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Research on uncertainty in measurement assisted alignment in aircraft assembly

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Abstract Operations in assembling and joining large size aircraft components are changed to novel digital and flexible ways by digital measurement assisted alignment. Positions and orientations (P&O) of aligned components are critical characters which assure geometrical positions and relationships of those components. Therefore, evaluating the P&O of a component is considered necessary and critical for ensuring accuracy in aircraft assembly. Uncertainty of position and orientation (U-P&O), as a part of the evaluating result of P&O, needs to be given for ensuring the integrity and credibility of the result; furthermore, U-P&O is necessary for error tracing and quality evaluating of measurement assisted aircraft assembly. However, current research mainly focuses on the process integration of measurement with assembly, and usually ignores the uncertainty of measured result and its influence on quality evaluation. This paper focuses on the expression, analysis, and application of U-P&O in measurement assisted alignment. The geometrical and algebraical connotations of U-P&O are presented. Then, an analytical algorithm for evaluating the multi-dimensional U-P&O is given, and the effect factors and characteristics of U-P&O are discussed. Finally, U-P&O is used to evaluate alignment in aircraft assembly for quality evaluating and improving. Cases are introduced with the methodology.

1. Introduction

Wide employments of digital assembly technologies and large scale metrologies provide novel digital and flexible approaches for aircraft assembly to improve quality and shorten leading-time; the changes of operations in assembling are mainly based on flexible adjusting devices and digital measurement instruments.[1–3] The large size components joining process, as a key stage of aircraft assembly, has been changed from the traditional process based on manual fixtures and operations to automatic alignment and connection in a digital way, which significantly improves aligning precision and efficiency.[4–8] In measurement assisted aircraft components alignment, digital measurement instruments, such as laser tracker, iGPS (indoor-GPS), and photogrammetry, are used to collect data of components' positions, orientations, and interfaces, and then an adjustment plan is carried out by a data processing system, to drive flexible adjusting devices (such as the Electronic Mating Alignment System, automated positioning systems based

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on POGOs, parallel adjusting platforms) for automatically adjusting positions and orientations of large size components.

Application of digital and automatic alignment technologies relies on the integration of digital measurement with an assembly process,[9–11] which is not a simple linkage of measuring and assembling, but more essentially the fuse of measured data, design data, and fixture data. In order to unify quality requirements, fixture data, and measurement demands, the concept of key measurement characteristics (KMCs),[12] as a set of geometrical datum and features, is introduced. KMCs in aircraft assembly provide unified carriers for storing and analyzing key assembly data from design, measurement, and other sources.

Based on KMCs, positions and orientations of components in measurement assisted assembly are measured and analyzed to guide adjustment and alignment. For example, assembly datum is defined as a set of geometrical features with optical target points (OTPs) surrounding the assembled components, coordinates of which in the measurement coordinate system are collected before alignment, fitted with their ideal positions in the product model to calculate the position and orientation of the assembly datum, and then the transformation from the measurement coordinate system to the global coordinate system is derived. Like assembly datum, positions and orientations of components and flexible locators are also measured and parsed to their geometrical status in the global coordinate system, in order to direct reasonable adjustment of their positions and orientations during sequential processes.

In measurement assisted alignment, measured objects are changed from coordinates of points and planar dimensions to positions and orientations; so it is critical to develop and apply new methodologies and approaches of data collecting and processing as well as components adjusting approaches.[13,14] Although a lot of methodologies and approaches have been studied for measuring, analyzing, and adjusting positions and orientations of aligned components,[15–17] there is still a lack of attention to the measurement uncertainty of position and orientation (U-P&O), which is as important as the measured value of position and orientation.

Measurement uncertainty is a non-negative parameter characterizing the dispersion of values attributed to a quantity, which has a probabilistic basis and reflects incomplete knowledge of the quantity. As a part of the measurement result of position and orientation, U-P&O ensures the integrity and credibility of measured value of position and orientation made out by a measuring process. Without the analysis of U-P&O, it would be difficult to trace the accumulated error in the data transfer process, which begins with assembly design and continues through adjustment, alignment, and verification; furthermore, the quality evaluation of alignment may not be correct without considering the uncertainty of measurement result. Position and orientation are multi-dimensional quantity, the expression of which is not given out in the *Guide to the Expression of Uncertainty in Measurement* (GUM)[18]; meanwhile, P&O and U-P&O of aligned components are specific and process-related quantities in measurement assisted aircrafts, and thus the studies on the expression, analysis, and application of U-P&O are necessary.

In this paper, mathematical definition and connotations of U-P&O are presented in the Section 2. Then, an analytical algorithm for measuring and calculating the multi-dimensional U-P&O is conducted in Section 3, with the discussion of its effect factors and data independency characteristic in measurement assisted aircraft assembly. In Section 4, a novel method of assembly quality evaluation based on uncertainty of position and orientation is proposed to present an application of U-P&O in measurement assisted assembly. Finally, future research on uncertainty of position and orientation is described in conclusions.

2. Mathematical definitions

2.1. Position and orientation

Position and orientation are key geometric features in defining the relationships between related components in aircraft assembly, especially in digital assembly processes. In order to conduct the definition and expression of uncertainty of position and orientation, it is necessary to define position and orientation from mathematical perspective.

Position and orientation describe the geometrical status of a rigid body in the global coordinate system (GCS), which should be predefined before digital measurement. As shown in Fig. 1(a), $O-XYZ$ represents the global coordinate system of a three-dimensional environment, then the point O' on the rigid body is selected as the original point to build a local coordinate system (LCS) represented by $O'-X'Y'Z'$, and finally the position and orientation of the rigid body relative to the GCS can be expressed by a six-dimensional variable, which includes the amount of rotation and displacement of the LCS relative to the GCS:

$$\boldsymbol{T} = \{\alpha, \beta, \gamma, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z\} \tag{1}$$

where α, β, γ are the rotation angles of each axis of the LCS; $\mathrm{d}x$, $\mathrm{d}y$, $\mathrm{d}z$ are the displacements of the original point of the LCS. Thus, the position and orientation of the rigid body is essentially the transformation from its LCS to the GCS, and its value depends on the selection of the LCS and the GCS.

In order to analytically solve position and orientation, it is necessary to conduct its mathematical expression. In a linear space, the transformation of coordinate systems can be expressed by a matrix: as shown in Fig. 1(b), the vectors directing from the original point O to any point in the GCS construct a linear space S, and the vectors directing from the original point O' to any point in the LCS construct another linear space S'. P_i is one of the points on the rigid body, and then the position of P_i in the GCS can be expressed by a vector of linear space as:

$$\boldsymbol{P}_i^{\mathrm{G}} = \left[\boldsymbol{p}_i^{\mathrm{G}}, 1\right]^{\mathrm{T}}, \quad \boldsymbol{P}_i^{\mathrm{G}} \in S,$$

where $\boldsymbol{p}_i^{\mathrm{G}} = (x_i^{\mathrm{G}}, y_i^{\mathrm{G}}, z_i^{\mathrm{G}})$ is the coordinate of the point P_i in the GCS.

The position of P_i in the LCS of the rigid body can be expressed by another vector of linear space as:

$$\boldsymbol{P}_i^{\mathrm{L}} = [\boldsymbol{p}_i^{\mathrm{L}}, 1]^{\mathrm{T}}, \quad \boldsymbol{P}_i^{\mathrm{L}} \in S',$$

where $\boldsymbol{p}_i^{\mathrm{L}} = (x_i^{\mathrm{L}}, y_i^{\mathrm{L}}, z_i^{\mathrm{L}})$ is the coordinate of the point P_i in the LCS.The two vectors are different descriptions of the same point, so there is a linear transformation, which enables that:

$$\boldsymbol{T} \bullet \boldsymbol{P}_i^{\mathrm{L}} = \boldsymbol{P}_i^{\mathrm{G}} \tag{2}$$

where $\boldsymbol{T}$ is a matrix with four rows and four lines.

Eqs. (1) and (2) display the geometrical and algebraical connotations of position and orientation of the rigid body, and the relation between them can be derived as:

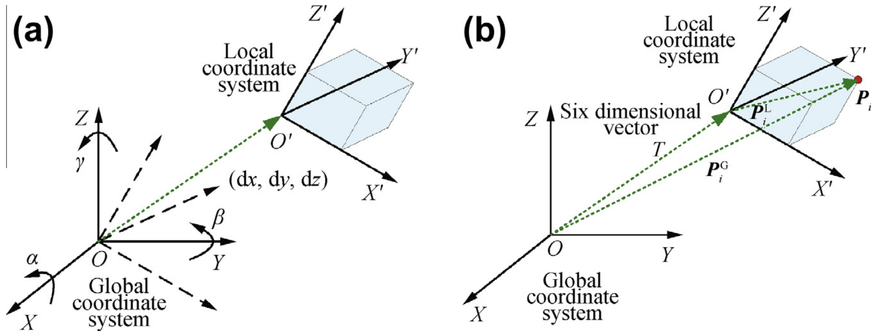

Fig. 1 Geometrical and algebraical connotations of position and orientation of a rigid body.

$$\boldsymbol{T}=\begin{bmatrix}\boldsymbol{R}_{3\times3} & \boldsymbol{M}_{3\times1}\\ \boldsymbol{0}_{1\times3} & \boldsymbol{1}\end{bmatrix} \tag{3}$$

where $\boldsymbol{R}_{3\times3}=\boldsymbol{R}(\alpha)\boldsymbol{R}(\beta)\boldsymbol{R}(\gamma)$ and $\boldsymbol{M}_{3\times1}=[\mathrm{d}x,\mathrm{d}y,\mathrm{d}z]^{\mathrm{T}}$. $\boldsymbol{R}_{3\times3}$ is called the rotation matrix, and $\boldsymbol{M}_{3\times1}$ is called the displacement vector.

Eq. (3) unifies the geometrical and algebraical connotations of position and orientation.

$$\boldsymbol{U}_T=\begin{bmatrix}\sigma_\alpha^2 & \mathrm{cov}(\alpha,\beta) & \mathrm{cov}(\alpha,\gamma) & \mathrm{cov}(\alpha,\mathrm{d}x) & \mathrm{cov}(\alpha,\mathrm{d}y) & \mathrm{cov}(\alpha,\mathrm{d}z)\\ \mathrm{cov}(\alpha,\beta) & \sigma_\beta^2 & \mathrm{cov}(\beta,\gamma) & \mathrm{cov}(\beta,\mathrm{d}x) & \mathrm{cov}(\beta,\mathrm{d}y) & \mathrm{cov}(\beta,\mathrm{d}z)\\ \mathrm{cov}(\alpha,\gamma) & \mathrm{cov}(\beta,\gamma) & \sigma_\gamma^2 & \mathrm{cov}(\gamma,\mathrm{d}x) & \mathrm{cov}(\gamma,\mathrm{d}y) & \mathrm{cov}(\gamma,\mathrm{d}z)\\ \mathrm{cov}(\alpha,\mathrm{d}x) & \mathrm{cov}(\beta,\mathrm{d}x) & \mathrm{cov}(\gamma,\mathrm{d}x) & \sigma_{\mathrm{d}x}^2 & \mathrm{cov}(\mathrm{d}x,\mathrm{d}y) & \mathrm{cov}(\mathrm{d}x,\mathrm{d}z)\\ \mathrm{cov}(\alpha,\mathrm{d}y) & \mathrm{cov}(\beta,\mathrm{d}y) & \mathrm{cov}(\gamma,\mathrm{d}y) & \mathrm{cov}(\mathrm{d}x,\mathrm{d}y) & \sigma_{\mathrm{d}y}^2 & \mathrm{cov}(\mathrm{d}y,\mathrm{d}z)\\ \mathrm{cov}(\alpha,\mathrm{d}z) & \mathrm{cov}(\beta,\mathrm{d}z) & \mathrm{cov}(\gamma,\mathrm{d}z) & \mathrm{cov}(\mathrm{d}x,\mathrm{d}z) & \mathrm{cov}(\mathrm{d}y,\mathrm{d}z) & \sigma_{\mathrm{d}z}^2\end{bmatrix} \tag{4}$$

2.2. Uncertainty of position and orientation

In large size aircraft assembly, uncertainty of position and orientation is a parameter describing the fluctuant scope of position and orientation of datum, components, and locators in the global coordinate system. $\boldsymbol{U}_T$ is used to mathematically express the value of U-P&O, which is valued by the variance of the error between the actual and ideal values of position and orientation. $\boldsymbol{T}^{\mathrm{d}}$ is used to represent the actual value of position and orientation, and $\boldsymbol{T}$ is the ideal position and orientation. If the distribution of $\boldsymbol{T}^{\mathrm{d}}$ is normal, then $\boldsymbol{T}^{\mathrm{d}}\sim U(\boldsymbol{T},\boldsymbol{U}_T)$.

Reference to its geometrical form, the value of position and orientation contains two aspects of information: the position of the original point of the LCS in the GCS, and the rotation of each axis of the LCS relative to the GCS; similarly, as shown in Fig. 2, uncertainty of position and orientation contains the uncertainty of the original point position and the uncertainty of directions of each axis, both of which are not independent. Apparently, it will not make sense to discuss uncertainty of position and orientation without consideration of the object's geometrical features being measured; assembly datum, components, and locators are rigid bodies with borders in the three-dimensional space, and their geometrical interface features randomly exist in a three-dimensional area which is determined by the uncertainty of their positions and orientations, as shown by the cuboids in Fig. 2, and has a max boundary and a min boundary, which form the envelop of their interfaces.

Reference to the algebraical form as shown by Eq. (1), position and orientation is a six-dimensional variable which includes the amount of rotation and displacement of the LCS relative to the GCS; therefore, uncertainty of position and orientation can be easily expressed by a six-dimensional vector as follows:

$$\boldsymbol{U}_T=\{\sigma_\alpha,\sigma_\beta,\sigma_\gamma,\sigma_{\mathrm{d}x},\sigma_{\mathrm{d}y},\sigma_{\mathrm{d}z}\}$$

Although the six-dimensional vector can be used to reflect the fluctuation of position and orientation, it cannot present the correlation of each dimension; thus a six-dimensional covariance matrix is derived as follows:

3. Analysis of U-P&O

3.1. Analytical algorithm

Usually, through collecting actual positions of three or more OTPs on the surface of a measured object, its position and orientation in the GCS can be calculated based on Eq. (2). In order to resolve the uncertainty of position and orientation based on measurement data, the analytical relation between $\boldsymbol{U_T}$ (the mathematical expression of uncertainty of position and orientation) and $\boldsymbol{U}_{\boldsymbol{P}_i^{\mathrm{G}}}$ (the mathematical expression of OTPs' position uncertainty) is conducted as follows.

According to Eqs. (2) and (3), the relation between OTPs' position and matrix of position and orientation can be expressed as a function $g(\bullet)$ as follows:

$$\boldsymbol{P}_i^{\mathrm{G}}=g\left(\boldsymbol{T},\boldsymbol{P}_i^{\mathrm{L}}\right) \tag{5}$$

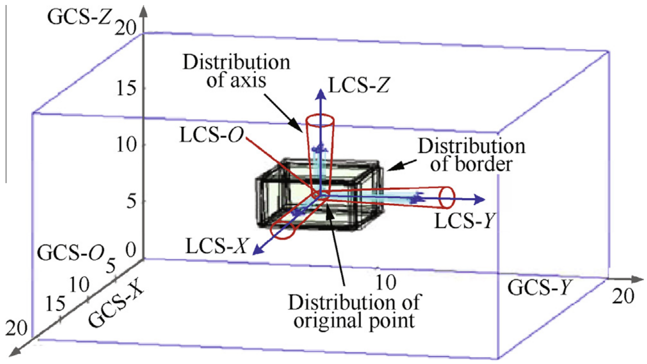

Fig. 2 Geometrical form of U-P&O.

Then, based on Eq. (3), the matrix of position and orientation in Eq. (5) is replaced by a six-dimensional variable:

$$\boldsymbol{P}_i^{\mathrm{G}} = g\left(\boldsymbol{h}, \boldsymbol{P}_i^{\mathrm{L}}\right) \tag{6}$$

where $\boldsymbol{h} = (\alpha, \beta, \gamma, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)^{\mathrm{T}}$.

If $\boldsymbol{h}_{\mathrm{est}}$ is a solution of Eq. (6), expand the function $g(\bullet)$ based on first-order Taylor series at $(\boldsymbol{h}_{\mathrm{est}}, \boldsymbol{P}_i^{\mathrm{L}})$:

$$\boldsymbol{P}_i^{\mathrm{G}} + \Delta\boldsymbol{P}_i^{\mathrm{G}} = g(\boldsymbol{h}_{\mathrm{est}} + \Delta\boldsymbol{h}, \boldsymbol{P}_i^{\mathrm{L}}) \approx g(\boldsymbol{h}_{\mathrm{est}}, \boldsymbol{P}_i^{\mathrm{L}}) + \left[\frac{\partial g}{\partial h}\right]^{\mathrm{T}}_{\boldsymbol{h}_{\mathrm{est}}, \boldsymbol{P}_i^{\mathrm{L}}} \Delta\boldsymbol{h}$$

Therefore, $\Delta\boldsymbol{P}_i^{\mathrm{G}} = \left[\frac{\partial g}{\partial h}\right]^{\mathrm{T}}_{\boldsymbol{h}_{\mathrm{est}}, \boldsymbol{P}_i^{\mathrm{L}}} \Delta\boldsymbol{h} = \boldsymbol{J}_i \Delta\boldsymbol{h}$, where $\boldsymbol{J}_i$ is the Jacobin matrix of the function $g(\bullet)$.

If n is the number of OTPs and $n \geqslant 3$, then

$$\begin{bmatrix} \Delta\boldsymbol{P}_1^{\mathrm{G}} \\ \vdots \\ \Delta\boldsymbol{P}_n^{\mathrm{G}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}_1 \\ \vdots \\ \boldsymbol{J}_n \end{bmatrix} \Delta\boldsymbol{h} \Rightarrow \Delta\boldsymbol{P}^{\mathrm{G}} = \boldsymbol{J}\Delta\boldsymbol{h} \tag{7}$$

Therefore, $\Delta\boldsymbol{h} = (\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}\Delta\boldsymbol{P}^{\mathrm{G}}$.

Finally, the covariance matrix of $\boldsymbol{h}$ can be derived as:

$$\begin{aligned} \mathrm{cov}(\boldsymbol{h}) &= E(\Delta\boldsymbol{h}\Delta\boldsymbol{h}^{\mathrm{T}}) \\ &= E((\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}}\Delta\boldsymbol{P}^{G}(\Delta\boldsymbol{P}^{G})^{\mathrm{T}}((\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}})^{\mathrm{T}}) \\ &= (\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}} \bullet E(\Delta\boldsymbol{P}^{G}(\Delta\boldsymbol{P}^{G})^{\mathrm{T}}) \bullet ((\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}})^{\mathrm{T}} \\ &= (\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}} \bullet \boldsymbol{U}_{P^G} \bullet ((\boldsymbol{J}^{\mathrm{T}}\boldsymbol{J})^{-1}\boldsymbol{J}^{\mathrm{T}})^{\mathrm{T}} \\ &= \boldsymbol{U}_T \end{aligned} \tag{8}$$

where $\boldsymbol{U}_{P^G} = \begin{bmatrix} \boldsymbol{U}_{P_1^G} & & \\ & \ddots & \\ & & \boldsymbol{U}_{P_n^G} \end{bmatrix}$.

The actual position and orientation of datum, components, and locators depends on the selection of the LCS and the deployment of OTPs on those objects; however, the transfer matrix between position uncertainty and U-P&O given by Eq. (8) embeds the influences of the LCS and OTPs' deployment on position and orientation measurement, which enables the calculation of U-P&O being independent from the measuring process and only relying on OTPs' position uncertainty.

3.2. Effect factors

According to its phenomenon, uncertainty of position and orientation can be divided into two sections: (i) variation of position and orientation generated by setting up and locating; and (ii) measurement uncertainty caused by OTPs' coordinates collection. As shown in Fig. 3, in setting up and locating of assembled components, influences of manufacturing variation, movement status, and structure deformation caused by gravity and assembly force are the main sources of components' position and orientation variation; and then, the positioning accuracy, stability, and datum variation of fixture which lead to the position variation of components are also important factors that need to be controlled. Meanwhile, some factors of assembly environment, especially the vibration of ground, also contribute to the variation of the actual position and orientation of components.

During the processes of coordinates collection and position and orientation fitting, precision characteristics, calibration,

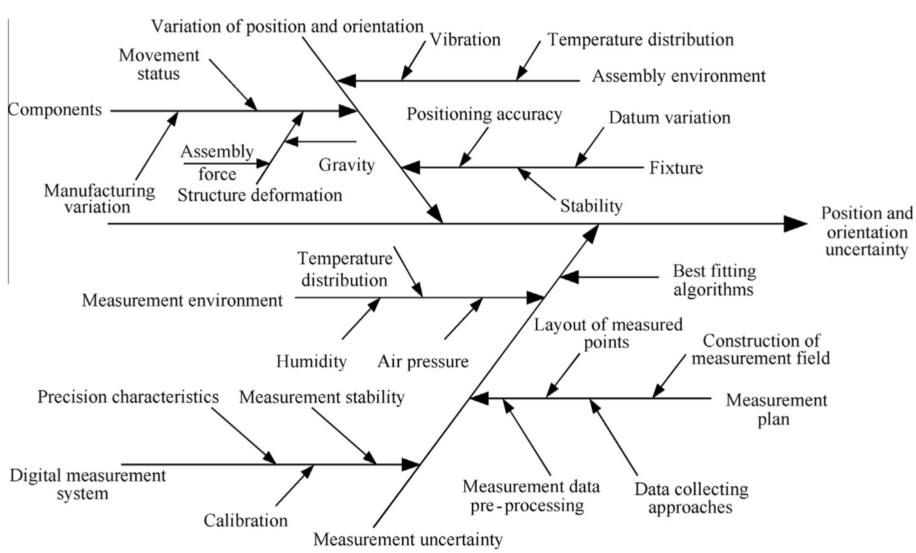

Fig. 3 Effect factors of U-P&O in digital measurement assisted aircraft assembly.

and measurement stability of the digital measurement system will significantly influence the uncertainty of coordinates measurement results[19]; even for the same digital measurement system, different measurement plans, which include construction of measurement field, data collecting approaches, measurement data pre-processing, and other aspects, will lead to different results; furthermore, the influences of assembly environment, such as humidity, air pressure, and temperature distribute, on measurement uncertainty are also significant.

The digital measurement system used in large components alignment is usually highly precise and well calibrated, and then the uncertainty of position and orientation is mainly determined by location uncertainty of the measured object; in order to analyze the location uncertainty independently and improve assembly process and assembly environment, a special measurement plan for collecting data and separating location uncertainty from measurement uncertainty contained in U-P&O are discussed as follows.

Measurement data of OTPs' coordinates is the only original data for fitting position and orientation and analyzing its uncertainty. Supposing there are n OTPs and the nominal, actual, and measured values of the position of the ith OTP P_i are expressed as $\boldsymbol{p}_i^{\mathrm{G}}, \boldsymbol{p}_i^{\mathrm{d}}$, and $\boldsymbol{p}_i^{\mathrm{m}}$, then the relation between them can be derived as:

$$\boldsymbol{p}_i^{\mathrm{m}} = \boldsymbol{p}_i^{\mathrm{d}} + \Delta\boldsymbol{p}_i^{\mathrm{m}} = \boldsymbol{p}_i^{\mathrm{G}} + \Delta\boldsymbol{p}_i^{\mathrm{d}} + \Delta\boldsymbol{p}_i^{\mathrm{m}} \tag{9}$$

where $\Delta\boldsymbol{P}_i^{\mathrm{d}}$ is the deviation between the actual and nominal values, which is supposed to be normal distribution as $\Delta\boldsymbol{P}_i^{\mathrm{d}} \sim U(0, \sigma_{\mathrm{d}}^2)$; $\Delta\boldsymbol{P}_i^{\mathrm{m}}$ is the deviation between the measured and actual values, which is also supposed to be normal distribution as $\Delta\boldsymbol{P}_i^{\mathrm{m}} \sim U(0, \sigma_{\mathrm{m}}^2)$.

Therefore, the measured value of the position of the OTP P_i in the GCS normally distributes as

$$\boldsymbol{p}_i^{\mathrm{m}} \sim U(\boldsymbol{p}_i^{\mathrm{G}}, \sigma_{\mathrm{d}}^2 + \sigma_{\mathrm{m}}^2)$$

When the value of σ_{d}^2 is 7.5 or more times of the value of σ_{m}^2, the PCA method can be used to separate two kinds of variance of normal distributed sample.[20]

The measurement plan for collecting data is:

(1) Measurement system selection: multi-station measurement net consisted of three or more laser trackers or photogrammetry is a feasible choice for implementing online and parallel measurement; furthermore, photogrammetry is more feasible and less costly for being used in a real assembly process.
(2) Sampling strategy: during t minutes (t is a positive integer), collect two coordinate data of each OTP every thirty seconds, and express each data as $\boldsymbol{p}_{1,j,k}^{\mathrm{m}}$, in which j is the serial number of measuring, $j = 1, 2, \cdots, 2t$, and k is the number of data collected at the jth measuring, $k = 1, 2$. Therefore, there will be $4t \times n$ coordinates data during t minutes being collected, which are the original data of uncertainty analysis.

During measurement assisted aircraft components alignment, assembly datum is usually setup on the ground of the assembly cite, which is more easily affected by assembly environment. Unfortunately, position and orientation of assembly datum is the original data of sequential data collecting and processing; therefore, uncertainty source analysis and separation is critical for ensuring the stability of assembly datum and precision of alignment.

3.3. Computational example

Uncertainty is a parameter that reflects the fluctuating range of a quantity, and thus its value doesn't change with the expressing form of the quantity. In the case of uncertainty of position and orientation, if the assembly environment, measurement system, measurement approach, and assembly process have been determined, the value of U-P&O will not change with the selection of the LCS and OTPs. This characteristic is termed data independency of U-P&O. Considering its geometrical connotation, as illustrated in Fig. 2, the uncertainty of position and orientation essentially describes the fluctuant scope of position and orientation of a rigid object in the assembly space. The scope is a connatural characteristic generated by the effect factors of assembly environment, measurement system, measurement approach, and alignment process. In order to expound and verify the independency of U-P&O, a measurement experiment of position and orientation of a wing in aircraft wing-fuselage alignment is carried out; in addition, the computational example verifies the analytical algorithm of U-P&O.

During aircraft wing-fuselage alignment, the measurement of position and orientation of the wing is the basis of adjusting trajectory planning. Two experiments of wing position and orientation measurement were carried out, as shown in Fig. 4. The experiments included three steps:

(1) Define the local coordinate system of the wing and select the corresponding OTPs.

An OTP on the structure of a component is not only a measured point, but also a hole or another feature for fixing the optical target. The selection of OTPs for position and orientation measurement largely follows the following three basic principles: (a) the selected OTPs should have the attributes of visibility, namely, the optical path between the OTPs and the measurement device is not affected or blocked by other factors; (b) the ideal place where the OTPs are to be located should be on the areas that show high structure rigidity, which is expected to diminish the influence resulting from the structure deformation, vibrations. and other potential environmental factors; (c) the existing holes together with other features should be well utilized when possible, which facilitates placing the optical target, whilst reducing the effect of the measuring process on the designs of the product structures.

In a simulation process, the layout of OTPs does not affect the measurement results. Therefore, based on the model of wing-fuselage alignment, OTPs of the wing are selected randomly from the points set on the surface of the wing and their nominal coordinates are extracted. In the first case, taking LCS1 as the local coordinate system of the wing, whilst using P_1, P_2, and P_3 as the OTPs for measuring; in the second case, LCS2 is the local coordinate system of the wing, while P_4, P_5, and P_6 are the OTPs.

(2) Setup the laser tracker and collect the coordinates of the OTPs.

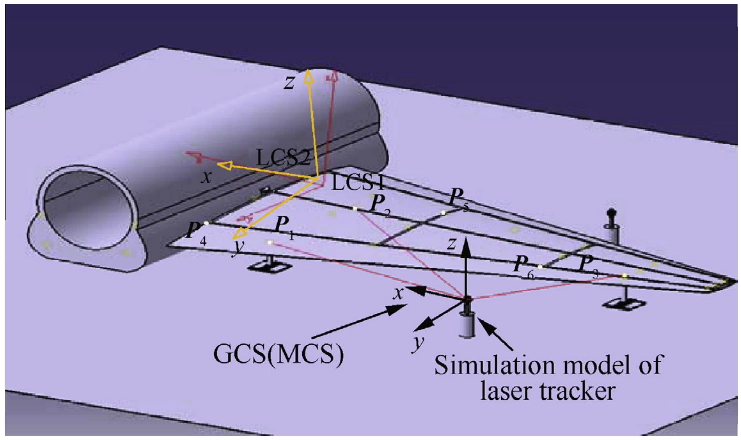

Fig. 4 Measuring the position and orientation of a wing in wing-fuselage alignment.

Given the hypothesis that the global coordinate system is the same as the measurement coordinate system, the model of a laser tracker is added to the simulation environment. In the experiments, a simulation measurement algorithm of laser tracker is used for collecting the coordinates of the OTPs. Then, the nominal and measured data of those OTPs can be derived as shown in Table 1.

(3) Calculate the position and orientation and U-P&O of the wing.

The position and orientation and U-P&O of the wing are calculated by the analytical algorithm presented in Section 3.1, and the results are showed in Table 2.

Obviously, although the designed LCSs and OTPs in the two experiments are different, the uncertainties of position and orientation based on the two datasets are the same. The experiments have verified the data independence of U-P&O.

4. Application study

4.1. Quality index based on uncertainty

The main purpose of assembly quality control is to ensure the assembly relationships between components are within the designed tolerances by methodologies based on quality evaluation. The quality of a product is evaluated according to the designed values, tolerances, and measurement results of one or more quality characteristics of the product. With the traditional quality evaluation method, the conclusion of a quality characteristic will be of conformance when the measurement result is within the designed tolerance; by contrast, the conclusion will be of non-conformance when the measurement result is out of the designed tolerance. However, the measurement result of the quality characteristic is uncertain. The uncertainty of measurement result is likely to cause that one measurement result is within the tolerance but another is out of the tolerance. As a result, the evaluation cannot exactly reflect the actual product quality.

Taking the quality characteristic A as an example, as shown in Fig. 5. x_0 is the nominal value of A, x_{0L} and x_{0U} are the lower and upper limits of the designed tolerance of A, x is the measured value of A, x_m is the average value of x, σ^2 is the variance of x, that is $x \sim N(x_m, \sigma^2)$, x_L and x_U are the lower and upper limits of the actual value of A, which is the fluctuant range. For the second case, the traditional method will determine A to be conformance, and for the third case, it will determine A to be non-conformance, but obviously, A may be conformance or nonconformance in those cases. Therefore, it is necessary to take into consideration the uncertainty during quality evaluation based on measured data.

In order to exactly evaluate product assembly quality, a new concept of quality index (QI) is proposed based on uncertainty, which is defined as: QI of a quality characteristic is the ratio of two probabilities, one is the probability of the result within the tolerance and the other is the probability of the result out of the tolerance. QI of the quality characteristic A in the above example can be derived in the equation below:

$$\mathrm{QI} = \frac{p\{x \in [x_{0L}, x_{0U}]\}}{p\{x \notin [x_{0L}, x_{0U}] | x \in [x_L, x_U]\}}$$

The values of x_L and x_U are determined by the confidence probability of measured data. Take the confidence probability

Table 1 Nominal and measured data of the OTPs. Unit: mm.

	Nominal value in LCS	Measured value in GCS
P_1	$[-3487.138, 11989.798, -702.421]$	$[3658.114, 3670.936, 1835.196]$
P_2	$[-3487.138, 1429.798, -702.421]$	$[3658.114, -6888.892, 1835.196]$
P_3	$[-28147.138, 6927.857, 1697.579]$	$[-21109.033, -1390.032, 2488.166]$
P_4	$[62.8614, 14239.1554, 920.9225]$	$[7084.8736, 5920.4025, 3704.6484]$
P_5	$[-10564.9264, 1349.6608, 1738.6847]$	$[-3574.3307, -6969.2918, 3770.4976]$
P_6	$[-22664.4987, 11856.6994, 2044.5322]$	$[-15665.3233, 3537.8311, 3221.1391]$

Table 2 Nominal and measured data of the positions and orientations and their uncertainty.

Items	1	2
Nominal value of position and orientation	$\begin{bmatrix} 0.996 & 3.805\times10^{-4} & -0.070 & 7086.937 \\ -3.912\times10^{-4} & 0.999 & -5.823\times10^{-5} & -8318.932 \\ 0.071 & 7.816\times10^{-5} & 0.997 & 2781.158 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.442 & 1.000 & -0.083 & 7186.715 \\ -1.032 & 0.499 & -0.428 & -8368.663 \\ 0.833 & 0.500 & 0.442 & 2832.158 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Measured value of position and orientation	$\begin{bmatrix} 0.997 & 3.812\times10^{-4} & -0.069 & 7086.825 \\ -3.857\times10^{-4} & 0.998 & -5.735\times10^{-5} & -8318.812 \\ 0.070 & 8.035\times10^{-5} & 0.997 & 2781.468 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.397 & 1.005 & -0.084 & 7186.623 \\ -1.028 & 0.399 & -0.419 & -8368.759 \\ 0.837 & 0.502 & 0.401 & 2832.052 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Uncertainty of position and orientation	[0.10, 0.16, 0.31, 5.24, 1.06, 1.32]	[0.11, 0.15, 0.31, 5.23, 0.98, 1.40]

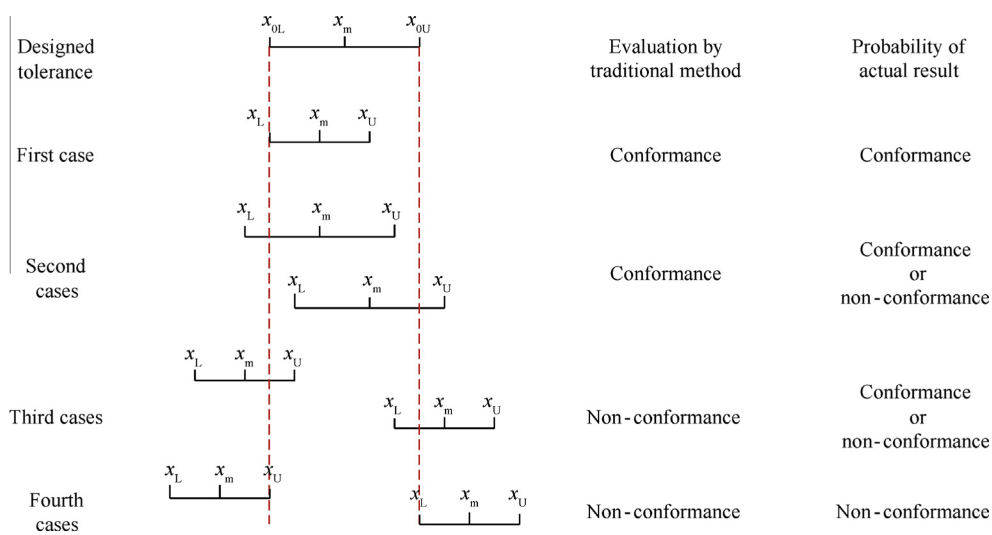

Fig. 5 Quality evaluation by the traditional method.

of 99.73% as an example, the fluctuant range of measured data is $\pm 3\sigma$, that is $x_L = x_m - 3\sigma$, $x_U = x_m + 3\sigma$. Then, in the case of bilateral tolerance, QI can be calculated using the following equations:

(1) If $x_m - 3\sigma > x_{0U}$ or $x_m + 3\sigma < x_{0L}$, QI = 0;
(2) If $x_m - 3\sigma > x_{0L}$ and $x_m + 3\sigma < x_{0U}$, QI = ∞;
(3) If $x_m - 3\sigma < x_{0L}$ and $x_m + 3\sigma > x_{0U}$,

$$\mathrm{QI} = \frac{p\{x_{0L} < x < x_{0U}\}}{0.9973 \times (p\{x_L < x < x_{0L}\} + p\{x_{0U} < x < x_U\})} = \frac{\Phi\left(\frac{x_{0U}-x_m}{\sigma}\right) - \Phi\left(\frac{x_{0L}-x_m}{\sigma}\right)}{0.9973 \times \left(\Phi\left(\frac{x_{0L}-x_m}{\sigma}\right) - \Phi\left(\frac{x_L-x_m}{\sigma}\right) + \Phi\left(\frac{x_U-x_m}{\sigma}\right) - \Phi\left(\frac{x_{0U}-x_m}{\sigma}\right)\right)}$$

(4) If $x_m - 3\sigma < x_{0L} < x_m + 3\sigma < x_{0U}$,

$$\mathrm{QI} = \frac{p\{x_{0L} < x < x_U\}}{0.9973 \times p\{x_L < x < x_{0L}\}} = \frac{\Phi\left(\frac{x_U-x_m}{\sigma}\right) - \Phi\left(\frac{x_{0L}-x_m}{\sigma}\right)}{0.9973 \times \left(\Phi\left(\frac{x_{0L}-x_m}{\sigma}\right) - \Phi\left(\frac{x_L-x_m}{\sigma}\right)\right)}$$

(5) If $x_{0L} < x_m - 3\sigma < x_{0U} < x_m + 3\sigma$,

$$\mathrm{QI} = \frac{p\{x_L < x < x_{0U}\}}{0.9973 \times p\{x_{0U} < x < x_U\}} = \frac{\Phi\left(\frac{x_{0U}-x_m}{\sigma}\right) - \Phi\left(\frac{x_L-x_m}{\sigma}\right)}{0.9973 \times \left(\Phi\left(\frac{x_U-x_m}{\sigma}\right) - \Phi\left(\frac{x_{0U}-x_m}{\sigma}\right)\right)}$$

According to the definition, a larger QI reflects better assembly quality. In order to draw a quality evaluation conclusion of a quality characteristic based on QI, it is required to identify a score table based on experience and knowledge of history data. The table maps the range of the QI value to a corresponding score, which represents the final conclusion of the measured quality characteristic. The flow chart for evaluating the quality of a quality characteristic is shown in Fig. 6.

4.2. Evaluating alignment based on U-P&O

In generally, there are more than one quality characteristics for aircraft components alignment quality evaluation and improvement. For example, during the wing-fuselage alignment, not only the mate relationship between the wing and the fuselage have to be ensured, but also the installation angle

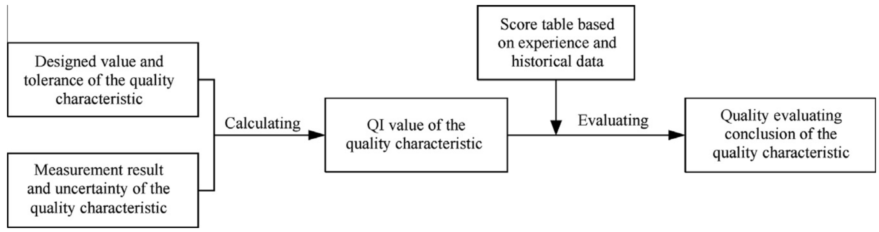

Fig. 6 The flow chart for evaluating the quality of a quality characteristic.

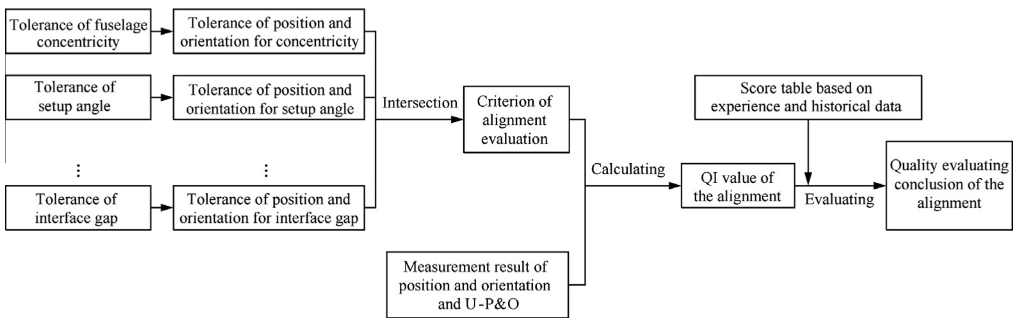

Fig. 7 Process for evaluating alignment based on U-P&O.

of the wing, the concentricity of the fuselage, and other characteristics have to be controlled within designed tolerances. In the traditional way of quality control for aircraft components alignment, those quality characteristics are measured and adjusted independently. As a result, the adjustment of one quality characteristic may bring another one out of the appropriate value. All of those quality characteristics cannot be simultaneously adjusted to meet the designed requirements, leading to increased assembly times and costs.

Quality characteristics in aircraft assembly are essentially the geometrical relationships of one component relative to another, and all of those geometrical relationships can be resolved and expressed by the positions and orientations of components which are relative to the global coordinate system in digital assembly; therefore, the designed values and tolerances of those quality characteristics can be converted into the designed value and tolerance of the position and orientation of the adjusted component. By doing so, the position and orientation of the adjusted component become the only quality characteristic for quality evaluation. Through measuring the position and orientation and resolving its uncertainty, the QI value can be calculated according to the designed value and tolerance of position and orientation. The quality evaluating conclusions can thus be drawn based on the score table.

In order to evaluate measurement assisted alignment, quality characteristics, such as fuselage concentricity, setup angle, and interface gap, are converted to position and orientation. Each mapping from a quality characteristic to position and orientation makes out a tolerance of the value of position and orientation. Then, the intersection of all the tolerances becomes the criterion of alignment evaluation. The process for evaluating alignment based on U-P&O is depicted in Fig. 7. Based on the measurement results of position and orientation and the U-P&O, the QI value of the alignment is calculated according to the criterion; finally, the evaluation conclusions can be derived for position and orientation adjustment planning and quality improvement in measurement assisted alignment.

5. Conclusions

(1) This paper presents the mathematical definition of uncertainty of position and orientation, with its connotations in both geometrical and algebraical aspects; an analytical algorithm based on measurement data is proposed for resolving uncertainty of position and orientation in aircraft components alignment.
(2) Effect factors of uncertainty of position and orientation are mainly divided into two sections: variation and measurement uncertainty of position and orientation, and a method for separating the two sections is discussed; the independency characteristic of uncertainty of position and orientation is presented based on measurement experiments, and then a novel method for evaluating alignment by quality index is proposed.
(3) The research on uncertainty of position and orientation is in its infancy. Future works will continually focus on the effect factors and characteristics analysis of uncertainty of position and orientation, and its further application in large size components alignment and other digital assembly processes.

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