



# $SU(4)_C \times SU(2)_L \times SU(2)_R$ models with $C$ -parity

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## Abstract

We construct supersymmetric  $SU(4)_C \times SU(2)_L \times SU(2)_R$  models with spontaneously broken left–right symmetry ( $C$ -parity). The minimal supersymmetric standard model (MSSM) can be recovered at low scales by exploiting the missing partner mechanism. The field content is compatible with realistic fermion masses and mixings, proton lifetime is close to or exceeds the current experimental bounds, and supersymmetric hybrid inflation can be implemented to take care of  $C$ -parity domain walls as well as magnetic monopoles, and to realize the observed baryon asymmetry via non-thermal leptogenesis.

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In another soon to be published [1] we constructed supersymmetric  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  models [2,3] in which the scalar (Higgs) sector respects a spontaneously broken discrete left–right symmetry ( $C$ -parity) [4,5]. A variety of symmetry breaking scales were discussed, and it was shown that for TeV scale breaking, a large number of new particles potentially much lighter than the  $SU(2)_R$  charged gauge boson could be found at the LHC. This current article is a continuation of [1] and here we will explore supersymmetric models based on the well known gauge symmetry  $G_{422} \equiv SU(4)_C \times SU(2)_L \times SU(2)_R$  [2]. Being a maximal subgroup of  $Spin(10)$  (also known as  $SO(10)$ ),  $G_{422}$  captures many of its most salient features. For instance,  $G_{422}$  gives rise to electric charge quantization, explains the standard model quantum numbers of each family, and predicts the existence of right handed neutrinos. However, there are also some important differences between  $SO(10)$  and  $G_{422}$  which can be experimentally tested. For instance, in  $G_{422}$  the lightest magnetic monopole carries two quanta of Dirac magnetic charge [6]. (In  $SO(10)$  the lightest monopole carries one quantum of Dirac magnetic charge, unless  $SO(10)$  breaks via  $G_{422}$ .) By the same token in the absence of  $SO(10)$ ,  $G_{422}$  predicts the existence of

$SU(3)$  color singlet states carrying electric charges  $\pm e/2$  [6–8]. Finally, gauge coupling unification and gauge boson mediated proton decay are a characteristic feature of  $SO(10)$  ( $C$ -parity reduces from three to two the number of independent gauge couplings in  $G_{422}$ ). Following [1], we construct  $G_{422}$  based supersymmetric models supplemented by  $C$ -parity. Ref. [9] considered similar models but there are some important differences. For instance, we exploit a missing partner mechanism [10] to realize MSSM at low energies without fine tuning. In our models, among other things, we also can realize supersymmetric inflation to avoid the monopole problem and  $C$ -parity domain walls.

In the simplest  $G_{422}$  models, the MSSM electroweak doublets come from a bidoublet  $H(\mathbf{1}, \mathbf{2}, \mathbf{2})$ , the matter fields are unified into three generations of  $\Psi(\mathbf{4}, \mathbf{2}, \mathbf{1})$ , the antimatter fields into three generations of  $\Psi^c(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$ , and the Yukawa couplings for matter come from  $H\Psi^c\Psi$ . In the simplest such mechanism, we are left with the unwanted relation  $Y^U = Y^D = Y^E = Y^{\text{Dirac}}$  between the Yukawa couplings, which can at best match experimental data for the third generation. We will discuss later how to get around this.

Two of the simplest ways of breaking  $G_{422}$  down to MSSM is to either use  $(\mathbf{4}, \mathbf{1}, \mathbf{2})/(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ , which we will call  $\Phi_R^c$  and  $\Phi_R$  respectively and/or  $(\mathbf{10}, \mathbf{1}, \mathbf{3})/(\bar{\mathbf{10}}, \mathbf{1}, \mathbf{3})$  superfields which we will call  $\Delta_R^c$  and  $\Delta_R$ , respectively. (This is analogous to the

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$\Phi$ 's and  $\Delta$ 's of Ref. [1], except that these fields also contain color triplets.) By invoking  $C$ -parity, we also ensure that these chiral superfields will come with their  $C$ -conjugates, namely  $\Phi_L(\mathbf{4}, \mathbf{2}, \mathbf{1})$ ,  $\Phi_L^c(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$ ,  $\Delta_L(\mathbf{10}, \mathbf{3}, \mathbf{1})$  and  $\Delta_L^c(\bar{\mathbf{10}}, \mathbf{3}, \mathbf{1})$ .

We begin by analyzing a model with only  $\Phi$ 's and no  $\Delta$ 's. Unlike the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  models we considered in [1], the  $\Phi$ 's here decompose into  $SU(3)_C$  singlets as well as triplets. The mechanisms presented there can serve to pair up the color singlet components but will fail to pair up the color triplet components. Let us see why this is the case. We double the number of bidoublets and consider the following terms in the superpotential  $W$ :

$$W \supset \kappa S(\Phi_L^c \Phi_L + \Phi_R^c \Phi_R - M^2), H_1 \Phi_L \Phi_R, H_2 \Phi_L^c \Phi_R^c, \quad (1)$$

where  $S$  is a gauge singlet superfield and  $H_1, H_2$  are the two bidoublets. Following [1], we will find that we get a solution with nonzero VEVs for the  $\Phi_R$ 's but not the  $\Phi_L$ 's and that as a result of this, the up-type Higgs component of the bidoublet  $H_1$  pairs up with the down-type Higgs component of  $\Phi_L$ , while the down-type Higgs component of  $H_2$  pairs up with the up-type Higgs component of  $\Phi_L^c$ . This is the so-called missing partner mechanism [10] because what remains at low energies of the bidoublets is the down-type component of  $H_1$  and the up-type component of  $H_2$ . Now, the up-type Yukawa couplings can come from  $H_2 \Psi^c \Psi$  and the down-type couplings from  $H_1 \Psi^c \Psi$ . Because of this, we no longer have any relation between  $Y^U$  and  $Y^D$ . Of course, it is possible, like what many other authors have done, to merely introduce two Higgs bidoublets and postulate some fine-tuning to get rid of the unwanted fields. However, we wish to avoid fine-tuning in our model, which is why we introduced the missing partner mechanism.

In  $G_{422}$ , the  $\Phi_L$ 's also contain  $(\mathbf{3}, \mathbf{2})_{\frac{1}{3}}$  and  $(\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{3}}$  components (in MSSM notation) and those still remain unpaired. Similarly, some linear combinations of the color singlet components of the  $\Phi_R$ 's pair up with  $S$  and the others become goldstone and sgoldstones. The color triplets  $(\mathbf{3}, \mathbf{1})_{\frac{4}{3}}$  and  $(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}}$  also become goldstones and sgoldstones but the other color triplets  $(\mathbf{3}, \mathbf{1})_{-\frac{2}{3}}$  and  $(\bar{\mathbf{3}}, \mathbf{1})_{\frac{2}{3}}$  do not.

A solution to this proposed in [7,11] is to introduce a  $(\mathbf{6}, \mathbf{1}, \mathbf{1})$  Higgs field and the couplings  $(\mathbf{6}, \mathbf{1}, \mathbf{1})\Phi_R\Phi_R$  and  $(\mathbf{6}, \mathbf{1}, \mathbf{1})\Phi_R^c\Phi_R^c$ . Here, however, we also have  $\Phi_L$  and  $\Phi_L^c$  Higgs fields containing  $(\mathbf{3}, \mathbf{2})_{\frac{1}{3}}$  and  $(\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{3}}$  components. Thus, instead of  $(\mathbf{6}, \mathbf{1}, \mathbf{1})$ , we introduce a  $(\mathbf{15}, \mathbf{1}, \mathbf{1})$  Higgs superfield and add the following terms to  $W$ :

$$W \supset \alpha H_{15}(\Phi_L \Phi_L^c + \Phi_R \Phi_R^c) + M' H_{15}^2. \quad (2)$$

This induces a nonzero VEV along the MSSM singlet direction for  $H_{15}$ .

By varying with respect to  $\Phi_R$  and  $\Phi_R^c$ , we find that the 44-component of  $\kappa S 1_{4 \times 4} + \alpha H_{15}$  has to be zero. This means that the color triplet components of  $\Phi_L$  and  $\Phi_R$  get paired up but the color singlet components do not. The nonzero VEVs are

as follows:

$$\langle \Phi_R \rangle = \langle \Phi_R^c \rangle = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ M & 0 \end{pmatrix}, \quad (3a)$$

$$\langle H_{15} \rangle = \alpha \frac{M^2}{M'} \begin{pmatrix} \frac{1}{8} & 0 & 0 & 0 \\ 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & -\frac{3}{8} \end{pmatrix}, \quad (3b)$$

$$\langle S \rangle = \frac{3}{8} \frac{\alpha^2 M^2}{\kappa M'}, \quad (3c)$$

$$\kappa S 1 + \alpha H_{15} = \alpha^2 \frac{M^2}{M'} \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (3d)$$

The  $(\mathbf{1}, \mathbf{1})_{\pm 2}$  components become the goldstone and the sgoldstones.  $S$  pairs up with a linear combination of  $(\mathbf{1}, \mathbf{1})_0$  and the orthogonal combination form the goldstone multiplet. The  $2 \times 2$  mass matrix for the  $(\mathbf{3}, \mathbf{1})_{\frac{4}{3}}$  and  $(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}}$  components in both  $\Phi_R$  and  $H_{15}$  is given by

$$\begin{pmatrix} \frac{1}{2} \alpha^2 \frac{M^2}{M'} & \alpha M \\ \alpha M & 2M' \end{pmatrix}, \quad (4)$$

which has a zero determinant. The direction with the zero eigenvalue is both the goldstone and sgoldstone direction.

Note that the unpaired set of weak doublets from  $\Phi_L$  and  $\Phi_L^c$  have a mass term which goes as  $\kappa \langle S \rangle 1 + \alpha H_{15}$ . This is where the missing partner mechanism involving two Higgs bidoublets comes in handy, as explained earlier. Because of the  $SU(4)_C$  symmetry, the model as it stands still suffers from the unwanted relation  $Y^D = Y^E$ . Now it turns out that there are a number of ways around this problem but we will only deal with the two most common strategies here; one of them is to introduce the nonrenormalizable coupling  $H_1 H_{15} \Psi^c \Psi / \Lambda$  and the other is to introduce a  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$  Higgs field [12]. Eq. (3b) tells us that the contribution of  $H_1 H_{15} \Psi^c \Psi / \Lambda$  to  $Y^D$  is  $-\frac{1}{3}$  that of the contribution to  $Y^E$ . If  $\Lambda$  corresponds to some value an order of magnitude or so larger than the GUT scale, and if the  $H_1 \Psi_3^c \Psi_3$  coupling for the 3rd generation turns out to be of order unity, then the latter coupling will dominate and we will have the approximate relation  $Y^b = Y^\tau$ . For the 1st and 2nd generations, the contributions from both couplings can be comparable. To get something more predictive, we may insist upon using certain texture ansatzes. For instance, the ansatzes in [13] which lead to realistic fermion masses and mixings can be realized with the field content listed in Table 1, possibly supplemented by  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$ . [The model needs to be augmented with the Majorana coupling  $(\Phi_R^c \Psi^c)^2$  and its  $C$ -conjugate to give superheavy masses to the right-handed neutrinos.]

We would like to make a remark on the MSSM  $\mu$  term. The required coupling is  $\mu H_1 H_2$ , with  $\mu$  on the order of the electroweak scale. In the present scheme, the Giudice–Masiero mechanism [14] is a plausible way to accomplish this.

Next we will look at the issue of proton decay in these  $G_{422}$  models. In order to get proton decay, we need  $SU(3)_C$

Table 1

The chiral superfield content of the  $G_{422}$  model with two bidoublets  $H_1, H_2$ . MSSM is recovered at low scales

Superfield	Representation	Superfield	Representation
$\Psi_i$	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\Psi_i^c$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$
$\Phi_L$	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\Phi_R$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$
$\Phi_L^c$	$(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$	$\Phi_R^c$	$(\mathbf{4}, \mathbf{1}, \mathbf{2})$
$S$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$		
$H_{15}$	$(\mathbf{15}, \mathbf{1}, \mathbf{1})$		
$H_1$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$		
$H_2$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$		

Table 2

The chiral superfield content of the  $G_{422}$  model with the  $\Delta$ 's and a single bidoublet

Superfield	Representation	Superfield	Representation
$\Psi_i$	$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\Psi_i^c$	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$
$\Delta_L$	$(\mathbf{10}, \mathbf{3}, \mathbf{1})$	$\Delta_R$	$(\bar{\mathbf{10}}, \mathbf{1}, \mathbf{3})$
$\Delta_L^c$	$(\bar{\mathbf{10}}, \mathbf{3}, \mathbf{1})$	$\Delta_R^c$	$(\mathbf{10}, \mathbf{1}, \mathbf{3})$
$S$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$		
$H_{15}$	$(\mathbf{15}, \mathbf{1}, \mathbf{1})$		
$T_L$	$(\mathbf{1}, \mathbf{3}, \mathbf{1})$	$T_R$	$(\mathbf{1}, \mathbf{1}, \mathbf{3})$
$B$	$(\mathbf{1}, \mathbf{3}, \mathbf{3})$		
$H$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$		

Table 3

$X$ -charge of the various superfields

$\Psi$	$\Psi^c$	$H_1$	$H_2$	$\Phi_L$	$\Phi_L^c$	$\Phi_R$	$\Phi_R^c$	$S$	$H_{15}$
1	-1	0	0	1	-1	-1	1	0	0

operators like  $\mathbf{333}$  or  $\overline{\mathbf{333}}$ . This would correspond to  $SU(4)_C$  operators like  $\langle \mathbf{4} \mathbf{444}, \langle \bar{\mathbf{4}} \mathbf{444}, \mathbf{644}$  or  $\overline{\mathbf{644}}$ . So far, we have not included any couplings of this nature. (Even the Majorana term  $\langle \Phi_R^c \rangle \Phi_R^c \Psi^c \Psi^c / \Lambda$  is not of this form.) In principle, these couplings can be forbidden by introducing a global symmetry  $U(1)_X$  which commutes with all the gauge symmetries. The  $X$ -charge assignments are given in Table 3. The proton in this case turns out to be essentially stable.

In the absence of  $U(1)_X$ , proton decay can occur via dimension five operators along the lines discussed in [13] and according to which, the dominant decay modes are  $\bar{\nu} K^+$  and  $\bar{\nu} \pi^+$  with lifetime  $\sim 10^{34-35}$  yrs.

Next, we consider an alternative model without the missing partner mechanism. Recall that the missing partner mechanism is needed for two things; to give masses to the color singlet components of  $\Phi_L$  and to break the  $Y^U = Y^D$  relation. The former can be taken care of by the nonrenormalizable coupling  $\Phi_L^c \Phi_L \Phi_R^c \Phi_R / \Lambda$  once the  $\Phi_R$ 's get a nonzero VEV. Introducing an  $SU(2)_R$  Higgs triplet  $T_R(\mathbf{1}, \mathbf{1}, \mathbf{3})$  together with its  $C$ -conjugate, we can arrange for  $T_R$  to get a nonzero VEV by employing the couplings  $T_R^2$  and  $T_R \Phi_R^c \Phi_R$  as well as their  $C$ -conjugates. Without the missing partner mechanism, we only have one bidoublet  $H$  which contains both  $H_u$  and  $H_d$ . The up–down relation between the Yukawa couplings can be broken by the nonrenormalizable coupling  $H \langle T_R \rangle \Psi^c \Psi / \Lambda$ . However, as the cutoff scale  $\Lambda$  will typically be an order of magnitude or so larger than the  $SU(2)_R$  breaking scale  $M$ , the splitting be-

tween the Yukawa couplings will only be significant for the first and second generations and not for the third. Thus, we will still have the approximate relation  $Y^i \simeq Y^b$ .

Let us note that we may also break the  $G_{422}$  symmetry with  $\Delta_L(\mathbf{10}, \mathbf{3}, \mathbf{1})$ ,  $\Delta_R(\bar{\mathbf{10}}, \mathbf{1}, \mathbf{3})$ ,  $\Delta_L^c(\bar{\mathbf{10}}, \mathbf{3}, \mathbf{1})$  and  $\Delta_R^c(\mathbf{10}, \mathbf{1}, \mathbf{3})$  instead of the  $\Phi$ 's [15] (see Table 2). (We note that the  $\mathbb{Z}_2$  matter parity of MSSM is automatically embedded within  $G_{422}$  in the absence of the  $\Phi$  fields.) The Majorana coupling will now be  $\Delta_R^c \Psi^c \Psi^c$ , and its  $C$ -conjugate. Under the decomposition from  $SU(4)_C$  to  $SU(3)_C$ ,  $\mathbf{10} \rightarrow \mathbf{6} \oplus \bar{\mathbf{3}} \oplus \mathbf{1}$  and  $\bar{\mathbf{10}} \rightarrow \bar{\mathbf{6}} \oplus \mathbf{3} \oplus \mathbf{1}$ . If we only have a  $(\mathbf{15}, \mathbf{1}, \mathbf{1})$  Higgs field and a singlet  $S$  and we use the same mechanism as our previous model, we find that the color sextet and triplet components of the  $\Delta$ 's will get nonzero masses because the Clebsch–Gordon contributions from  $\langle S \rangle$  and  $\langle H_{15} \rangle$  do not cancel but the mass contribution to the color singlet components cancel. (This is a consequence of setting the  $F$ -terms to zero.) This includes the  $(\mathbf{1}, \mathbf{3})_{\pm 2}$  components of  $\Delta_L$  and  $\Delta_L^c$ . Since the  $\Delta_R$ 's are  $SU(2)_R$  triplets, the color singlet components decompose into three once  $G_{422}$  is broken. One linear combination of the  $(\mathbf{1}, \mathbf{1})_0$  components pairs up with  $S$  and the other linear combination becomes goldstone and sgoldstone bosons and goldstinos. The  $(\mathbf{1}, \mathbf{1})_{\pm 2}$  components also become goldstone and sgoldstone bosons and goldstinos. This leaves us with the  $(\mathbf{1}, \mathbf{1})_{\pm 4}$  components of  $\Delta_R$  and  $\Delta_R^c$ . To realize MSSM at low scales, we introduce the Higgs fields  $(\mathbf{1}, \mathbf{3}, \mathbf{1})$ ,  $(\mathbf{1}, \mathbf{1}, \mathbf{3})$  and  $(\mathbf{1}, \mathbf{3}, \mathbf{3})$  and the renormalizable couplings

$$\begin{aligned} & (\mathbf{1}, \mathbf{3}, \mathbf{1})^2, \quad (\mathbf{1}, \mathbf{1}, \mathbf{3})^2, \quad (\mathbf{1}, \mathbf{3}, \mathbf{1}) \Delta_L^c \Delta_L, \\ & (\mathbf{1}, \mathbf{1}, \mathbf{3}) \Delta_R^c \Delta_R, \quad (\mathbf{1}, \mathbf{3}, \mathbf{3})^2, \\ & (\mathbf{1}, \mathbf{3}, \mathbf{3}) \Delta_L \Delta_R, \quad (\mathbf{1}, \mathbf{3}, \mathbf{3}) \Delta_L^c \Delta_R^c. \end{aligned} \quad (5)$$

The  $(\mathbf{1}, \mathbf{1}, \mathbf{3})$  field acquires an induced VEV from the  $(\mathbf{1}, \mathbf{1}, \mathbf{3}) \langle \Delta_R^c \rangle \langle \Delta_R \rangle$  coupling and this pairs up the  $(\mathbf{1}, \mathbf{1})_{\pm 4}$  components via  $\langle (\mathbf{1}, \mathbf{1}, \mathbf{3}) \rangle \Delta_R^c \Delta_R$ . The  $(\mathbf{1}, \mathbf{3})_{-2}$  component is paired up via  $(\mathbf{1}, \mathbf{3}, \mathbf{3}) \Delta_L \langle \Delta_R \rangle$  and the  $(\mathbf{1}, \mathbf{3})_2$  component is paired up via  $(\mathbf{1}, \mathbf{3}, \mathbf{3}) \Delta_L^c \langle \Delta_R^c \rangle$ . The  $(\mathbf{1}, \mathbf{3})_0$  component of  $(\mathbf{1}, \mathbf{3}, \mathbf{3})$  is self-paired. (The  $(\mathbf{1}, \mathbf{3})_2$  and  $(\mathbf{1}, \mathbf{3})_{-2}$  components of  $B$  also pair up and so, what we really have is a chain of pairings in which all the components become massive.)

The up–down Yukawa relation is broken not by the missing partner mechanism but by the nonrenormalizable coupling  $H \langle (\mathbf{1}, \mathbf{1}, \mathbf{3}) \rangle \Psi^c \Psi / \Lambda$ . (The  $C$ -conjugate of this coupling is also included.) For the same reason as before, this difference is suppressed by  $\langle (\mathbf{1}, \mathbf{1}, \mathbf{3}) \rangle / \Lambda$  and so,  $Y^i \simeq Y^b$  for the third generation. Let us summarize by putting together all the terms in the superpotential:

$$\begin{aligned} W \supset & S(\Delta_L^c \Delta_L + \Delta_R^c \Delta_R - M^2), \quad H_{15}(\Delta_L^c \Delta_L + \Delta_R^c \Delta_R), \\ & H_{15}^2, \quad H \Psi^c \Psi, \quad (H T_L \Psi^c \Psi + H T_R \Psi^c \Psi) / \Lambda, \\ & H H_{15} \Psi^c \Psi / \Lambda, \quad \Delta_L^c \Psi \Psi + \Delta_R^c \Psi^c \Psi^c, \\ & T_L^2 + T_R^2, \quad T_L \Delta_L^c \Delta_L + T_R \Delta_R^c \Delta_R, \\ & B^2, \quad B \Delta_L \Delta_R + B \Delta_L^c \Delta_R^c. \end{aligned} \quad (6)$$

One consequence of  $C$ -parity is the existence of  $\mathbb{Z}_2$  domain walls once it is spontaneously broken. The scale at which this occurs happens to be same as the  $G_{422}$  breaking scale. Such

domain walls can give rise to cosmological problems unless they are inflated away. In addition, when  $G_{422}$  breaks down to MSSM, magnetic monopoles carrying two quanta of Dirac magnetic charge [6] can be generated. Astrophysical and cosmological bounds on such monopoles are fairly stringent and the standard solution is to inflate them away. One way to do this in our case is to invoke shifted hybrid inflation [16] where a nonrenormalizable term ( $S[(\Phi_L^c \Phi_L)^2 + (\Phi_R^c \Phi_R)^2]$ ) is added to the superpotential. With such a suitably altered model, it is possible to start inflation with a trajectory where  $C$ -parity as well as  $G_{422}$  are already spontaneously broken. In such a scenario, domains do not form once inflation ends and neither do monopoles. During inflation itself, the inflationary trajectory is identical to that analyzed in [16] with  $\Phi_L = \Phi_L^c = 0$  throughout. Although the postinflationary trajectory will be different, the end result is that both  $C$ -domain walls and monopoles are eliminated.

The end of inflation is followed by the decay of the inflaton fields  $\Phi_R$ ,  $\Phi_R^c$  and  $S$  into right handed neutrinos ( $\nu^c$ ) and sneutrinos ( $\tilde{\nu}^c$ ). As discussed in [17,18], following [19], the subsequent out of equilibrium decay of  $\nu^c$  and  $\tilde{\nu}^c$  generates lepton asymmetry, which is then partially converted to the observed baryon asymmetry by the electroweak sphalerons.

Note that hybrid inflation requires that the  $G_{422}$  breaking scale is comparable to  $M_{\text{GUT}} \sim 10^{16}$  GeV. If  $G_{422}$  breaks at significantly lower (such as intermediate) scales, an alternative scenario for suppressing monopoles should be employed (see, for instance, [20]).

In conclusion, we have constructed realistic supersymmetric  $G_{422}$  models in which the scalar (Higgs) sector respects a discrete left–right symmetry ( $C$ -parity) which is spontaneously broken at the same scale as the  $SU(2)_R$  gauge symmetry. We have shown how the MSSM is recovered without fine tuning at low scales. The scalar fields we employ enable us to reproduce, following [13], the observed fermion masses and mixings, implement a missing partner mechanism [1], and realize inflation followed by non-thermal leptogenesis.

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