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ORIGINAL ARTICLE

# A novel solution technique for two dimensional Burger's equation

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**Abstract** In this paper, the Laplace decomposition method (LDM) is proposed to solve the two-dimensional nonlinear Burgers' differential equations. Two test problems are considered to illustrate the accuracy of the proposed algorithm. It is shown that the numerical results are in good agreement with the exact solutions for each problem.



## 1. Introduction

The Burger's differential equation displays a sample model for describing the communication between reaction apparatuses, acoustic waves, convection effects, diffusion transports, heat conduction and modeling of dynamics. Burger's prototype of turbulence is a very significant fluid dynamics archetype. The study of this model plays a vital role in understanding different numerical and analytical techniques. Also, this model provides a basis for theory of shock waves. Several authors have investigated Burger's model for various physical flow problem in fluid dynamics. The mathematical structure of Burger's equation was introduced by Johannes Martinus Burgers [1–5]. The general form of two-dimensional Burger's equations is given as follows:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

with subjected initial and boundary conditions are

$$u(x,y,0)=f_1(x,y),\quad v(x,y,0)=f_2(x,y),\quad x,y\in\Omega, \tag{3}$$

$$u(x,y,t)=f_3(x,y,t),\quad v(x,y,t)=f_4(x,y,t),\quad x,y\in\partial\Omega,\quad t>0, \tag{4}$$

where $u(x,y,t)$ and $v(x,y,t)$ are velocity components in the direction of $x$ and $y$-axes, Re is a Reynold number, $f_i$ $i=1,2\ldots4.$ are given function on specified points, $\Omega=\{(x,y)\mid a\leqslant x\leqslant b, a\leqslant y\leqslant b\}$ is a domian and $\partial\Omega$ is a boundary. Numerical schemes for the solutions of Burger's equation normally categorize into the following classes: finite difference, finite element and spectral methods [6–8]. In [9], two dimensional Burger's equation is solved with fully implicit finite-difference form. The analytical approximate solution schemes for nonlinear differential equations are of great significance in physical problems. Several analytical techniques such as Adomian decomposition method, homotopy analysis method, homotopy perturbation method and variational iterative

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method are devised for the solution of Burger's differential equation [10–16].

From last few years significant devotion has been given to Laplace decomposition method (LDM) and its modifications for resolving physical model equations [17–30]. To the best of our information, LDM is not so far applied to solve Burgers' equation. Motivated by the work described in [31–40], we have used the LDM scheme to solve the two-dimensional Burgers' equations.

The structure of the article is as follows. In section 2, LDM is given. In Section 3, numerical applications of LDM are illustrated. The final remarks are given in the last section.

## 2. General mechannism of Laplace decomposition method

Consider equation $F(u(x,t)) = g(x,t)$, where $F$ represents a general nonlinear ordinary or partial differential operator including both linear and nonlinear terms. The linear terms are decomposes into $L + R$, where $L$ is the highest order linear operator and $R$ is the remaining of the linear operator. Thus, the equation can be written as

$$Lu + Ru + Nu = g(x,t), \tag{5}$$

where $Nu$, indicates the nonlinear terms. By applying Laplace transform on both sides of Eq. (5), we get

$$\mathcal{L}[Lu + Ru + Nu] = \mathcal{L}[g(x,t)]. \tag{6}$$

Using the differential property of Laplace transform, we have

$$s^n\mathcal{L}[u] - \sum_{k=1}^{n} s^{k-1}u^{(n-k)}(x,0) + \mathcal{L}[Ru] + \mathcal{L}[Nu] = \mathcal{L}[g(x,t)]. \tag{7}$$

Operating inverse Laplace transform on both sides of Eq. (7), we get

$$u = G(x,t) - \mathcal{L}^{-1}\left[\frac{1}{s^n}[\mathcal{L}[Nu] + \mathcal{L}[Ru]]\right]. \tag{8}$$

The Laplace decomposition method assumes the solution $u$ can be expanded into infinite series as

$$u = \sum_{m=0}^{\infty} u_m. \tag{9}$$

Also the nonlinear term $Nu$ can be written as

$$Nu = \sum_{m=0}^{\infty} A_m, \tag{10}$$

where $A_m$ are the Adomian polynomials [17]. By substituting Eqs. (9) and (10) in Eq. (8), the solution can be written as

$$\sum_{m=0}^{\infty} u_m(x,t) = G(x,t) - \mathcal{L}^{-1}\left[\frac{1}{s^n}\left[\mathcal{L}\left[\sum_{m=0}^{\infty} A_m\right] + \mathcal{L}\left[R\left(\sum_{m=0}^{\infty} u_m\right)\right]\right]\right]. \tag{11}$$

In Eq. (11), the Adomian polynomials can be generated by several means. Here we used the following recursive formulation:

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad m = 0,1,2,\ldots. \tag{12}$$

In general, the recursive relation is given by

$$u_0(x,t) = G(x,t), \tag{13}$$

$$u_{m+1}(x,t) = -\mathcal{L}^{-1}\left[\frac{1}{s^n}[\mathcal{L}[A_m] + \mathcal{L}[R(u_m)]]\right], m \geqslant 0, \tag{14}$$

where $G(x,t)$ represents the term arising from source term and prescribe initial conditions. The proposed method does not resort to linearization, assumptions of weak nonlinearity and it is more realistic compared to the method of simplifying the physical problems.

## 3. Numerical Implementation of Laplace decomposition method

**Example 1.** Consider two dimensional nonlinear Burger's differential equation [16]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{15}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{16}$$

with initial conditions

$$u(x,y,0) = x + y, \qquad v(x,y,0) = x - y. \tag{17}$$

Applying Laplace transform algorithm, we get

$$su(x,y,s) - u(x,y,0) + \mathcal{L}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 u)\right], \tag{18}$$

$$sv(x,y,s) - v(x,y,0) + \mathcal{L}\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = \mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 v)\right], \tag{19}$$

$$u(x,y,s) = \frac{u(x,y,0)}{s} - \frac{1}{s}\mathcal{L}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 u)\right], \tag{20}$$

$$v(x,y,s) = \frac{v(x,y,0)}{s} - \frac{1}{s}\mathcal{L}\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 v)\right]. \tag{21}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Using given initial conditions Eqs. (20) and (21), becomes

$$u(x,y,s) = \frac{x+y}{s} - \frac{1}{s}\mathcal{L}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 u)\right], \tag{22}$$

$$v(x,y,s) = \frac{x-y}{s} - \frac{1}{s}\mathcal{L}\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 v)\right]. \tag{23}$$

Applying inverse Laplace transform to Eqs. (22) and (23), we have

$$u(x,y,t) = x + y - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 u)\right]\right], \tag{24}$$

$$v(x,y,t) = x - y - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}(\nabla^2 v)\right]\right]. \tag{25}$$

The Laplace decomposition method (LDM) [1–3] assumes that the series solution of $u(x,y,t)$ and $v(x,y,t)$ are given by

$$u = \sum_{n=0}^{\infty} u_n(x, y, t), \tag{26}$$

$$v = \sum_{n=0}^{\infty} v_n(x, y, t). \tag{27}$$

Utilizing Eqs. (26) and (27), into Eqs. (24) and (25) yields:

$$\sum_{n=0}^{\infty} u_n(x, y, t) = x + y - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n(u) + \sum_{n=0}^{\infty} B_n(u, v)\right]\right] + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}\left(\nabla^2\left(\sum_{n=0}^{\infty} u_n\right)\right)\right]\right], \tag{28}$$

$$\sum_{n=0}^{\infty} v_n(x, y, t) = x - y - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} C_n(u, v) + \sum_{n=0}^{\infty} D_n(v)\right]\right] + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}\left(\nabla^2\left(\sum_{n=0}^{\infty} v_n\right)\right)\right]\right]. \tag{29}$$

In above Eqs. (28) and (29), $A_n(u), B_n(u, v)$, $C_n(u, v)$ and $D_n(v)$ are Adomian polynomials that represent nonlinear tearms given as follow:

$$\sum_{n=0}^{\infty} A_n(u) = uu_x, \tag{30}$$

$$\sum_{n=0}^{\infty} B_n(u, v) = vu_y, \tag{31}$$

$$\sum_{n=0}^{\infty} C_n(u, v) = uv_x, \tag{32}$$

$$\sum_{n=0}^{\infty} D_n(u, v) = uv_y. \tag{33}$$

The few components of the Adomian polynomials are given below:

$$A_0(u) = u_{0x}, \tag{34}$$

$$A_1(u) = 2u_{0x}^2 u_{1x}, \tag{35}$$

$$\vdots$$

$$A_n(u) = \sum_{i=0}^{n} u_{ix} u_{(n-i)x}, \tag{36}$$

$$B_0(u) = v_0 u_{0y}, \tag{37}$$

$$B_1(u) = v_0 u_{1y} + v_1 u_{0y}, \tag{38}$$

$$\vdots$$

$$B_n(u, v) = \sum_{i=0}^{n} v_i u_{(n-i)x}, \tag{39}$$

$$C_0(u, v) = u_0 v_{0x}, \tag{40}$$

$$C_1(u, v) = u_0 u_{1x} + u_1 v_{0x}, \tag{41}$$

$$\vdots$$

$$C_n(u, v) = \sum_{i=0}^{n} u_i v_{(n-i)x}, \tag{42}$$

$$D_0(u, v) = u_0 v_{0y}, \tag{43}$$

$$D_1(u, v) = u_0 v_{1y} + u_1 v_{0y}, \tag{44}$$

$$\vdots$$

$$D_n(u, v) = \sum_{i=0}^{n} u_i v_{(n-i)y}. \tag{45}$$

From Eqs. (30)–(39), our required recursive relation is given by

$$u_0(x, y, t) = x + y, \tag{46}$$

$$u(x, y, t) = -\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} A_n(u) + \sum_{n=0}^{\infty} B_n(u, v)\right]\right] + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}\left(\nabla^2\left(\sum_{n=0}^{\infty} u_n\right)\right)\right]\right], \tag{47}$$

$$v_0(x, y, t) = x - y, \tag{48}$$

$$v(x, y, t) = -\mathcal{L}^{-1}\frac{1}{s}\mathcal{L}\left[\sum_{n=0}^{\infty} C_n(u, v) + \sum_{n=0}^{\infty} D_n(u, v)\right] + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left[\frac{1}{\mathrm{Re}}\left(\nabla^2\left(\sum_{n=0}^{\infty} v_n\right)\right)\right]\right]. \tag{49}$$

The first few components of $u_n(x, y, t)$ and $v_n(x, y, t)$ follows immediately upon setting $\mathrm{Re} = 1$, we have

$$u_1(x, y, t) = -2xt, \tag{50}$$

$$v_1(x, y, t) = -2yt, \tag{51}$$

$$u_2(x, y, t) = 2xt^2 + 2yt^2, \tag{52}$$

$$v_2(x, y, t) = 2xt^2 - 2yt^2, \tag{53}$$

$$u_3(x, y, t) = 4yt^4 - 4xt^3, \tag{54}$$

$$v_3(x, y, t) = 4xt^4 - 4yt^3. \tag{55}$$

Therefore solution obtained by LDM is given below:

$$\begin{aligned} u(x, y, t) &= \sum_{n=0}^{\infty} u_n(x, y, t) \\ &= x + y - 2xt + 2xt^2 + 2yt^2 + 4yt^4 - 4xt^3 + \cdots \\ &= x(1 + 2t^2 + 4t^4 + \ldots) - 2xt(1 + 2t^2 + \cdots) \\ &\quad + y(1 + 2y^2 + 4t^4 + \cdots), \\ &= (x - 2xt + y)(1 + 2t^2 + 4t^4 + \ldots), \\ &= \frac{x - 2xt + y}{1 - 2t^2}. \end{aligned} \tag{56}$$

$$\begin{aligned} v(x,y,t) &= \sum_{n=0}^{\infty} v_n(x,y,t) = x - y - 2yt + 2xt^2 - 2yt^2 + 4xt^4 - 4yt^3 + \ldots \\ &= x(1 + 2t^2 + 4t^4 + \ldots) - 2yt(1 + 2t^2 + \ldots) - y(1 + 2y^2 + 4t^4 + \ldots), \\ &= (x - 2yt - y)(1 + 2t^2 + 4t^4 + \ldots), \\ &= \frac{x - 2yt - y}{1 - 2t^2}. \end{aligned} \tag{57}$$

which is the exact solution of two dimensional Burger's equations [16].

**Example 2.** Let us consider another system of two dimensional Burger's equations with the following initial conditions

$$u(x, y, 0) = \frac{3}{4} - \frac{1}{4(1 + e^{\mathrm{Re}(-x+y)/8})}, \qquad v(x, y, 0) = \frac{3}{4} + \frac{1}{4(1 + e^{\mathrm{Re}(-x+y)/8})}. \tag{58}$$

The exact solution of problem is

$$u(x, y, t) = \frac{3}{4} - \frac{1}{4(1 + e^{\mathrm{Re}(-t-4x+4y)/32})}, \tag{59}$$

$$v(x, y, t) = \frac{3}{4} + \frac{1}{4(1 + e^{\mathrm{Re}(-t-4x+4y)/32})}. \tag{60}$$

**Table 1** Comparison of LDM solution (Eight order solution) with exact solution for Re = 1, with mesh points $x = 0.1$ and $y = 0.1$.

| $t$ | Exact $u(x, y, t)$ | LDM $u(x, y, t)$ | Exact $v(x, y, t)$ | LDM$v(x, y, t)$ |
|---|---|---|---|---|
| 0.05 | 0.190955 | 0.190955 | −0.0100503 | −0.0100503 |
| 0.10 | 0.183673 | 0.183673 | −0.0204082 | −0.0204082 |
| 0.15 | 0.178010 | 0.178010 | −0.0314136 | −0.0314136 |
| 0.20 | 0.173913 | 0.173913 | −0.0434783 | −0.0434783 |
| 0.25 | 0.171429 | 0.171429 | −0.0571429 | −0.0571429 |
| 0.30 | 0.170732 | 0.170734 | −0.0731707 | −0.0731709 |
| 0.35 | 0.172185 | 0.172187 | −0.0927152 | −0.0927154 |
| 0.40 | 0.176471 | 0.176473 | −0.117647 | −0.117647 |
| 0.45 | 0.184874 | 0.184876 | −0.151261 | −0.151264 |
| 0.50 | 0.200000 | 0.200000 | −0.200000 | −0.200000 |

**Table 2** Comparison of LDM solution (Eight order solution) with exact solution for Re = 1, with mesh points $x = 0.3$ and $y = 0.1$.

| $t$ | Exact $u(x, y, t)$ | LDM $u(x, y, t)$ | Exact $v(x, y, t)$ | LDM $v(x, y, t)$ |
|---|---|---|---|---|
| 0.05 | 0.371859 | 0.371859 | 0.190955 | 0.190955 |
| 0.1 | 0.346939 | 0.346939 | 0.183673 | 0.183673 |
| 0.15 | 0.324607 | 0.324607 | 0.178012 | 0.178012 |
| 0.20 | 0.304348 | 0.304348 | 0.173913 | 0.173913 |
| 0.25 | 0.285714 | 0.285715 | 0.171429 | 0.171430 |
| 0.30 | 0.268293 | 0.268295 | 0.170732 | 0.170734 |
| 0.35 | 0.251656 | 0.251659 | 0.172184 | 0.172185 |
| 0.40 | 0.235294 | 0.235296 | 0.176470 | 0.176471 |
| 0.45 | 0.218487 | 0.218488 | 0.184873 | 0.184874 |
| 0.50 | 0.200000 | 0.200000 | 0.200000 | 0.200000 |

**Table 3** Comparison of LDM solution (Eight order solution) with exact solution for Re = 1, with mesh points $x = 0.1$ and $y = 0.3$.

| $t$ | Exact $u(x, y, t)$ | LDM $u(x, y, t)$ | Exact $v(x, y, t)$ | LDM $v(x, y, t)$ |
|---|---|---|---|---|
| 0.05 | 0.39196 | 0.39196 | −0.231156 | −0.231156 |
| 0.1 | 0.387755 | 0.387755 | −0.265306 | −0.265306 |
| 0.15 | 0.387435 | 0.387435 | −0.303665 | −0.303665 |
| 0.20 | 0.391304 | 0.391304 | −0.347826 | −0.347827 |
| 0.25 | 0.400000 | 0.400000 | −0.400000 | −0.400011 |
| 0.30 | 0.414634 | 0.414636 | −0.463415 | −0.463417 |
| 0.35 | 0.437086 | 0.437088 | −0.543046 | −0.543048 |
| 0.40 | 0.470588 | 0.470589 | −0.647059 | −0.647061 |
| 0.45 | 0.521008 | 0.521009 | −0.789916 | −0.789919 |
| 0.50 | 0.600000 | 0.600000 | −1.000000 | −1.000000 |

**Table 4** Comparison of LDM solution (Eight order solution) with exact solution for Re = 1, with mesh points $x = 0.1$ and $y = 0.5$.

| $t$ | Exact $u(x, y, t)$ | LDM $u(x, y, t)$ | Exact $v(x, y, t)$ | LDM $v(x, y, t)$ |
|---|---|---|---|---|
| 0.05 | 0.532084 | 0.532084 | 0.967916 | 0.967916 |
| 0.1 | 0.530712 | 0.530712 | 0.969288 | 0.969288 |
| 0.15 | 0.529391 | 0.529391 | 0.97061 | 0.97061 |
| 0.20 | 0.528118 | 0.528118 | 0.971882 | 0.971882 |
| 0.25 | 0.526894 | 0.526894 | 0.973106 | 0.973106 |
| 0.30 | 0.525717 | 0.525717 | 0.974283 | 0.974283 |
| 0.35 | 0.524587 | 0.524587 | 0.975413 | 0.975413 |
| 0.40 | 0.523511 | 0.523511 | 0.9765 | 0.9765 |
| 0.45 | 0.522457 | 0.522457 | 0.977543 | 0.977543 |
| 0.50 | 0.521456 | 0.521456 | 0.978544 | 0.978544 |

**Table 5** Comparison of LDM solution (Eight order solution) with exact solution for Re = 1, with mesh points $x = 0.3$ and $y = 0.5$.

| $t$ | Exact $u(x, y, t)$ | LDM $u(x, y, t)$ | Exact $v(x, y, t)$ | LDM $v(x, y, t)$ |
|---|---|---|---|---|
| 0.05 | 0.515513 | 0.515513 | 0.984487 | 0.984487 |
| 0.1 | 0.514801 | 0.514801 | 0.985199 | 0.985199 |
| 0.15 | 0.514120 | 0.514120 | 0.985880 | 0.985880 |
| 0.20 | 0.513468 | 0.513468 | 0.986532 | 0.986532 |
| 0.25 | 0.512845 | 0.512845 | 0.987155 | 0.987155 |
| 0.30 | 0.512250 | 0.512250 | 0.987750 | 0.987750 |
| 0.35 | 0.511680 | 0.511680 | 0.988320 | 0.988320 |
| 0.40 | 0.511136 | 0.511136 | 0.989864 | 0.989864 |
| 0.45 | 0.510616 | 0.510616 | 0.989384 | 0.989384 |
| 0.50 | 0.510119 | 0.510119 | 0.989881 | 0.989881 |

**Table 6** Comparison of LDM solution (Eight order solution) with exact solution for Re = 1, with mesh points $x = 0.1$ and $y = 0.9$.

| $t$ | Exact $u(x, y, t)$ | LDM $u(x, y, t)$ | Exact $v(x, y, t)$ | LDM $v(x, y, t)$ |
|---|---|---|---|---|
| 0.05 | 0.605429 | 0.605429 | 0.894571 | 0.894571 |
| 0.1 | 0.602393 | 0.602393 | 0.897607 | 0.897607 |
| 0.15 | 0.599384 | 0.599384 | 0.900616 | 0.900616 |
| 0.20 | 0.596406 | 0.596406 | 0.903594 | 0.903594 |
| 0.25 | 0.593462 | 0.593462 | 0.906538 | 0.906538 |
| 0.30 | 0.590555 | 0.590555 | 0.909445 | 0.909445 |
| 0.35 | 0.587688 | 0.587688 | 0.912312 | 0.912312 |
| 0.40 | 0.584863 | 0.584863 | 0.915137 | 0.915137 |
| 0.45 | 0.582083 | 0.582083 | 0.917917 | 0.917917 |
| 0.50 | 0.579351 | 0.579351 | 0.920649 | 0.920649 |

The few components of series solution obtained with the help of Laplace decomposition method are as follow:

$$u_1(x,y,t) = -\frac{e^{-x+y}\mathrm{Re}t}{64\left(1+\frac{1}{8}e^{-x+y}\mathrm{Re}\right)^3} - \frac{e^{-2x+2y}\mathrm{Re}t}{64\left(1+\frac{1}{8}e^{-x+y}\mathrm{Re}\right)^3} + \frac{e^{-x+y}t}{16\left(1+\frac{1}{8}e^{-x+y}\mathrm{Re}\right)^3}, \tag{61}$$

$$v_1(x,y,t) = \frac{e^{-x+y}\mathrm{Re}t}{64\left(1+\frac{1}{8}e^{-x+y}\mathrm{Re}\right)^3} + \frac{e^{-2x+2y}\mathrm{Re}t}{64\left(1+\frac{1}{8}e^{-x+y}\mathrm{Re}\right)^3} - \frac{e^{-x+y}t}{16\left(1+\frac{1}{8}e^{-x+y}\mathrm{Re}\right)^3}, \tag{62}$$

The efficiency of Laplace decomposition method for system of two dimensional Burger's equations for above two examples have a closed agreement with exact solution. The comparison between Laplace decomposition method with exact solution is listed in Tables 1–6 for different values of Renoyld number in case of above examples. The numerical results show that the LDM may serve as a replacement to the solution of nonlinear problems in physical sciences.

## 4. Conclusion

The aim here is to provide the exact and series solution of a Burger's equation by using Laplace decomposition method (LDM). The convergence of LDM is also shown in Tables 1–6. The results of LDM are compared with exact solution. The results of LDM have a closed agreement with exact solution. The analysis given here shows further confidence on LDM. Therefore, this method can be applied to other non-linear equations arises in physical sciences and does not require linearization, discretization or perturbation and occupy less memory space in execution of a recursive relation.

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