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Corrigendum

# Corrigendum to "On functions preserving levels of approximation: a refined model construction for various lambda calculi" [Theoret. Comput. Sci. 212 (1999) 261–303] ☆

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Volume 212 (1999), for the article "On functions preserving levels of approximation: a refined model construction for various lambda calculi" by Dieter Spreen, pages 261–303: The proof of Lemma 15(2) is not correct. In what follows we give a corrected proof. To this end Property (4) in Lemma 14 has to be strengthened.

**Lemma 14.** *Let $f : D \to E$ be a projection morphism. Then* $\mathrm{Tr}(f)$ *satisfies the following properties:*

(1) *If $\{u_1, \dots, u_n\}$ is compatible and $(u_1, v_1) \dots, (u_n, v_n) \in \mathrm{Tr}(f)$,* then $\{v_1, \dots, v_n\}$ *is also compatible and $(u_1 \sqcup \cdots \sqcup u_n, v_1 \sqcup \cdots \sqcup v_n) \in \mathrm{Tr}(f)$.*

(2) *If $(u, v) \in \mathrm{Tr}(f)$ and $v' \sqsubseteq v$, then there is some $u' \sqsubseteq u$ such that $(u', v') \in \mathrm{Tr}(f)$.*

(3) *If $(u, v)$, $(u', v) \in \mathrm{Tr}(f)$ with $u \uparrow u'$ then $u = u'$.*

(4) *If $(u, v) \in \mathrm{Tr}(f)$, then $u \in D_i$ exactly if $v \in E_i$, for all $i \in \omega$.*

**Proof.** (1)–(3) follow as in [1]. For the proof of (4) let $(u, v) \in \mathrm{Tr}(f)$ and $i \in \omega$. If $u \in D_i$ then $v \sqsubseteq f(u) = f([u]_i^D) = [f(u)]_i^E$. Hence $v \in E_i$, by [1, Condition 4(2)].

Now, conversely, let $v \in E_i$. Since $(u, v) \in \mathrm{Tr}(f)$ we have that $v \sqsubseteq f(u)$. It follows that $v = [v]_i^E \sqsubseteq [f(u)]_i^E = f([u]_i^D)$, which implies that there is some $u' \sqsubseteq [u]_i^D$ so that $(u', v) \in \mathrm{Tr}(f)$. With Property (3) we obtain that $u' = u$. Thus $u = [u]_i^D$, i.e., $u \in D_i$. □

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For $u \in \bigcup_i D_i$ let $\mathrm{rk}(u)$, the *rank* of $u$, be the smallest number $n$ so that $u \in D_n$. Then Property (4) is equivalent to the statement that, if $(u, v) \in \mathrm{Tr}(f)$ then $\mathrm{rk}(u) = \mathrm{rk}(v)$.

As is easy to see, the Property (4) used in [1, Lemma 14] is a consequence of Properties (2) and (4) above.

**Lemma 15.** (1) *From* Tr( $f$ ) *one can compute $f$ via the following formula:*

$$f(x) = \bigsqcup \{v \mid (\exists u \sqsubseteq x)(u, v) \in \mathrm{Tr}(f)\}.$$

(2) *Each set of pairs of compact elements satisfying Conditions* (1)–(4) *of Lemma* 14 *is the trace of the projection morphism defined by the formula above.*

**Proof.** (1) follows as in [1]. For the proof of (2) let $X$ be a subset of $D^0 \times E^0$ with Properties (1)–(4) of the above Lemma and define

$$f(x) = \bigsqcup \{v \mid (\exists u \sqsubseteq x)(u, v) \in X\}.$$

Then $f$ is a stable function with $\mathrm{Tr}(f) = X$ [1]. It remains to check that $f$ commutes with the projections. We have that

$$f([x]_i^D) = \bigsqcup \{v \mid (\exists u \sqsubseteq [x]_i^D)(u, v) \in X\}$$

and

$$[f(x)]_i^E = \bigsqcup \{[\hat{v}]_i^E \mid (\exists \hat{u} \sqsubseteq x)(\hat{u}, \hat{v}) \in X\}.$$

If $(u, v) \in X$ with $u \sqsubseteq [x]_i^D$ then $u \in D_i$. With Property (4) it follows that $v \in E_i$. Thus $v = [v]_i^E$ and $u \sqsubseteq x$. This shows that $f([x]_i^D) \sqsubseteq [f(x)]_i^E$.

For the proof of the converse inequality let $(\hat{u}, \hat{v}) \in X$. with $\hat{u} \sqsubseteq x$. As a consequence of Properties (2) and (4) we obtain that for some $u \sqsubseteq [\hat{u}]_i^D, (u, [\hat{v}]_i^E) \in X$. Since $[\hat{u}]_i^D \sqsubseteq [x]_i^D$, it follows that also $[f(x)]_i^E \sqsubseteq f([x]_i^D)$. $\square$

The above result has been applied in the proof of [1, Lemma 16]. It is readily verified that also the new Condition 14(4) is satisfied in that case.

## References

[1] D. Spreen, On functions preserving levels of approximation: a refined model construction for various lambda calculi, Theoret. Comput. Sci. 212 (1999) 261–303.