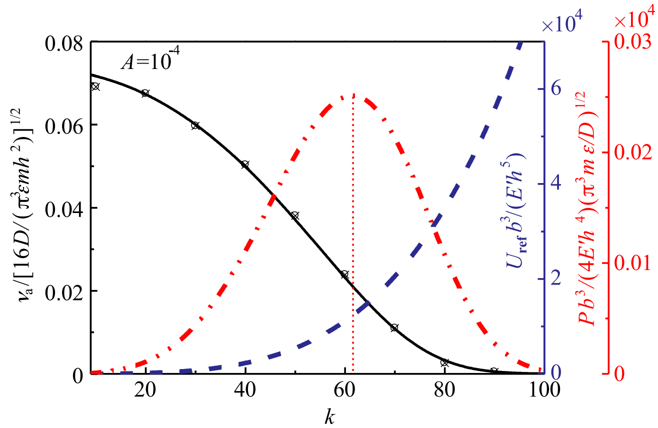

Fig. 4. Dependence of the stationary ratio of expectation crossing ν_a, referred deformation energy U_{ref} and harvestable power P on the curvature parameter k. Disperse markers denote the results from Monte Carlo simulation.

$$U_{ref} = U\big|_{q=k/8} = \frac{E'h^5}{b^3}\left[\left(\frac{2k^2}{\pi^2} + \frac{\pi^4}{48}\right)\left(\frac{k}{8}\right)^2 - \frac{\pi}{2}k\left(\frac{k}{8}\right)^3 + \frac{\pi^4}{32}\left(\frac{k}{8}\right)^4\right]. \quad (5)$$

The referred deformation energy increases with the curvature parameter k, as shown in Fig. 4. To balance the stationary ratio of expectation crossing and the referred deformation energy, the concept of the harvestable power is introduced as

$$P = U_{ref} \cdot \nu_a. \quad (6)$$

The curvature parameter k is then optimized by maximizing the harvestable power. By substituting Eqs. (4) and (5) into Eq. (6), the harvestable power is explicitly expressed as

$$P\Bigg/\left(\frac{4E'h^4}{b^3}\sqrt{\frac{D}{\pi^3m\varepsilon}}\right) = \frac{1}{I}\left[\left(\frac{2k^2}{\pi^2} + \frac{\pi^4}{48}\right)\cdot\frac{k^2}{64} - \frac{\pi k^4}{1024} + \frac{1}{32}\left(\frac{\pi k}{8}\right)^4\right] \cdot \exp\left\{-A\cdot\left[\left(\frac{2k^2}{\pi^2} + \frac{\pi^4}{48}\right)\cdot\frac{k^2}{64} - \frac{\pi k^4}{1024} + \frac{1}{32}\left(\frac{\pi k}{8}\right)^4\right]\right\} \quad (7)$$

with $I = \int_{-\infty}^{\infty}\exp\{-A[(2k^2/\pi^2 + \pi^4/48)q^2 - \pi/2kq^3 + \pi^4/32q^4]\}dq$.

Careful observation shows that the non-dimensional harvestable power $P/\left[\left(4E'h^4/b^3\right)\sqrt{D/\left(\pi^3m\varepsilon\right)}\right]$ on the left hand of Eq. (7) depends on the non-dimensional curvature parameter $k = b^2/(Rh)$ and one non-dimensional constructed parameter $A = \varepsilon\pi^2E'h^5/\left(4Db^4\right)$. For a given value of constructed parameter $A = \varepsilon\pi^2E'h^5/\left(4Db^4\right)$, the optimal curvature parameter $k^* = b^2/(R^*h)$ and the associated optimal harvestable power $P^*/\left[\left(4E'h^4/b^3\right)\sqrt{D/\left(\pi^3m\varepsilon\right)}\right]$ can be easily calculated through Eq. (7). The relations of the optimal curvature parameter and the associated optimal harvestable power to the constructed parameter are shown in Fig. 5. Due to the dependence of the constructed parameter on the geometric configuration, material property and noise intensity, the logarithm coordinates are adopted to describe the large variation range of parameter values. Fig. 5 depicts the relations between the non-dimensional quantities and exhibits the

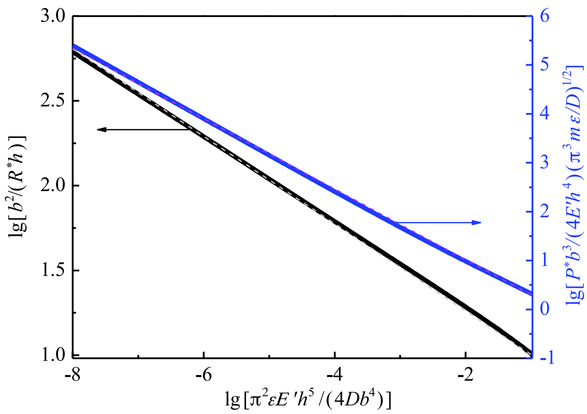

Fig. 5. Dependence of the optimal curvature parameter $b^2/(R^*h)$ and the associated optimal harvestable power $P^*/\left[\left(4E'h^4/b^3\right)\sqrt{D/\left(\pi^3 m\varepsilon\right)}\right]$ on the constructed parameter $\varepsilon\pi^2E'h^5/\left(4Db^4\right)$. Dashed lines denote the results of linear fitting.

universal design curves which are invariant with any change of the geometric, material and excitation parameters.

In the double logarithmic coordinate, the optimal curvature parameter $k^* = b^2/(R^*h)$ and the associated optimal harvestable power $P^*/\left[\left(4E'h^4/b^3\right)\sqrt{D/\left(\pi^3 m\varepsilon\right)}\right]$ almost linearly decrease with the increase of the constructed parameter $A = \varepsilon\pi^2E'h^5/\left(4Db^4\right)$. The linear fittings on the function relations between the optimal curvature parameter, the optimal harvestable power and the constructed parameter give the semi-analytical formulas

$$\lg\left[b^2/\left(R^*h\right)\right] \doteq -0.26\lg\left[\varepsilon\pi^2E'h^5/\left(4Db^4\right)\right] + 0.75, \tag{8a}$$

$$\lg\left\{P^*\Big/\left[\left(4E'h^4\Big/b^3\right)\sqrt{D/\left(\pi^3 m\varepsilon\right)}\right]\right\} = -0.72\cdot\lg\left[\varepsilon\pi^2E'h^5/\left(4Db^4\right)\right] - 0.42, \tag{8b}$$

in which the values of slope and intercept are universal constants although they are derived by linear fitting. Once the geometric parameters h and b, material properties E' and ε, and noise intensity $2D$ are assigned, the optimal curvature radius R^* can be directly derived through Eq. (8a). The optimal harvestable power P^* associated with the optimal curvature radius R^*, which is determined by Eq. (8b), can be used to evaluate the upper bound of the mean output power by piezoelectric patches attached.

In summary, this letter investigated the optimum design of the cylindrical shell-type bistable harvester with the goal of maximizing the performance. By introducing the concept of harvestable power to balance the frequency of snap through and the referred output energy associated with each snap through, the universal dependence of the optimal curvature parameter and the associated optimal harvestable power on one constructed parameter is analytically established. The universal relations can be directly used to design the curvature radius of the cylindrical shell under arbitrarily given geometric, material and excitation parameters. It is worth pointing out that the optimum design was established based on the structural point of view and neglecting the influence of piezoelectric patches and the harvesting circuit. As a result, the optimal design parameter provided by the universal relations is only a sub-optimal result.

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