Method of Designing an Infinite Impulse Response Equalizer for Magnetic Recording Channels

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Abstract

A finite impulse response (FIR) equalizer is practically employed in conjunction with the Viterbi detector for data detection process in magnetic recording channels. However, the FIR equalizer with a large number of taps is required at high density storage channels. It is well-known that an infinite impulse response (IIR) filter with a small number of taps can closely approximate such an FIR filter. In this paper, we present three methods of designing the IIR equalizer for magnetic recording channels. Results indicate that, when the number of equalizer taps is small (e.g., 3 taps) and the normalized recording density is high, the IIR equalizer designed based on the minimum mean-squared error approach performs much better than the FIR equalizer, especially in longitudinal recording channels.

1. Introduction

Partial-response maximum-likelihood (PRML) [1] is a technique of using an equalizer in conjunction with the Viterbi detector [2] for data detection process in magnetic recording channels. In practice, a finite impulse response (FIR) equalizer is employed in today's hard disk drives. At high normalized recording densities (NDs), the FIR equalizer with a large number of taps is required to sufficiently shape the read-back signal into a predetermined target [1, 3]. However, the total number of equalizer taps is generally limited by the maximum allowable loop delay in the timing recovery loop [3] because a small loop delay provides more robust phase locking, which in turn improves the overall system performance. Furthermore, the benefits of the equalizer with fewer taps are three folds: 1) a smaller area on the silicon

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Keywords: Equalizer design, infinite impulse response, magnetic recording
chip, 2) a shorter optimization time of read-channel chip during production, and 3) a small delay in the timing loop. It has been known that an infinite impulse response (IIR) filter with a small number of taps can closely approximate the FIR filter.

The partial response (PR) targets of the form \((1 - D)(1 + D)^n\) and \((1 + D)^n\) are suitable for longitudinal and perpendicular recording channels, respectively, where \(D\) is a delay operator and \(n\) is an integer. Given the PR target, its corresponding FIR equalizer can be derived based on the minimum mean-squared error (MMSE) approach [4]. In this paper, we propose three methods of designing the IIR equalizer for PR channels. The proposed methods may converge to a local minimum of the multi-model performance surface and usually require computationally expensive pole monitoring to ensure stability. However, based on extensive simulations, we have been able to conclude that the proposed IIR equalizer is highly stable for PR channels.

Many works related to the IIR equalizers have been studied and analyzed in the literature. For example, Park and Carley [5] investigated the performance of employing continuous-time adaptive IIR equalizers for EPR4 channels. The IIR modeling was considered in the design of decision feedback equalizers to reduce the number of filter taps [6]. In addition, Kim and Moon [7] approximated a high density storage channel with a digital IIR filter so that the detector can incorporate this knowledge to improve the performance of noise-predictive maximum-likelihood (NPML) detection. Nonetheless, in this paper, we propose three methods of designing the IIR equalizer and compare their performances with the FIR equalizer.

The rest of this paper is organized as follows. After describing our system model in Section 2, we explain the methods of designing the IIR equalizers for PR channels in Section 3. Numerical results are given in Section 4. Finally, Section 5 concludes this paper.

2. System description

Consider the system model shown in Fig. 1, where a binary input sequence \(x_k \in \{\pm 1\}\) with bit period \(T\) is filtered by an ideal differentiator \((1 - D)/2\) to form a transition sequence \(c_k \in \{-1, 0, 1\}\), where \(c_k = \pm 1\) corresponds to a positive or a negative transition, and \(c_k = 0\) corresponds to the absence of a transition. The transition sequence \(c_k\) passes through the magnetic recording channel represented by \(g(t)\). The transition response \(g(t)\) for longitudinal recording is given by [3] \(g(t) = 1/(1 + (2\pi/\text{PW}_{50})^2)\), where \(\text{PW}_{50}\) is the width of \(g(t)\) at half its maximum, whereas that for perpendicular recording is expressed as [8] \(g(t) = \text{erf}(2\sqrt{\ln 2}/\text{PW}_{50})\), where \(\text{erf}(x)\) is an error function, and \(\text{PW}_{50}\) determines the width of the derivative of \(g(t)\) at half its maximum. In the context of magnetic recording, a normalized recording density is defined as \(\text{ND} = \text{PW}_{50}/T\), which determines how many data bits can be packed within the resolution unit \(\text{PW}_{50}\). The media jitter noise, \(\Delta t_k\), is modeled as a random shift in the transition position with a Gaussian probability distribution function with zero mean and variance \(\sigma_j^2\) [9] (i.e., \(\Delta t_k \sim N(0, \sigma_j^2)\)) truncated to \(T/2\), where \(|a|\) takes the absolute value of \(a\), and \(\sigma_j\) is specified as a percentage of \(T\). Clearly, the severity of media jitter noise depends on how large the value of \(\sigma_j\) is.

The read-back signal can then be expressed as [8] \(p(t) = \sum_{k=-\infty}^{\infty} c_k g(t - kT + \Delta t_k) + n(t)\), where \(n(t)\) is additive white Gaussian noise with two-sided power spectral density \(N_0/2\). The read-back signal \(p(t)\) is filtered by a seventh-order Butterworth low-pass filter (LPF) and is then sampled at time \(t = kT\), assuming perfect synchronization. The sampler output \(s_k\) is equalized by an equalizer, \(F(D)\), such that the output sequence, \(y_k\), resembles the desired sequence, \(d_k\). Eventually, the Viterbi detector performs sequence detection to determine the most likely input sequence.
3. Design of IIR equalizers

The PRML system practically utilizes a \((2K+1)\)-tap FIR equalizer of the form \(F_{\text{FIR}}(D) = \sum_{k=-K}^{K} f_k D^k\), where \(K\) is an integer, and \(f_k\) is the \(k\)-th coefficient of \(F_{\text{FIR}}(D)\). The design of the target and its corresponding FIR equalizer based on the MMSE approach is given by Moon and Zeng [4]. This paper proposes the methods of designing the IIR equalizer for a given PR channel.

For simplicity, we consider the IIR equalizer of the form

\[
F(D) = \frac{B(D)}{A(D)} = \frac{\sum_{k=-N}^{N} b_k D^k}{\sum_{k=-M}^{M} a_k D^k},
\]

where \(N\) and \(M\) are integers, and \(b_k\) and \(a_k\) are the \(k\)-th coefficient of the numerator and the denominator of \(F(D)\), respectively. For a given PR target, the aim is to find the suitable coefficients, \(a_k\)'s and \(b_k\)'s, such that the resulting IIR equalizer performs better than the FIR equalizer, especially when the number of equalizer taps is small and the ND is high. This can be accomplished by the three following methods.

3.1. Based on structure verification

This method is based on structure verification [10], which attempts to approximate the FIR equalizer with the IIR equalizer so as to reduce the number of equalizer taps, while maintaining similar performance. This means we want \(F(D) = B(D)/A(D) = F_{\text{FIR}}(D)\) or, equivalently,

\[
b_k = f_k \star a_k = \sum_{i=0}^{M} a_i f_{k-i},
\]

where \(\star\) denotes the convolution operator. For \(k = \{-K, -K + 1, \ldots, K + M\}\), (2) can be written as a partitioned matrix of the form

\[
\begin{bmatrix}
\mathbf{b}_1 \\
\mathbf{b}_2 \\
\mathbf{b}_3
\end{bmatrix}
= \begin{bmatrix}
\mathbf{F}_1 \\
\mathbf{F}_2 \\
\mathbf{F}_3
\end{bmatrix}
\mathbf{a},
\]

where

\[
\mathbf{b}_1 = [b_{-K}, b_{-K+1}, \ldots, b_{-N-1}]^T, \quad \mathbf{b}_2 = [b_{-N+1}, \ldots, b_0, \ldots, b_N]^T, \quad \mathbf{b}_3 = [b_{N+1}, b_{N+2}, \ldots, b_{N+M}]^T, \quad \text{and} \quad \mathbf{a} = [a_0, a_1],
\]

Fig. 1. A system model with equalizer design
..., a_M^T$, are $(K - N)$-, $(2N + 1)$-, $(K + M - N)$-, and $(M + 1)$-element column vectors, respectively, $[.]^T$ is the transpose operation, and

$$
F_1 = 
\begin{bmatrix}
    f_{-K} & 0 & 0 & \cdots & 0 \\
    f_{-K+1} & f_{-K} & 0 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    f_{-N-1} & f_{-N-2} & f_{-N-3} & \cdots & f_{-N-M}
\end{bmatrix}
$$

$$
F_2 = 
\begin{bmatrix}
    f_{-N} & f_{-N-1} & f_{-N-2} & \cdots & f_{-N-M} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    f_0 & f_{-1} & f_{-2} & \cdots & f_{-M} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    f_N & f_{N-1} & f_{N-2} & \cdots & f_{N-M}
\end{bmatrix}
$$

$$
F_3 = 
\begin{bmatrix}
    f_{N+1} & f_N & f_{N-1} & \cdots & f_{N+1-M} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    f_K & f_{K-1} & f_{K-2} & \cdots & f_{K-M} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & \cdots & f_K
\end{bmatrix}
$$

are $(K-N)$-by-$(M+1)$, $(2N+1)$-by-$(M+1)$, and $(K+M-N)$-by-$(M+1)$ toeplitz matrices, respectively. Apparently, (3) gives

$$
b_2 = F_2 a, \quad (4)
$$

$$
b_1 = F_1 a \quad \text{and} \quad b_3 = F_3 a, \quad (5)
$$

Note that this method requires that $K > N$ and $K + M - N > 0$. Then, the denominator's coefficients of $F(D)$, i.e., $a$, can be obtained by solving (4) and (5), where $b_1$ and $b_3$ must be zero vectors. Since it is not possible to find the vector $a$ to satisfy $b_1 - b_3 = 0$, we then optimize $a$ by minimizing the error $E$ defined as

$$
E = b_1^T b_1 + b_3^T b_3 = a^T F_1^T F_1 a + a^T F_3^T F_3 a. \quad (6)
$$

During the minimization process, we impose a constraint $a_0 = 1$ to avoid reaching the trivial solution of $a = 0$. Therefore, adding this constraint to (6) yields

$$
E = a^T F_1^T F_1 a + a^T F_3^T F_3 a - 2\lambda (I^T a - 1), \quad (7)
$$

where $\lambda$ is the Lagrange multiplier, and $I$ is an $(M + 1)$-element column vector whose first element is one and the rest is zero. By differentiating (9) with respect to $\lambda$ and $a$, and setting the results to zero, we obtain

$$
\lambda = \frac{1}{I^T (F_1^T F_1 + F_3^T F_3)^{-1} I} \quad \text{and} \quad a = \lambda \left( F_1^T F_1 + F_3^T F_3 \right)^{-1} I. \quad (8)
$$

Finally, the numerator’s coefficients of $F(D)$, i.e., $b_2$, is obtained by substituting $a$ into (4). Clearly, this method is simple because it only requires the knowledge of $F_{\text{FIR}}(D)$, $N$ and $M$. 

3.2. Minimizing a filtered error sequence

This method has been proposed Warisan et al. [11], which designs the IIR equalizer based on Fig. 2. Assuming that the PR target $H(D)$ is known, meaning that $d_k$ is also known. From Fig. 1, it is clear that

$$y_k * a_k = s_k * b_k,$$  \hspace{1cm} (9)

where $s_k$ and $y_k$ is the input and the output of the IIR equalizer $F(D)$, respectively. Because $y_k = d_k + w_k$, where $w_k$ is an error sequence, substituting $y_k$ into (9) gives

$$(d_k + w_k) * a_k = s_k * b_k,$$  \hspace{1cm} (10)

$$v_k = s_k * b_k - d_k * a_k,$$  \hspace{1cm} (11)

where $v_k = w_k * a_k$ is a filtered error sequence. Equation (11) can be rewritten into a matrix form of $v_k = s^T b - d^T a$, where $s = [s_{k+N}, ..., s_k, ..., s_{k-N}]^T$, $b = [b, b_0, ..., b_N]^T$, $d = [d_k, d_k, ..., d_k, ..., d_{k-M}]^T$, and $a = [a_0, a_1, ..., a_M]^T$ are $(2N+1)$-, $(2N+1)$-, $(M+1)$-, and $(M+1)$-element column vectors.

The IIR equalizer is designed such that $E\left\{v_k^2\right\}$ is minimized, where $E\{\cdot\}$ is an expectation operator. Again, during the minimization process, we introduce the constraint $a_0 = 1$ to avoid reaching the trivial solutions of $b = a = 0$. Hence, this minimization process yields

$$\lambda = \frac{1}{\mathbf{I}^T (\mathbf{D} - \mathbf{P}^T \mathbf{R}^{-1} \mathbf{P})^{-1} \mathbf{I}}, \quad \mathbf{a} = \lambda (\mathbf{D} - \mathbf{P}^T \mathbf{R}^{-1} \mathbf{P})^{-1} \mathbf{I}, \quad \text{and} \quad b = \mathbf{R}^{-1} \mathbf{Pa},$$ \hspace{1cm} (12)

where $\lambda$ is the Lagrange multiplier, $\mathbf{I}$ is an $(M+1)$-element column vector whose first element is one and the rest is zero, $\mathbf{R} = E\{ss^T\}$ is an $(2N+1)$-by-$(2N+1)$ autocorrelation matrix of a sequence $s_k$, $\mathbf{P} = E\{sd^T\}$ is an $(2N+1)$-by-$(M+1)$ cross-correlation matrix of sequences $s_k$ and $d_k$, and $\mathbf{D} = E\{dd^T\}$ is an $(M+1)$-by-$(M+1)$ autocorrelation matrix of a sequence $d_k$.

3.3. Minimizing an error sequence

This method has been proposed by Kovintavewat et al. [12]. Instead of minimizing the filtered error sequence $v_k$, this method directly minimizes the error sequence $w_k$ (see Fig. 2). Assuming that $a_0 = 1$, (10)
can be rewritten as \( w_k = s^T b - d^T a - \hat{w}^T \hat{a} - d_k \), where \( d = [d_{k-1}, d_{k-2}, ..., d_{k-M}]^T \), \( \hat{a} = [\hat{a}_1, \hat{a}_2, ..., \hat{a}_M]^T \), and \( \hat{w} = [\hat{w}_{k-1}, \hat{w}_{k-2}, ..., \hat{w}_{k-M}]^T \) are \( M \)-element column vectors.

The IIR equalizer \( F(D) \) can then be obtained by minimizing \( \mathbb{E} \{ w_k^2 \} \) with respect to \( \hat{a} \) and \( b \). This minimization process gives

\[
\begin{bmatrix}
R \\
-X^T \\
D + V + V^T + W
\end{bmatrix}
\begin{bmatrix}
b \\
ar \\
z
\end{bmatrix}
= \begin{bmatrix}
c \\
-q
\end{bmatrix},
\tag{13}
\]

where \( X = \mathbb{E} \{ sd^T \} + \mathbb{E} \{ s \hat{w}^T \} \), \( D = \mathbb{E} \{ d \hat{d}^T \} \), \( V = \mathbb{E} \{ \hat{d} \hat{w}^T \} \), and \( W = \mathbb{E} \{ \hat{w} \hat{w}^T \} \) are \((2N+1)\)-by-\( M \), \( M \)-by-\( M \), \( M \)-by-\( M \), and \( M \)-element column vectors, respectively. From (16), because \( A \) is a square matrix, the coefficients of \( F(D) \), i.e., \( z \), is easily obtained by solving \( z = A^{-1} y \).

4. Numerical results

We consider the PR target \( H(D) = 1 + 2D - 2D^3 - D^4 \) for longitudinal recording and \( H(D) = 1 + 4D + 6D^2 + 4D^3 + D^4 \) for perpendicular recording. The \((2K+1)\)-tap FIR equalizer is designed based on the MMSE approach [4], which also yields an error sequence \( w_k \) that will be used to design the IIR equalizer. The signal-to-noise ratio (SNR) is defined as \( \text{SNR} = 10 \log_{10} \left( \frac{E_i}{N_0} \right) \) in decibel (dB), where \( E_i \) is the energy of the channel impulse response. All equalizers are designed at \( \text{SNR} \) required to achieve bit-error rate (BER) of \( 10^{-5} \). Each BER point is computed using as many 4096-bit data sectors as needed to collect 500 error bits, whereas only one data sector is used to design the equalizers.

As investigated by Kovintavewat et al. [12], the IIR equalizer with only one pole is sufficient to be employed in magnetic recording channels. Therefore, only the 3-tap IIR equalizer with one pole (i.e., \( N = 1 \) and \( M = 1 \)) is considered in this paper. Furthermore, we denote “IIR-Mx” as the IIR equalizer, where \( M1 (x = 1) \), \( M2 (x = 2) \), and \( M3 (x = 3) \) are the methods presented in Section 3.1, 3.2, and 3.3, respectively.

4.1. No jitter noise

In this section, we compare the performance of different equalizers in the absence of jitter noise (i.e., \( \sigma_j = 0\% \)). Fig. 3 (Left) compares the performance of the FIR and IIR equalizers for longitudinal recording channels by plotting the SNR required to achieve \( \text{BER} = 10^{-4} \) as a function of NDs. It is clear that the IIR-M2 and IIR-M3 equalizers perform better than the FIR equalizer, especially when ND is high. However, the IIR-M1 equalizer performs worse than other equalizers, including the FIR equalizer. This might be because the design method of the IIR-M1 equalizer directly attempts to approximate a large number of FIR equalizer taps (e.g., 11 taps) into a small number of IIR equalizer taps (e.g., 3 taps), instead of minimizing the equalizer output and the desired output as used in IIR-M2 and IIR-M3 design. Additionally, we observed that there is no significant performance improvement by employing the IIR equalizer when the number of equalizer taps is large (not shown here).

Similarly, we also compare the performance of different equalizers in perpendicular recording channels in Fig. 3 (Right). Similar results can be obtained as in longitudinal recording channels. That is, the IIR-M3 equalizer yields the best performance if compared to other equalizers, and the IIR-M2 and IIR-M3 equalizers perform better than the FIR equalizer when ND is high.
The reason that the IIR-M2 and IIR-M3 equalizers provide better performance than the FIR equalizer is because they can shape the read-back signal to the PR target better than the FIR equalizer does, especially when the number of equalizer taps is small. This can be found by looking at the frequency responses of different equalizers. Assume that the 11-tap FIR equalizer is the best, the 3-tap equalizer whose frequency response closely matches the frequency response of the 11-tap equalizer will yield the best performance among 3-tap equalizers. We found that (not shown here) the IIR-M2 and IIR-M3 equalizers give a better match to the frequency response of the 11-tap equalizer than the 3-tap FIR equalizer, especially at low frequencies where most data’s energy concentrates here. This is why the IIR-M3 performs the best, followed by the IIR-M2 equalizer, and both outperform the 3-tap FIR equalizer.

4.2. With jitter noise

This section compares the performance of different equalizers in the presence of jitter noise in perpendicular recording channels, because perpendicular recording is the current technology used in today’s hard disk drives. As discussed in Section 4.1, since the IIR-M1 equalizer performs worse than both the IIR-M2 and IIR-M3 equalizers, we then ignore the IIR-M1 equalizer in this section. Therefore, Fig. 4 plots the SNR (in dB) required to achieve BER = 10^{-4} as a function of the amount of jitter noises in perpendicular recording channels at ND = 3. It is obvious that the IIR-M3 equalizer performs better than the IIR-M2 equalizer, and both outperform the 3-tap FIR equalizer, especially when the jitter noise is large. Thus, it is worth employing the IIR equalizer in perpendicular recording channels.

5. Conclusion

This paper proposed three methods of designing the IIR equalizers for PR channels. The first method (i.e., IIR-M1) is based on structure verification, where only the FIR equalizer’s coefficients and the number of zeros and poles of the IIR equalizer are required. The second method (i.e., IIR-M2) is based on minimizing the filtered error sequence, and the last method (i.e., IIR-M3) directly minimizes the error sequence, where both methods require the knowledge of the target.

Simulation results has illustrated that, when the number of equalizer taps is small (e.g., 3 taps) and ND is high, the IIR-M2 and IIR-M3 equalizers perform better than the FIR equalizer, especially in longitudinal recording channels. Although the IIR filter has an issue about stability, we found that the proposed IIR equalizer is highly stable for PR channels.
Fig. 4. Performance comparison in perpendicular recording with different amounts of jitter noise

References


