The Application of $p$-Version Finite-Element Methods to Fracture-Dominated Problems Encountered in Engineering Practice

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Abstract—This paper details experience applying higher-order ($p$-version) finite-element methods to highly specialized and difficult problems in structural life predictions, namely in predictions of aircraft structural fatigue life using crack growth analyses. Crack growth analyses require reliable estimates of the Mode I stress intensity factors (SIFs) for given load, boundary conditions, and geometry. Fortunately, accurate and reliable stresses and stress intensity factors, obtained quickly and efficiently from finite-element simulations performed with higher-order methods, enable basic procedures to be successfully applied to the development of advanced life prediction methods.

Many FEA software packages could have been used to obtain engineering data such as SIFs and stresses. However, advanced numerical methods incorporated into higher-order finite-element methods allow the engineering analyst to have confidence in the results. This confidence comes from being able to obtain more accurate and reliable results using convenient error checking and numerical convergence of engineering data such as maximum stress and strain and SIFs, stress contours, and deformations. In addition, greatly shortened analysis schedules are realized through the use of procedures such as the automatic extraction of SIF parameters using the contour integral method (CIM). The versatility of higher-order methods is demonstrated by some examples encountered in engineering practice—multisite damage scenarios and corroded plates. © 2003 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

The demands for continued use of the aging aircraft fleets around the world require specialized methods for assessing the safety, readiness, and costs associated with corrosion. The Aloha Airlines catastrophic decompression in 1987 forced the aircraft industry to focus on the burgeoning age degradation problem. The focus is to use the available state-of-the-art technology in fracture, finite-element analysis, and crack growth analysis tools to account for age degradation effects. Attention is focused on transport fuselage lap joints, Figure 1, which are known to exhibit various types of damage, including multisite cracks or damage (MSD) at the fastener holes.
crevice corrosion between the lap skins, and a unique mechanical phenomenon known as corrosion pillowing. Corrosion pillowing causes out-of-plane bending of the lap splice skins, so that the skins undergo sustained bending stresses that could hasten the eventual failure of the splice [1]. Multisite damage has been investigated by numerous researchers both experimentally [2,3] and numerically [2–4]. Many of these models are quite complicated, including the effects of friction and interactive contact between the fasteners and lap skins. However, models of MSD here were intentionally kept simple so that the effects of corrosion on damage tolerance could be isolated from other mechanical effects such as friction.

Fatigue, crack growth, and finite-element modeling algorithms, each modeling corrosion interaction effects, are tools that can be readily adapted to provide adequate assessments to determine the magnitude of the corrosion problem. Corrosion by itself is a difficult structural problem to solve, requiring innovative solutions in order to overcome substantial challenges. Recently developed models include a myriad of structural effects, such as stress risers caused by corrosion topography variations, bending stresses induced by corrosion by-product build-up between adjacent parts, and multiple cracks in the structure induced by corrosion and fatigue cycling. Advances in higher-order (p-version) finite-element methods have begun to provide essential inputs to crack growth analyses used to predict the effects of corrosion on aircraft structural life.

Fundamental fracture mechanics principles are used to assess present damage levels in the aircraft structure as well as estimate future effects of damage on the damage tolerance of the structure. A crack growth analysis requires input of

- the structure’s geometry,
- flaw and cracking scenario,
- the cyclic load and environmental (if appropriate) spectra,
- the Mode I stress intensity factors (SIFs) if the structure is not a ‘menu’ geometry, and
- material-specific data such as material crack growth rates ($\frac{dC}{dN}$) and static properties such as Young’s modulus and critical SIFs (i.e., $K_{IC}$ and $K_I$, plane strain and plane stress fracture toughness, respectively).

Starting from an assumed initial crack length and configuration, the crack growth analysis proceeds by evaluating the SIFs for the given crack and geometry configuration, looking up the corresponding $\frac{dC}{dN}$, checking for failure, and then incrementing the crack lengths for a chosen
number of fatigue cycles, thereby resulting in a new crack configuration and length, and hence, a new set of SIFs.

Corrosion has been studied by a number of researchers who have experimentally investigated corrosion and corrosion pitting [5,6]. However, we have been able to find very few instances in which numerical methods such as finite elements have been used to predict and quantify corrosion effects on structural life. A proposed procedure for assessing corrosion/age degradation effects, called the holistic life prediction methodology (HLPM) [7], requires reliable and accurate estimations of the structure’s stress states and its SIF states. The HLPM uses available state-of-the art fracture mechanics and structural analysis tools to evolve an engineering method for integrating life impacts due to corrosion degradation into structural life assessments. With these tools, it becomes feasible to determine the economic impact of corrosion and to improve the structural airworthiness of all aircraft systems. This paper provides a description of only the higher-order finite-element tool and its role in the HLPM. Finite element models used to characterize the corroded surface topography stress states and multisite damage (MSD) of the lap skins are described.

A novel aspect of this approach is that what has often in the past been considered and used as academic finite-element methods (that is, higher-order methods and the p-version) can actually be considered and used as ‘expert’ technology routinely in a professional engineering practice. Moreover, ease of use should encourage other engineers to integrate this p-version approach into their own structural analysis activities.

This paper is organized in the following manner. First, in Section 2, as a vehicle for discussing our particular finite-element analysis approach, we present numerical error estimation procedures and results for a typical through-crack model. We present error checks for only one problem here; however, we discuss every subsequent problem we successfully accomplished later in the paper. In Section 3, three examples designed to illustrate p-version method’s ability to obtain meaningful and useful results at two length scales, that is, micro- and macro-scales, are presented. The examples presented are intentionally kept simple, in order to isolate and more closely study the corrosion effects on a structure’s damage tolerance. Example 1 is a model of a typical corroded skin extracted from an actual transport aircraft lap splice that shows corrosion effects on a microscale. Example 2 illustrates a proposed model of the evolution of corrosion pits into cracks and shows corrosion effects on a microscale. Example 3 is a simulation of a full bay (frame to frame) of a lap joint without stiffeners that models corrosion effects on a macroscale. Section 4 discusses limitations of the present modeling approach and subjects for proposed improvements to the models. Finally, Section 5 summarizes important results from our analyses.

2. ERROR CHECKING

Each engineering analysis contains many sources of error. In the case of finite-element analyses, three significant error sources can be identified: modeling errors, numerical errors (or approximation errors), and idealization errors. Because modeling errors, which arise from techniques needed to implement the idealization, are highly individualistic and depend on the skill of the analyst, these types of errors will not be discussed. Idealization errors and corresponding model limitations for this particular structure are discussed near the end of this paper. Numerical errors and ways to minimize these errors are discussed in this section.

As is well known, any numerical approximation is subject to sources of error—too few degrees-of-freedom, poor meshing techniques, and poor modeling and implementation of load and boundary constraints each can significantly affect the numerical results, and hence, interpretation of those results. Obtaining reliable and accurate numerical results is always a major goal of any simulation. The engineering goal of the computation (a.k.a. the data of interest) in this case is to obtain reliable and accurate estimates of the Mode I stress intensity factor (SIF) $K_I$. In each case discussed below, this goal was always met, and often with minimal analysis effort.
Why higher-order methods? We have found that there are few robust error checking procedures that can be easily used by engineering analysts. Many procedures that are available commercially use a black box mentality that computes 'error parameters' of unknown origin using transparent or proprietary methods and then leave it to the analyst to speculate as to what goes on inside the box. Fortunately, advancements in higher-order methods such as the p-version of the finite-element method facilitate routine error checking in the finite-element models here. Routine error checks include convergence of the Mode I stress intensity factor (SIF) $K_1$, continuity of stresses across adjacent element boundaries, and load resultants for accurate representation of the intended loads. The example given here shows the procedure used routinely when performing all other finite-element analyses described in this paper. The example is an engineering idealization of a typical transport fuselage lap joint, Figure 1. A series of equal length cracks at the upper skin's top row of the joint has been modeled by introducing two equal length cracks on each side of the fastener hole. For reference, the crack lengths are 0.1 in (2.54 mm) and the fastener spacing is 1 in (25.4 mm). The plate thickness is 0.04 in (1.016 mm). The hoop stress resulting from fuselage pressurization is 10 ksi (68.93 MPa). The fasteners, modeled with normal springs distributed 180 degrees around the hole, are 0.1875 in (4.76 mm) in diameter. A scenario that could approximate a 'multisite damage' (MSD) condition, in which every fastener in the top row has two cracks oriented perpendicular to the hoop load, is modeled with two equal length cracks emanating at the 0 and 180 degree positions (that is, $y = \text{constant}$) of the fastener. Symmetry of this geometry, loads, boundary conditions (B.C.) and crack configuration can be modeled with symmetry B.C. enforced on the two long vertical edges in the model of Figure 2. All modeling is accomplished in two dimensions (plane stress).

Figure 2. Demonstration model for error checking and estimates. Symmetry boundary conditions (circles) are depicted on left and right vertical surfaces. Planar tension load depicted with arrows on top. Normal springs simulating fastener loading depicted in centers of three holes.
For each example given in this paper, a series of eight $p$-levels was used to estimate the numerical error. Numerical convergence is verified and substantiated with checks of

- the error in the energy norm, Figure 3 and Table 1 (a measure of global error);
- displacement error, Figure 4 (a measure of model input error);
- stress contour continuity, Figure 5 (a measure of the 'goodness' of the mesh refinement);
- load application error (discussed below); and,
- error in the engineering data of interest, Figure 6 (in this case the Mode I stress intensity factor, $K_I$).

Acceptable to excellent results were obtained during each type of error estimation technique. The convergence of the error in the energy norm is a critical indicator of global error convergence—both the rate of convergence and estimates of the percent error are important. For fracture mechanics problems, it is not unusual to discover that this error is one of the most difficult types to minimize. Figure 3 and Table 1 are illustrations of this difficulty. In the figure and table, the acronym 'DOF' stands for 'degrees-of-freedom' for the model. Nonquadratic convergence rates and relatively high % estimated error in the 1% to 5% range are typical and acceptable.

![Figure 3](image.png)

**Figure 3.** Convergence of error in energy norm acceptable for fracture mechanics problems.

<table>
<thead>
<tr>
<th>p-level</th>
<th>DOF</th>
<th>Total Potential Energy</th>
<th>Rate of Convergence</th>
<th>Estimated % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>142</td>
<td>-5.6240307141402e-004</td>
<td>0.00</td>
<td>32.86</td>
</tr>
<tr>
<td>2</td>
<td>514</td>
<td>-5.0477122389150e-004</td>
<td>0.55</td>
<td>16.21</td>
</tr>
<tr>
<td>3</td>
<td>1110</td>
<td>-5.1276949045546e-004</td>
<td>0.57</td>
<td>10.41</td>
</tr>
<tr>
<td>4</td>
<td>1930</td>
<td>-5.1511064078236e-004</td>
<td>0.49</td>
<td>7.96</td>
</tr>
<tr>
<td>5</td>
<td>2974</td>
<td>-5.1018237561593e-004</td>
<td>0.46</td>
<td>6.53</td>
</tr>
<tr>
<td>6</td>
<td>4242</td>
<td>-5.1678532926070e-004</td>
<td>0.45</td>
<td>5.57</td>
</tr>
<tr>
<td>7</td>
<td>5734</td>
<td>-5.1716769216687e-004</td>
<td>0.45</td>
<td>4.86</td>
</tr>
<tr>
<td>8</td>
<td>7450</td>
<td>-5.1742465581107e-004</td>
<td>0.45</td>
<td>4.32</td>
</tr>
</tbody>
</table>

Displacement plots are used primarily to check for proper application of boundary conditions and loads, Figure 4. Typically there is more interest in stresses as engineering data than displacements, whereas displacements are normally an easily measured quantity. The region around the
The top hole with two cracks is shown in Figure 4, in which proper boundary conditions (B.C.) are verified—the cracks "open" as expected. Verifying the load magnitude is a critical part of any error checking procedure. The force applied was 400 lbs (= 10 ksi * 0.04 in thick skin * 1 in wide model). Checking the force resultants by integrating the stresses on the applied load boundary yields 399.999 lbs, a difference of less than 0.00025%.

The efficacy of the mesh discretization can be estimated by checking stress contour continuity across adjacent element edges. Plotted in Figure 5 are the von Mises stress contours near the top hole. As expected, there are very high stress gradients near the crack tips. However, the reader will note that the continuity across adjacent element edges is quite good, especially considering no smoothing or averaging of the edge stresses from the computation over adjacent elements was used.

The real power of the higher-order methods like the p-version is revealed in the check for numerical error in the engineering data of interest, the Mode I stress intensity factor, $K_I$. In higher-order methods, fracture mechanics parameters like $K_I$ can be efficiently extracted from the finite-element solution by using the contour integral method (CIM). This procedure computes fracture mechanics parameters using a superconvergent (that is, converges as the energy)
Fracture-Dominated Problems

Figure 6. Mode I stress intensity factor excellent convergence, to within 0.07% with 7450 DOF.

Table 2. Convergence of the error in Mode I stress intensity factor \( K_I \).

<table>
<thead>
<tr>
<th>p-level</th>
<th>DOF</th>
<th>Radius (in)</th>
<th>( K_I ) ksi-in(^{1/2} )</th>
<th>( K_{II} ) ksi-in(^{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>142</td>
<td>4.799e-002</td>
<td>8.584489031038721e+000</td>
<td>4.320034242542275e-001</td>
</tr>
<tr>
<td>2</td>
<td>514</td>
<td>4.799e-002</td>
<td>1.025105982688355e+001</td>
<td>2.504406590296494e-002</td>
</tr>
<tr>
<td>3</td>
<td>1110</td>
<td>4.799e-002</td>
<td>9.52121972581062e+000</td>
<td>1.753611052927837e-002</td>
</tr>
<tr>
<td>4</td>
<td>1930</td>
<td>4.799e-002</td>
<td>9.903283642039678e+000</td>
<td>2.578146960708403e-003</td>
</tr>
<tr>
<td>5</td>
<td>2974</td>
<td>4.799e-002</td>
<td>1.004005489420370e+001</td>
<td>-4.018220233246161e-003</td>
</tr>
<tr>
<td>6</td>
<td>4242</td>
<td>4.799e-002</td>
<td>9.491395379480142e+000</td>
<td>-8.25319092077591e-003</td>
</tr>
<tr>
<td>7</td>
<td>5734</td>
<td>4.799e-002</td>
<td>1.000134621307378e+000</td>
<td>-1.00191492699032e-002</td>
</tr>
<tr>
<td>8</td>
<td>7450</td>
<td>4.799e-002</td>
<td>1.00022175725585e+001</td>
<td>-1.002650162276887e-002</td>
</tr>
</tbody>
</table>

Estimated limit \( K_I \) 9.994841e+000
Estimated limit \( K_{II} \) -1.002666e-002

3. RESULTS

Three typical examples that illustrate the manner in which higher-order methods can be used to study corrosion in lap joints are discussed here. The examples were also chosen to illustrate how higher-order methods can be used to analyze corrosion effects on two length scales: micro- and macro-scales. Example 1 is a model of a typical corroded skin extracted from an actual transport aircraft lap splice that shows corrosion effects on a microscale. Example 2 illustrates a proposed model of the evolution of corrosion pits into cracks and shows corrosion effects on a microscale. Example 3 is a simulation of a full bay (frame to frame) of a lap joint without stiffeners that models corrosion effects on a macroscale.

**EXAMPLE 1. CORRODED SURFACE FROM LAP JOINT SKIN.** A fuselage lap joint from a transport aircraft was dismantled and sectioned. A section of the top skin in the lap joint is shown in
Figure 7. Crevice corrosion and degradation caused by liquids trapped between the two parallel plates occurred in this joint. Laser profilometry of the corroded area is taken between points A and B marked in Figure 7. To reduce modeling effort and to quickly estimate the effect of the corrosion topography on the SIF field, the results from a two-dimensional model were used as an indication of the three-dimensional effects of the topography. The resulting laser profile is shown in Figure 8. Cubic splines are used to “connect the dots” which describe the surface. This cubic spline is then used to define the upper surface of a finite-element model, Figure 9. The hoop stress in the lap splice was then simulated with normal tractions applied at the two ends shown in Figure 9. The engineering data of interest is again the Mode I SIF. The numerical convergence of this parameter is excellent, as illustrated in Figure 10.

![Figure 7](image1)

**Figure 7.** Crevice corrosion in a transport fuselage lap joint skin.

![Figure 8](image2)

**Figure 8.** Laser profile of a portion of corroded surface in Figure 7.

![Figure 9](image3)

**Figure 9.** Finite-element model introduced cracks at the deepest point of the surface in Figure 8. This crack tip is 0.02 in (0.508 mm) from the surface.

![Figure 10](image4)

**Figure 10.** Mode I stress intensity factor excellent convergence, to within 0.07% with 4039 DOF. Error estimation for this calculation is discussed in [2].
One essential goal of the analysis was to study the effect of the corroded surface on the stress and stress intensity fields in the skin. In a hypothetical damage tolerance analysis, corrosion losses could be accounted for by merely determining the area lost to corrosion and then amplifying the stress by a corresponding amount. It was hypothesized that local (near surface) stress amplifications are caused by the variation in the corroded skin topography due to the roughness of the surfaces, and that in the early stages of cracking, this effect would be important. To illustrate the effect this particular corroded topography has on the SIF field, these corroded topography results were compared with the analytic smooth surface SIF solution. The solution for a smooth surface with edge crack is well known. 

The average thickness of the corroded plate, \( t = 0.0593 \text{in} \), was used in the analytic solution and these results are compared to the corroded plate results in Table 3. As can be seen, there are significant amplifications of the SIF solutions caused by the corroded topography which become less amplified as the crack gets larger (that is, the crack tip moves out of the influence of the topography) and then becomes more amplified as the crack tip approaches the back face (because the crack reaches the back face more quickly due to the 'local' thickness losses). There are two mechanical effects that account for the differences in the SIFs: the stresses are amplified by the corroded topography, and the stresses in the corroded topography models have an additional bending stress caused by the offset in the load vectors in the far field and near crack field planes.

### Table 3. Effect of corroded topography on the Mode I stress intensity factor \( K_I \). The solution for crack length 0.02 in (in bold below) is from the finite-element model of Figure 9.

<table>
<thead>
<tr>
<th>Crack in</th>
<th>Corroded ( K_I ) psi-in(^{1/2} )</th>
<th>Smooth ( K_I ) psi-in(^{1/2} )</th>
<th>Ratio ( K_I\text{(corroded)}/K_I\text{(smooth)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>2525</td>
<td>1066</td>
<td>2.369</td>
</tr>
<tr>
<td>0.002</td>
<td>3146</td>
<td>1519</td>
<td>2.071</td>
</tr>
<tr>
<td>0.005</td>
<td>4449</td>
<td>2489</td>
<td>1.787</td>
</tr>
<tr>
<td>0.010</td>
<td>6449</td>
<td>3889</td>
<td>1.658</td>
</tr>
<tr>
<td>0.020</td>
<td>12324</td>
<td>7608</td>
<td>1.620</td>
</tr>
<tr>
<td>0.030</td>
<td>25566</td>
<td>14903</td>
<td>1.715</td>
</tr>
<tr>
<td>0.040</td>
<td>70609</td>
<td>33476</td>
<td>2.109</td>
</tr>
</tbody>
</table>

#### Example 2. Pits as Cracks. The mechanical effects of a typical corrosion pit are discussed in this example. The modeling goals were to determine how closely pits approach the geometry and mechanical response of a crack. A corrosion experiment, conducted by Aeronautical and Maritime Research Laboratory (AMRL) on aluminum alloy 7050-T7451, used 3.5% NaCl solution to cause pitting in the bores of holes drilled into test coupons. For this example, finite-element models similar to the one shown in Figure 9 are constructed for

1. a notch in the edge of a smooth plate (Figure 11),
2. a corrosion pit whose geometry was determined during the AMRL experiment after tear-down of the failed specimens (Figure 12), and
3. an edge crack in a smooth plate (Figure 13).

Von Mises stress contours are shown for each model at the same applied stress levels. To remove the influence of the "back face" (i.e., the face which is the opposite of the crack origin in Figure 13), the back face is moved far enough away so it does not appreciably affect the results.

Consider each of the geometric features in the figures a ‘flaw’ that can be measured. In Figure 11, for instance, the flaw size is the radius of the notch \( R \). In Figure 12, the flaw size is the depth of the pit \( d \), and in Figure 13, the flaw size is the crack length \( a \). Each of the flaws is the very same length, 0.0075 in (0.19 mm). The shape and color distributions of the von Mises stress contours in Figures 11–13 suggest an ‘evolution’ of the stresses as the round “pit” evolves into an elongated pit and finally into a crack. These stress distributions suggest a concept of
“pit as crack”—as the pit sharpens the stress field evolves from a characteristic “peanut” shape of Figure 11 into shapes that more resemble two distinct stress “lobes” in Figures 12 and 13. Even in the absence of a crack, mechanically the corrosion pit model in Figure 12 looks much closer to the edge crack model in Figure 13 than it does to the round notch model in Figure 11, leading to the hypothesis that cracks develop from pits because pits are closer mechanically to cracks than they are to smooth notches. If confirmed, this hypothesis could have interesting and useful implications for future studies of pitting corrosion and cracks. One implication is that it appears likely that the modeling of pitting and its effects on crack growth is greatly simplified, as the pit depth can be superimposed directly with the crack length in the crack growth analysis. Another implication is that less emphasis on the topological description of the pits is necessary for robust crack growth analyses. Of course, further study is necessary to confirm the hypothesis—for instance, it may occur that pits in this particular alloy are completely different from pits in other alloys; perhaps pits in other alloys are round.
EXAMPLE 3. LAP JOINT MODEL. A model that approximates a full bay of a typical transport fuselage lap joint is described here. The splice has three rows of 20 fasteners each between stiffeners 20 in (508 mm) apart. Due to symmetry in the geometry, loads, boundary conditions, and crack configurations, only half of the bay was modeled. Similar to the model shown in Figure 2, interactions between the top skin (modeled here) and the bottom skin in the splice are modeled with normal springs distributed 180 degrees around the fastener hole, Figure 14. Fuselage pressurization and the resulting hoop stress are treated as planar and are modeled with a normal traction applied on an edge away from the fasteners. Two opposing cracks, perpendicular to the applied normal traction, are introduced on each hole in the top row. In contrast to the MSD model of Figure 2, this particular joint model allows the independent variation of the lengths of both the right and left cracks located at each hole. The MSD scenario is but one possible result of widespread corrosion damage in a lap joint. This model will then allow different combinations of crack lengths to be studied, and thereby study other crack scenarios and failure modes. The goal of the computation was to obtain accurate and reliable $K_I$ data for all crack configurations (i.e., crack lengths and locations).

Eight finite-element simulations (with increasingly higher-order DOFs) were performed for each crack configuration. For illustration purposes here, all crack lengths were the same, 0.01 inches. The von Mises stress distribution at $p = 8$ for the 0.01 in crack length is shown in Figure 15 for the region close to the left-most hole of Figure 14. The stress contour continuity across adjacent element edges everywhere is excellent. Convergence in the SIF for the left-most crack (near the symmetry B.C. in Figure 14) is shown in Figure 16 and is also excellent. Figure 17 shows the spectrum of geometry factors ($\beta$s) for all 20 cracks. The geometry factor $\beta$ is defined as

$$\beta = \frac{K_I}{\sigma_\infty \sqrt{\pi a}},$$

where $K_I$ is the Mode I stress intensity factor, units ksi-in$^{1/2}$, $\sigma_\infty$ is the applied far field stress, ksi, and $a$ is the crack length, in.

Figure 17 illustrates the importance of the proper specification of the B.C.s—the free edge B.C. that avoided the complexities of the stiffener/skin interactions caused the SIF to be higher on the edges than in the middle of the joint. It is known from experiments that crack SIFs decrease...
Figure 14. Multisite damage (MSD) in a fuselage lap joint modeled with multiple cracks in top row of fasteners. Symmetry of bay illustrated with rollers at the left edge. Inset displays the mesh near a top row fastener. Note the aspect ratio of the cracks: hole diameter is approximately 0.027:1.

Figure 15. Von Mises stress distribution for multisite damage (MSD) in a fuselage lap joint.

as the cracks approach the stiffener (discussed further in Section 4 below). Future models will include the stiffener effects.

4. MODEL LIMITATIONS

As with any engineering model, assumptions were made in order to make the mechanical problem of corrosion in lap joints more tractable and hence allow the development of a robust and useful engineering analysis tool for performing trade studies that allow variation of corrosion and geometry parameters in corrosion-fatigue interaction problems. The difference between reality and the engineering model is the idealization error referenced in the introduction. The present engineering model described here uses a first-order model of the fastener-hole interaction and the corrosion topography effects and ignores frictional effects, the effects of boundary conditions on the part-through crack SIFs, and secondary bending. Each of the limitations is described more completely below.
Figure 16. Mode I stress intensity factor excellent convergence, to within 0.04% with 79053 DOF.

Figure 17. Geometry factors for all cracks. In this model, the βs are increasing the farther one moves away from the centerline, in contrast to what is known to occur in an actual lap joint with stiffening structure.

Perhaps the most difficult mechanical interaction to accurately model in lap joints is the load transfer between the fasteners and corresponding holes and the frictional load transfer caused by large skin surface areas that rub against each other in the lap part of the splice. The fastener-hole load transfer mechanism is very complicated—fastener-hole frictional contact, fastener torque, fastener type, hole tolerance, and installation procedure are all critical elements of a complete description of the mechanical interactions. The frictional load transfer between the two lap skins can be quite significant, and is not typically accounted for in a traditional damage tolerance analyses. This can create a 'reserve' fatigue life (that is, by ignoring the frictional effects, life predictions become more conservative); however, the introduction of corrosion preventive compounds (CPCs) in between the lap skins has been shown to decrease the life of the lap joint,
as friction is minimized, and hence, all load transfer between the two skins now goes almost completely through the fasteners alone. This often hastens the failure of the fasteners that have not been properly sized to take the entire load transfer between the skins.

Secondary bending effects arise from the offset of the two skins in the lap splice—the load planes of the top and bottom skins are off-center by the average of the two skin thickness, therefore causing a moment with corresponding bending stress to be realized. It is possible that the secondary bending is significant, as some researchers have seen stresses 40% higher than the fuselage pressurization hoop stress [10].

The effects of various boundary conditions that are known to exist in these types of structures have not been addressed. For instance, computed only were multisite damage (MSD) effects for through (1-D) cracks. Preliminary analyses indicate that the MSD effects can significantly affect the growth of these types of cracks. Until recently, it has been very difficult and time-consuming to obtain reliable and accurate part-through (2-D) crack SIFs. Fortunately, higher-order finite-element software such as StressCheck [11] has recently addressed this shortcoming with the addition of automatic three-dimensional CIM extraction procedures. Preliminary analyses had shown these CIM procedures to be easy-to-use and extremely robust.

A typical transport lap joint can have many different structural subcomponents:

1. thin (typically 0.04 to 0.063 in) skins are supported by circumferential stiffeners (frames) and longitudinal stiffeners (longerons).

In addition to taking pressurization loads, these stiffeners are designed to take fuselage bending loads caused by air loads and fuselage weight. The frames generally are designed for one other important function—damage tolerance. One failure mode considered in a lap joint is the MSD scenario—many cracks form in the top row of the lap splice fastener array and eventually link up if undetected. The frame structure is designed to arrest the rapid propagation of the MSD cracks and slow down or prevent the cracks from propagating into the next lap joint bay, thereby preventing catastrophic failure of the lap joint and possible loss of the aircraft and its passengers. An analysis of the stiffener effects, therefore, is very important to a complete damage tolerance analysis. Stiffener effects were not addressed in the present models—when time permits, stiffener effects will be addressed. At best, any crack growth analyses with the models detailed in this paper must be considered conservative estimates of the structure’s predicted life.

5. CONCLUSIONS

This paper summarizes recent experience using higher-order finite-element methods and fracture mechanics extraction procedures to study corrosion in fuselage lap joints. The analysis described here thus far supports the following conclusions.

- Higher-order methods can and are being efficiently implemented into routine engineering practice.
- Efficient procedures for extraction of fracture mechanics parameters that are incorporated into higher-order finite-element methods are critical to reliable engineering analyses.
- Proper modeling of corrosion thickness loss effects in a lap joint requires much more than an area loss correction—topography variations caused by liquids that unevenly corrode surfaces result in significant stress field modifications and secondary bending. Area out calculations alone are not enough to properly account for increases in SIFs and significantly underestimate SIF distributions when a corroded surface is present in a lap skin.
- ‘Sharp’ pits appear to behave mechanically like cracks. The implication is that the topography or geometric description of pits is not as important as was presupposed, and that the most important physical parameter that characterizes pits and their mechanical behavior is the pit depth.
REFERENCES


11. Users Manual for StressCheck, Version 5.0, Engineering Software Research and Development (ESRD), St. Louis, MO.