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Time-dependent CP asymmetry in $B \to K^*\gamma$ as a (quasi)null test of the Standard Model

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Abstract

We calculate the dominant Standard Model contributions to the time-dependent CP asymmetry in $B^0 \to K^{*0}\gamma$, which is $O(1/m_b)$ in QCD factorisation. We find that, including all relevant hadronic effects, in particular from soft gluons, the asymmetry S is very small, $S = -0.022 \pm 0.012 \pm 0.01^{+0}_{-0.01}$, and smaller than suggested recently from dimensional arguments in a $1/m_b$ expansion. Our result implies that any significant deviation of the asymmetry from zero, and in particular a confirmation of the current experimental central value, $S_{\mathrm{HFAG}} = -0.28 \pm 0.26$, would constitute a clean signal for new physics.

1. Introduction

The radiative decay $b \to s\gamma$ has been extensively studied as a probe of both the flavour structure of the Standard Model (SM) and new physics beyond the SM (see Ref. [1] for a review). While the vast majority of studies has focused on the prediction of the decay rate for exclusive and both spectra and decay rate for inclusive $b \to s\gamma$ decays, there is one rather peculiar feature of this process which has attracted far less attention, namely that, in the SM, the emitted photon is predominantly left-handed in b, and right-handed in $\bar{b}$ decays. This is due to the fact that, in the language of effective field theories, the dominant contribution is from the chiral-odd dipole operator $\bar{s}_{L(R)}\sigma_{\mu\nu}b_{R(L)}$. As only left-handed quarks participate in the weak interaction, an effective operator of this type necessitates, in the SM, a helicity flip on one of the external quark lines, which results in a factor m_b (and a left-handed photon) for $b_R \to s_L\gamma_L$ and a factor m_s (and a right-handed photon) for $b_L \to s_R\gamma_R$. Hence, the emission of right-handed photons is suppressed by roughly a factor m_s/m_b. This suppression can easily be alleviated in a large number of new physics scenarios where the helicity flip occurs on an internal line, resulting in a factor m_i/m_b instead of m_s/m_b. A prime example are left–right symmetric models [2], whose impact on the photon polarisation was discussed in Ref. [3]. These models also come in a supersymmetric version whose effect on $b \to s\gamma$ was investigated in Ref. [4]. Supersymmetry with no left–right symmetry can also provide large contributions to $b \to s\gamma_R$, see Ref. [5] for recent studies. Other potential sources of large effects which have been studied are warped extra dimensions [6] or anomalous right-handed top couplings [7]. Unless the amplitude for $b \to s\gamma_R$ is of the same order as the SM prediction for $b \to s\gamma_L$, or the enhancement of $b \to s\gamma_R$ goes along with a suppression of $b \to s\gamma_L$, the impact on the branching ratio is small, as the two helicity amplitudes add incoherently. This implies there can be a substantial contribution of new physics to $b \to s\gamma$ escaping detection when only branching ratios are measured.

Although the photon helicity is, in principle, an observable, it is very difficult to measure directly. It can, however, be accessed indirectly, for instance in the time-dependent CP asymmetry in $B^0 \to K^{*0}\gamma$, which relies on the interference of both left and right helicity amplitudes and vanishes if one of them is absent. This method was first suggested in Ref. [3] and later discussed in

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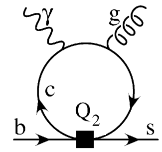

Fig. 1. Dominant contribution to $b \to s\gamma g$. A second diagram with photon and gluon vertices exchanged is implied.

more detail in Refs. [8,9]. It is rather special in the sense that usually new physics modifies the SM predictions for time-dependent CP asymmetries by affecting the mixing phase (as in $B_s \to J/\psi\phi$, see for instance Ref. [10]), introducing new weak phases or moderately changing the size of the decay amplitudes which, in the absence of precise calculational tools, makes it very hard to trace its impact. In contrast, the time-dependent CP asymmetry in $B^0 \to K^{*0}\gamma$ is very small in the SM, irrespectively of hadronic uncertainties, by virtue of the helicity suppression of one decay amplitude, and new physics enters by relieving that suppression. The smallness of the asymmetry in the SM, and the possibility of large effects from new physics, makes it one of the prime candidates for a so-called "null test" of the SM, as recently advertised in Ref. [11].

Other channels and methods to probe the photon helicity have been discussed in Refs. [12,13]. In this Letter, however, we focus on the time-dependent CP asymmetry in $B^0 \to K^{*0}\gamma$. It is given by

$$A_{\mathrm{CP}} = \frac{\Gamma(\bar{B}^0(t) \to \bar{K}^{*0}\gamma) - \Gamma(B^0(t) \to K^{*0}\gamma)}{\Gamma(\bar{B}^0(t) \to \bar{K}^{*0}\gamma) + \Gamma(B^0(t) \to K^{*0}\gamma)} = S\sin(\Delta m_B t) - C\cos(\Delta m_B t), \tag{1}$$

where K^{*0} and $\bar{K}^{*0}$ are observed via their decay into the CP eigenstate $K_S\pi^0$. The term involving an interference of photons with different polarisation is S, for which the following experimental results are available from the B factories:

$$\begin{aligned} S_{\mathrm{BaBar}} &= -0.21 \pm 0.40\ (\mathrm{stat}) \pm 0.05\ (\mathrm{syst}) \quad \text{BaBar [14] } (232 \times 10^6\ B\bar{B} \text{ pairs}), \\ S_{\mathrm{Belle}} &= -0.32^{+0.36}_{-0.33}\ (\mathrm{stat}) \pm 0.05\ (\mathrm{syst}) \quad \text{Belle [15] } (535 \times 10^6\ B\bar{B} \text{ pairs}), \end{aligned} \tag{2}$$

with the HFAG average $S_{\mathrm{HFAG}} = -0.28 \pm 0.26$ [16]. While these results are compatible with zero at the 1σ level, the central values of both BaBar and Belle are in agreement and interestingly large. A drastic reduction of the experimental uncertainty will probably be difficult at the LHC, but can be achieved at a super-B factory, with an anticipated statistical uncertainty of S of 0.07 with 10 ab^{-1} of data [17] and 0.04 with 50 ab^{-1} [18].

In order to clearly distinguish any new physics signal from the SM background, one needs to know the latter as precisely as possible. As discussed above, one contribution comes from $b_L \to s_R\gamma_R$, with a helicity flip on the s quark line; it generates the contribution

$$S^{\mathrm{SM},s_R} = -\sin(2\beta)\frac{m_s}{m_b}\big(2 + O(\alpha_s)\big) \tag{3}$$

to the CP asymmetry, with β being one of the angles of the CKM unitarity triangle. At leading order in α_s, S^{SM,s_R} is free from hadronic uncertainties. As pointed out in Ref. [8], another mechanism to remove the helicity suppression of $b \to s\gamma_R$ is to emit an additional gluon. The dominant contribution to this mechanism is via a c-quark loop and is shown in Fig. 1. In inclusive decays this is a bremsstrahlung correction and can be calculated in perturbation theory [8]. In exclusive decays, on the other hand, the gluon can be either hard or soft. If it is hard, it attaches to the spectator quark, which induces $O(\alpha_s)$ corrections to (3). If it is soft, it has to be interpreted as a parton in one of the external hadrons. Stated differently, if the gluon is soft, the amplitude involves higher Fock states of the B and K^*. A data-driven method to distinguish this contribution from that of the dipole operator $\bar{s}_{L(R)}\sigma_{\mu\nu}b_{R(L)}$ was discussed in Ref. [13] and relies on the Dalitz-plot analysis of decays of type $B^0 \to \gamma K_S +$ neutrals, where neutrals stands for π^0, $\eta^{(\prime)}$, K_S, light vector mesons or any combination of these particles. In Ref. [8] it was shown, in the framework of soft-collinear effective theory (SCET), that contributions from $b \to s\gamma g$ are suppressed by one power of m_b with respect to the left-handed photon emission, which confirms the results obtained in QCD factorisation [19,20], where an explicit $O(\alpha_s)$ calculation demonstrated, to leading order in $1/m_b$, the absence of right-handed photons in $\bar{B}^0 \to \bar{K}^{*0}\gamma$. A SCET analysis of the CP asymmetry in Ref. [9] estimated the size of the $1/m_b$ corrections to S^{SM} induced by $b \to s\gamma g$ as $\sim \pm 0.1$, but is based on dimensional counting of the operators involved rather than a calculation of the relevant matrix elements. Another calculation, in perturbative QCD, gives $S^{\mathrm{SM}}_{\mathrm{pQCD}} = -(3.5 \pm 1.7)\%$ [21], including effects mainly from hard gluons; the contribution of soft gluons is treated in a model-dependent way.

The purpose of this Letter is to provide a calculation of the soft-gluon contribution to the time-dependent CP asymmetry in $B \to K^*\gamma$ induced by the c-quark loop shown in Fig. 1. The method we use are QCD sum rules on the light-cone. It turns out that the relevant hadronic parameters were calculated already previously, in 1997, in Ref. [22], using the method of local QCD sum rules. The motivation at the time was to estimate long-distance corrections to the branching ratio of $B \to K^*\gamma$. In fact those corrections were first discovered for the inclusive process [23]. In this Letter, we show that the same parameters also enter the time-dependent CP asymmetry in $B \to K^*\gamma$ and present a new calculation of their values.

2. The CP asymmetry

Let us define the amplitudes of the decay of B mesons into K^* and left- or right-handed photons in the following way:

$$\bar{\mathcal{A}}_{L(R)} = \mathcal{A}\big(\bar{B}^0 \to \bar{K}^{*0}\gamma_{L(R)}\big), \qquad \mathcal{A}_{L(R)} = \mathcal{A}\big(B^0 \to K^{*0}\gamma_{L(R)}\big). \tag{4}$$

Neglecting, as usual, the small width difference between B^0 and $\bar{B}^0$, the time-dependent CP asymmetry is then given by (1) with

$$S = \frac{2\,\mathrm{Im}(\frac{q}{p}(\mathcal{A}_L^*\bar{\mathcal{A}}_L + \mathcal{A}_R^*\bar{\mathcal{A}}_R))}{|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 + |\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_R|^2}, \qquad C = \frac{|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 - |\bar{\mathcal{A}}_L|^2 - |\bar{\mathcal{A}}_R|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2 + |\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_R|^2}. \tag{5}$$

Here q/p is given in terms of the B^0–$\bar{B}^0$ mixing matrix M_{12}, in the standard convention for the parametrisation of the CKM matrix, by

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}} = e^{-2i\beta}.$$

Extending in an obvious way the notations introduced in Ref. [20] in the context of QCD factorisation, the decay amplitudes can be written as

$$\begin{aligned}\bar{\mathcal{A}}_{L(R)} &= \frac{G_F}{\sqrt{2}}\big(\lambda_u a_7^u\big(\bar{K}^*\gamma_{L(R)}\big) + \lambda_c a_7^c\big(\bar{K}^*\gamma_{L(R)}\big)\big)\langle\bar{K}^*\gamma_{L(R)}\big|Q_7^{L(R)}|\bar{B}\rangle \\ &\equiv \frac{G_F}{\sqrt{2}}\big(\lambda_u a_{7L(R)}^u + \lambda_c a_{7L(R)}^c\big)\langle\bar{K}^*\gamma_{L(R)}\big|Q_7^{L(R)}|\bar{B}\rangle.\end{aligned} \tag{6}$$

In QCD factorisation, $a_{7L}^{c,u}$ are of order 1 in a $1/m_b$ expansion [20],[1]

$$a_{7L}^{c,u} = C_7 + O(\alpha_s, 1/m_b), \tag{7}$$

with C_7 being the Wilson coefficient of the operator Q_7. The complete set of operators and formulas for the Wilson coefficients can be found in Ref. [24]. $a_{7R}^{c,u}$, on the other hand, are of order $1/m_b$ [8,9]. $\lambda_p = V_{ps}^* V_{pb}$ and the operators $Q_7^{L(R)}$ are given by

$$Q_7^{L(R)} = \frac{e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1 \pm \gamma_5) b F^{\mu\nu};$$

$Q_7^{L(R)}$ generates left- (right-)handed photons in the decay $b \to s\gamma$. The matrix element in (6) can be expressed in terms of the form factor $T_1^{B\to K^*}$ as

$$\begin{aligned}\big\langle\bar{K}^*(p,\eta)\gamma_{L(R)}(q,e)\big|Q_7^{L(R)}|\bar{B}\rangle &= -\frac{e}{2\pi^2} m_b T_1^{B\to K^*}(0)\big[\epsilon^{\mu\nu\rho\sigma} e_\mu^*\eta_\nu^* p_\rho q_\sigma \pm i\big\{\big(e^*\eta^*\big)(pq) - \big(e^*p\big)\big(\eta^*q\big)\big\}\big] \\ &\equiv -\frac{e}{2\pi^2} m_b T_1^{B\to K^*}(0) S_{L(R)},\end{aligned} \tag{8}$$

where $S_{L,R}$ are the helicity amplitudes corresponding to left- and right-handed photons, respectively, and $e_\mu(\eta_\mu)$ is the polarisation four-vector of the photon (K^*). The definition of $T_1^{B\to K^*}$ can be found in Ref. [25], and an updated value in Ref. [26]. In the Wolfenstein parametrisation of the CKM matrix, $\lambda_u \sim \lambda^4$ and is doubly Cabibbo suppressed with respect to $\lambda_c \sim \lambda^2$, so we drop this contribution from now on. With λ_u set to zero, the direct CP asymmetry C in (5) vanishes.

3. Calculation of a_{7R}^c in the SM

One contribution to $\bar{\mathcal{A}}_R$ is very well known and comes from the m_s dependent part of the full electromagnetic dipole operator Q_7,

$$Q_7 = \frac{e}{8\pi^2}\big[m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b + m_s\bar{s}\sigma_{\mu\nu}(1-\gamma_5)b\big]F^{\mu\nu} \equiv Q_7^L + \frac{m_s}{m_b}Q_7^R. \tag{9}$$

Hence, a_{7R}^c is given by

$$a_{7R}^c = \frac{m_s}{m_b}C_7 + O\bigg(\frac{1}{m_b}, \frac{\alpha_s}{m_b}\bigg). \tag{10}$$

[1] The $a_7^{c,u}$ calculated in Ref. [20], to leading order in $1/m_b$, coincide with our $a_{7L}^{c,u}$, whereas $a_{7R}^{c,u}$ are set zero in [20]. Our expression (6) is purely formal and does not imply that $a_{7R(L)}^{c,u}$ factorise at order $1/m_b$. As a matter of fact, they do not.

As discussed above, all contributions to $\bar{A}_R$ must include a helicity flip of the s quark, which in the above is done by including the effects from a nonvanishing s quark mass. Another possibility to relieve the helicity suppression of right-handed photons is by considering, at parton level, a three-particle final state with an additional gluon. The dominant contribution (with the largest Wilson coefficient) to this process comes from the operator

$$Q_2^c = [\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{c}\gamma^\mu(1-\gamma_5)b]$$

and is shown in Fig. 1. As the c quark has sufficiently large virtuality in the loop (the photon is on-shell and the gluon nearly so), the diagram is dominated by short distances and can be expanded in inverse powers of m_c. To do so, we follow Ref. [22] and rewrite Q_2^c as

$$Q_2^c = \frac{1}{3}[\bar{c}\gamma_\mu(1-\gamma_5)c][\bar{s}\gamma^\mu(1-\gamma_5)b] + 2\left[\bar{s}\gamma_\mu(1-\gamma_5)\frac{\lambda^a}{2}c\right]\left[\bar{c}\gamma^\mu(1-\gamma_5)\frac{\lambda^a}{2}b\right]. \tag{11}$$

Confirming the result of Ref. [22], we find that the short-distance expansion of the diagram in Fig. 1 yields

$$Q_F = ie^{*\mu}\int d^4x\, e^{iqx}\mathrm{T}\{[\bar{c}(x)\gamma_\mu c(x)]Q_2^c(0)\} = -\frac{1}{48\pi^2 m_c^2}\left(D^\rho F^{\alpha\beta}\right)\left[\bar{s}\gamma_\rho(1-\gamma_5)g\tilde{G}^a_{\alpha\beta}\frac{\lambda^a}{2}b\right] + \cdots, \tag{12}$$

where $F^{\alpha\beta} = i(q^\alpha e^{*\beta} - q^\beta e^{*\alpha})$ corresponds to an outgoing photon and the dots denote terms of higher order in $1/m_c$. Note that the contribution of the first term in (11) vanishes for an on-shell photon. The contribution of Q_F to the decay amplitude is

$$\mathcal{A}_{Q_F}\left(\bar{B}\to\bar{K}^*\gamma\right) = -\frac{2e}{3}\langle\bar{K}^*\gamma|Q_F|\bar{B}\rangle,$$

where $2/3$ is the electric charge of the c quark and the minus sign comes from the EM interaction operator. At this point we would also like to make explicit our conventions for the strong and electromagnetic couplings. We use the covariant derivative

$$D_\mu = \partial_\mu + ieQ_f B_\mu - igA^a_\mu\frac{\lambda^a}{2}$$

for a fermion with electric charge Q_f. Here $e = +\sqrt{4\pi\alpha}$ which is consistent with the sign-convention for Q_7, Eq. (9).[2] The contribution of Q_2^c to a^c_{7R} is hence governed by the matrix element $\langle K^*\gamma|(D^\rho F^{\alpha\beta})[\bar{s}\gamma_\rho(1-\gamma_5)g\tilde{G}^a_{\alpha\beta}\frac{\lambda^a}{2}b]|B\rangle$, which, again following Ref. [22], can be parametrised as

$$\begin{aligned}
&\left\langle\bar{K}^*(p,\eta)\gamma(q,e)\right|\left(D^\rho F^{\alpha\beta}\right)\left[\bar{s}\gamma_\rho(1-\gamma_5)g\tilde{G}^a_{\alpha\beta}\frac{\lambda^a}{2}b\right]\left|\bar{B}(p+q)\right\rangle\\
&\quad = 2\left\langle\bar{K}^*(p,\eta)\right|\bar{s}\gamma_\mu q^\mu(1-\gamma_5)g\tilde{G}_{\alpha\beta}b\left|\bar{B}(p+q)\right\rangle e^{*\alpha}q^\beta\\
&\quad = 2\left\{L\epsilon_{\mu\nu\rho\sigma}e^{*\mu}\eta^{*\nu}p^\rho q^\sigma + i\tilde{L}\left[\left(e^*\eta^*\right)(pq) - \left(e^*p\right)\left(\eta^*q\right)\right]\right\}\\
&\quad = (L+\tilde{L})S_L + (L-\tilde{L})S_R,
\end{aligned} \tag{13}$$

where $S_{L,R}$ are the photon helicity structures defined in (8). The operator Q_2^c thus induces power corrections of type $(L\pm\tilde{L})/(m_c^2 m_b)$ to a^c_{7L} and a^c_{7R}, respectively. As already mentioned before, these power corrections have previously been considered in Ref. [22]. Before we present a new calculation of L and $\tilde{L}$, let us finally give their contribution to a^c_{7R}:

$$a^c_{7R} = C_7\frac{m_s}{m_b} - C_2\frac{L-\tilde{L}}{36m_c^2 m_b T_1^{B\to K^*}(0)}. \tag{14}$$

Corrections to this expression are of order α_s/m_b, come with smaller (penguin) Wilson coefficients or are of higher order in $1/m_{b,c}$. What about the convergence of the $1/m_c$ expansion? For the inclusive decay $b\to s\gamma$ this question was studied in Ref. [27]. Higher terms in the short-distance expansion of (12) generate operators with higher order derivatives acting on $F^{\alpha\beta}$, generating powers of the photon momentum q, and on $\tilde{G}_{\alpha\beta}$, generating new hadronic matrix elements. A complete calculation of these additional contributions to (14) is not possible with the presently available methods, but we can try to give an estimate. As found in Ref. [27], the expansion parameter of the $1/m_c$ expansion is $t = (q\cdot D)/(2m_c^2)$ with D acting on the gluon field strength tensor. The hadronic matrix elements with additional powers of D can be estimated as

$$\left\langle K^*\right|\bar{s}D^n\tilde{G}b|B\rangle \sim (\Lambda_{\mathrm{QCD}})^n\left\langle K^*\right|\bar{s}\tilde{G}b|B\rangle, \tag{15}$$

and hence $t\sim(m_B/2)\Lambda_{\mathrm{QCD}}/(2m_c^2)\approx 0.2$. Using (15), the $1/m_c$ series can be resummed and enhances the term in (12) by a factor 1.1 for $t = 0.2$, and 1.3 for $t = 0.4$.[3] Although (15) is only a crude estimate of the true value of these matrix elements, this

[2] The sign of the strong coupling g differs with respect to Ref. [20], which however does not matter as all final expressions contain only factors g^2.

[3] The enhancement factor is given by the function $\bar{F}(t,t)$ defined in the last reference of [27].

result suggests that the $1/m_c$ expansion converges well. We also would like to mention, as noted in [27], that besides the derivative expansion in the gluon field there are further higher-twist contributions from e.g. two gluon fields. These contributions, however, are suppressed by additional powers of $\Lambda_{\rm QCD}^2/(m_c^2)$ [27]. We shall include the effect of truncating the $1/m_c$ expansion by doubling the theoretical error of our final result for the CP asymmetry.

4. Non-factorisable soft gluon effects: L and $\tilde{L}$

The following results for L and $\tilde{L}$ were obtained in 1997, in Ref. [22], using the method of local QCD sum rules and neglecting the effects of $SU(3)$ breaking:

$$L = (0.55 \pm 0.1)\ \mathrm{GeV}^3, \qquad \tilde{L} = (0.70 \pm 0.1)\ \mathrm{GeV}^3. \tag{16}$$

Since then, a number of studies [28] have demonstrated that the appropriate method to calculate B decay form factors from QCD sum rules is to use QCD sum rules on the light-cone [29,30]. In this Letter, we cannot give any account of the method itself, but refer to the relevant literature. Suffice it to say that one of the main ingredients in the method are light-cone distribution amplitudes (DAs) of two- and three-particle Fock states of the final-state meson. These have been known for some time for ρ mesons [31,32], but complete expressions for K^* mesons will become available only later in 2006 [33] (see also [34,35]). In this Letter, we include the first (preliminary) results of this ongoing study in our calculation of L and $\tilde{L}$. The light-cone sum rules read:

$$\begin{aligned}\frac{f_B m_B^2}{m_b} L e^{-m_B^2/M^2} = m_b^4 \int_0^{1-m_b^2/s_0} & d\alpha_2\, e^{-m_b^2/(\bar\alpha_2 M^2)} \\ \times \Bigg\{ \frac{1}{2\bar\alpha_2^2} \int_0^{1-\alpha_2} & d\alpha_1 \left[\left(\frac{m_{K^*}}{m_b}\right) f_{K^*}^{\parallel} \Phi_{3;K^*}^{\parallel}(\underline{\alpha}) + \left(\frac{m_{K^*}}{m_b}\right)^2 f_{K^*}^{\perp} \bar\alpha_2 \big(\Psi_{4;K^*}^{\perp}(\underline{\alpha}) + \Phi_{4;K^*}^{\perp(1)}(\underline{\alpha})\big)\right] \\ - \left(\frac{m_{K^*}}{m_b}\right)^2 & f_{K^*}^{\perp} \left[\frac{1}{4\bar\alpha_2^2} \mathrm{I}\big[\Phi_{3;K^*}^{\perp} + 2\big(\Phi_{4;K^*}^{\perp(3)} + \Phi_{4;K^*}^{\perp(4)}\big)\big] + \frac{d}{d\alpha_2}\left(\frac{1}{2\bar\alpha_2}\mathrm{I}\big[\Phi_{4;K^*}^{\perp(1)} + \Phi_{4;K^*}^{\perp(4)}\big]\right)\right]\Bigg\},\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{f_B m_B^2}{m_b} \tilde{L} e^{-m_B^2/M^2} = m_b^4 \int_0^{1-m_b^2/s_0} & d\alpha_2\, e^{-m_b^2/(\bar\alpha_2 M^2)} \\ \times \Bigg\{ \frac{1}{2\bar\alpha_2^2} \int_0^{1-\alpha_2} & d\alpha_1 \left[\left(\frac{m_{K^*}}{m_b}\right) f_{K^*}^{\parallel} \tilde\Phi_{3;K^*}^{\parallel}(\underline{\alpha}) - \left(\frac{m_{K^*}}{m_b}\right)^2 f_{K^*}^{\perp} \bar\alpha_2 \big(\tilde\Psi_{4;K^*}^{\perp}(\underline{\alpha}) + \Phi_{4;K^*}^{\perp(2)}(\underline{\alpha})\big)\right] \\ + \left(\frac{m_{K^*}}{m_b}\right)^2 & f_{K^*}^{\perp} \frac{1}{4\bar\alpha_2^2} \mathrm{I}\big[\Phi_{3;K^*}^{\perp} - 2\big(\Phi_{4;K^*}^{\perp(1)} + 2\Phi_{4;K^*}^{\perp(2)} + \Phi_{4;K^*}^{\perp(3)}\big)\big]\Bigg\}.\end{aligned} \tag{18}$$

The above expressions are accurate up to terms of order $(m_{K^*}/m_b)^3$, which are of higher twist, and $O(\alpha_s)$ corrections. Here $\Phi(\underline{\alpha}) = \Phi(\alpha_1, \alpha_2)$ are three-particle DAs of the K^* of twist 3 or 4 (as indicated by the index). The (rather lengthy) definition of these DAs is given in Ref. [33]. The variable α_1 can be interpreted as the longitudinal momentum fraction carried by the quark in the meson, whereas α_2 is the momentum fraction carried by the antiquark. $\mathrm{I}[\Phi]$ is a functional acting on the DA $\Phi(\underline{\alpha})$, which is defined as

$$\mathrm{I}[\Phi] = \int_0^{\alpha_2} dx \int_0^{1-x} d\alpha_1\, \Phi(\alpha_1, x).$$

The DAs can be described in a systematic way using conformal expansion [31]; here, we restrict ourselves to the leading terms in that expansion and use the expressions [33] ($\alpha_3 = 1 - \alpha_1 - \alpha_2$)

$$\begin{aligned}\Phi_{3;K^*}^{\parallel}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{\kappa_{3K}^{\parallel} + \omega_{3K}^{\parallel}(\alpha_1 - \alpha_2) + \lambda_{3K}^{\parallel}\frac{1}{2}(7\alpha_3 - 3)\right\}, \\ \tilde\Phi_{3;K^*}^{\parallel}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{\zeta_{3K}^{\parallel} + \tilde\lambda_{3K}^{\parallel}(\alpha_1 - \alpha_2) + \tilde\omega_{3K}^{\parallel}\frac{1}{2}(7\alpha_3 - 3)\right\}, \\ \Phi_{3;K^*}^{\perp}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{\kappa_{3K}^{\perp} + \omega_{3K}^{\perp}(\alpha_1 - \alpha_2) + \lambda_{3K}^{\perp}\frac{1}{2}(7\alpha_3 - 3)\right\},\end{aligned}$$

$$\Phi^{\perp(1)}_{4;K^*}(\underline{\alpha}) = 120\alpha_1\alpha_2\alpha_3\left(\frac{1}{4}\kappa^{\perp}_{3K} + \frac{1}{2}\kappa^{\perp}_{4K}\right),$$
$$\Phi^{\perp(2)}_{4;K^*}(\underline{\alpha}) = -30\alpha_3^2\left\{(1-\alpha_3)\left(-\frac{1}{4}\kappa^{\perp}_{3K} + \frac{1}{2}\kappa^{\perp}_{4K}\right) - (\alpha_1-\alpha_2)\tilde{\zeta}^{\perp}_{4K}\right\},$$
$$\Phi^{\perp(3)}_{4;K^*}(\underline{\alpha}) = -120\alpha_1\alpha_2\alpha_3\left(\frac{1}{4}\kappa^{\perp}_{3K} - \frac{1}{2}\kappa^{\perp}_{4K}\right),$$
$$\Phi^{\perp(4)}_{4;K^*}(\underline{\alpha}) = 30\alpha_3^2\left\{(1-\alpha_3)\left(-\frac{1}{4}\kappa^{\perp}_{3K} - \frac{1}{2}\kappa^{\perp}_{4K}\right) - (\alpha_1-\alpha_2)\zeta^{\perp}_{4K}\right\},$$
$$\Psi^{\perp}_{4;K^*}(\underline{\alpha}) = 30\alpha_3^2\left\{(1-\alpha_3)\zeta^{\perp}_{4K} + (\alpha_1-\alpha_2)\left(\frac{1}{4}\kappa^{\perp}_{3K} + \frac{1}{2}\kappa^{\perp}_{4K}\right)\right\},$$
$$\tilde{\Psi}^{\perp}_{4;K^*}(\underline{\alpha}) = 30\alpha_3^2\left\{(1-\alpha_3)\tilde{\zeta}^{\perp}_{4K} - (\alpha_1-\alpha_2)\left(-\frac{1}{4}\kappa^{\perp}_{3K} + \frac{1}{2}\kappa^{\perp}_{4K}\right)\right\}. \tag{19}$$

Preliminary numerical results for the various hadronic parameters ζ, κ, ω and λ are collected in Table 1; they will be discussed in more detail in Ref. [33]. The DAs defined above are related to those introduced in Ref. [31,32] as

$$\Phi^{\parallel}_{3;K^*} = \mathcal{V}, \qquad \tilde{\Phi}^{\parallel}_{3;K^*} = \mathcal{A}, \qquad \Phi^{\perp}_{3;K^*} = \mathcal{T}, \qquad \Phi^{\perp(i)}_{4;K^*} = T^{(i)}, \qquad \Psi^{\perp}_{4;K^*} = S, \qquad \tilde{\Psi}^{\perp}_{4;K^*} = \tilde{S}. \tag{20}$$

Although the introduction of new notations may, at first, look unmotivated, it actually extends the labelling scheme introduced, in Ref. [36], for pseudoscalar mesons, to vector mesons and aims to provide a systematic way to label the multitude of two- and three-particle pseudoscalar and vector meson DAs, replacing the slightly ad-hoc notations introduced in our previous papers on the subject [31,32]. As for the other hadronic parameters entering (17) and (18), we use $m_b = (4.7 \pm 0.1)$ GeV, $f_B = (200 \pm 30)$ MeV, $f^{\parallel}_{K^*} = (217 \pm 5)$ MeV [39] and $f^{\perp}_{K^*}(1\ \text{GeV}) = (185 \pm 10)$ MeV [34]. All scale-dependent parameters are evaluated at the scale $\mu^2 = m_B^2 - m_b^2 \pm 1\ \text{GeV}^2$, see Ref. [25]. s_0 and M^2 are sum rule specific parameters which do not acquire sharp values, but have to be varied in a certain range. Based on our experience with B decay form factors [25] we choose $s_0 = (35 \pm 2)\ \text{GeV}^2$ and $M^2 = (10 \pm 3)\ \text{GeV}^2$. We then obtain

$$L = (0.2 \pm 0.1)\ \text{GeV}^3, \qquad \tilde{L} = (0.3 \pm 0.2)\ \text{GeV}^3, \qquad L - \tilde{L} = -(0.1 \pm 0.1)\ \text{GeV}^3. \tag{21}$$

It turns out that the contribution of the $(m_{K^*}/m_b)^2$ terms to the sum rules is tiny, so that the result and its uncertainty is entirely dominated by m_b, f_B and the twist-3 DAs $\Phi^{\parallel}_{3;K^*}$ and $\tilde{\Phi}^{\parallel}_{3;K^*}$. We repeat that the parameters describing these DAs, collected in Table 1, are preliminary.

The results in (21) refer to the renormalisation scale $\mu^2 = m_B^2 - m_b^2 \approx (2.2\ \text{GeV})^2$. Unfortunately, the dependence of L and $\tilde{L}$ on μ is unknown. We can, however, estimate the potential impact of a change of scale by evaluating the light-cone sum rules at the higher scale $\mu = m_b$, although this is, strictly speaking, incorrect in that framework. Nonetheless, L and $\tilde{L}$ itself decrease by about 20% by this procedure, whereas $L - \tilde{L}$ decreases by 10%, which is well within the quoted errors.

Table 1

Three-particle twist-3 and 4 hadronic parameters. All results labelled "new" are preliminary and will be finalised in Ref. [33]. Note that the absolute sign of all these parameters depends on the sign convention chosen for the strong coupling g. The above results correspond to the choice $D_\mu = \partial_\mu - igA^a_\mu(\lambda^a/2)$ of the covariant derivative

	$\mu = 1$ GeV	Remarks
$\zeta^{\parallel}_{3K}$	0.033 ± 0.007	new; $\zeta^{\parallel}_{3\rho}$ determined in [37]
$\tilde{\lambda}^{\parallel}_{3K}$	0.06 ± 0.03	G-odd, new
$\tilde{\omega}^{\parallel}_{3K}$	-0.06 ± 0.02	new; $\tilde{\omega}^{\parallel}_{3\rho}$ determined in [37]
$\kappa^{\parallel}_{3K}$	0.001 ± 0.001	G-odd; previously determined in [38]
$\omega^{\parallel}_{3K}$	0.14 ± 0.03	new; $\omega^{\parallel}_{3\rho}$ determined in [37]
$\lambda^{\parallel}_{3K}$	-0.02 ± 0.01	G-odd, new
$\kappa^{\perp}_{3K}$	0.006 ± 0.003	G-odd, new
$\omega^{\perp}_{3K}$	0.4 ± 0.1	new; $\omega^{\perp}_{3\rho}$ determined in [31]
$\lambda^{\perp}_{3K}$	-0.05 ± 0.02	G-odd, new
$\zeta^{\perp}_{4K}$	0.10 ± 0.05	quoted from [29]; no $SU(3)$ breaking; to be updated in [33]
$\tilde{\zeta}^{\perp}_{4K}$	$= -\zeta^{\perp}_{4K}$	quoted from [29]; no $SU(3)$ breaking; to be updated in [33]
$\kappa^{\perp}_{4K}$	0.012 ± 0.004	G-odd; quoted from [35]

Comparing with the results obtained in Ref. [22], Eq. (16), we find that our central values are considerably smaller. As mentioned before, the authors of [22] used local sum rules, which are of only limited value for determining B decay form factors at maximum recoil, i.e. for maximum energy of the final state meson, see the discussion in the first two references in [28]. On the other hand, in 1997 not much was known about three-particle twist-3 DAs of vector mesons, so local QCD sum rules were the best tool at hand at the time. We also find that our errors are larger than those in (16), which is due to the fact that the uncertainties quoted in (16) are obtained by varying only the sum rule parameters s_0 and M^2, but not the hadronic input parameters.

5. Results and conclusions

We are now finally ready to present results for the CP asymmetry S in (1). The m_s-dependent terms in (3) yield

$$S^{\mathrm{SM},s_R} = -0.027 \pm 0.006(m_{s,b}) \pm 0.001\big(\sin(2\beta)\big), \tag{22}$$

where we use $m_s(2\ \mathrm{GeV}) = (100 \pm 20)$ MeV [40], $m_b(m_b) = (4.20 \pm 0.04)$ GeV [41] and $\sin(2\beta) = 0.685 \pm 0.032$ [16]. One can estimate the impact of radiative corrections on that result by comparing it with the perturbative QCD calculation of Ref. [21]. The authors of [21] obtain the same central value for S^{SM,s_R} and also quote, very helpfully, results obtained for neglecting various sources of corrections, in particular -0.034 ± 0.013 if all long-distance contributions are neglected. From this we conclude that the impact of radiative corrections on (22) is likely to slightly increase the asymmetry, but not by more than 0.01. As for the contribution of $L - \tilde{L}$, it is given, to leading order in α_s, by

$$S^{\mathrm{SM,\ soft\ gluons}} = -2\sin(2\beta)\left(-\frac{C_2}{C_7}\frac{L-\tilde{L}}{36 m_b m_c^2 T_1^{B\to K^*}(0)}\right) = 0.005 \pm 0.01. \tag{23}$$

Here we use $C_2(m_b) = 1.02$, $C_7(m_b) = -0.31$, which are the leading-order values, $m_c = 1.3$ GeV and $T_1^{B\to K^*}(0) = 0.31 \pm 0.04$ [26], and have doubled the error to account for neglected higher-order terms in the $1/m_c$ expansion. That is: the contribution of soft gluons to S is much smaller numerically than that in m_s/m_b, Eq. (22). This result has to be compared with the dimensional estimate presented in Ref. [9], from a SCET-based analysis,

$$\big|S_{[9]}^{\mathrm{SM,\ soft\ gluons}}\big| = 2\sin(2\beta)\left|\frac{C_2}{3C_7}\right|\frac{\Lambda_{\mathrm{QCD}}}{m_b} \approx 0.06. \tag{24}$$

Our result (23) suggests that the true value of the soft gluon contributions is much smaller. Comparing (23) and (24), it becomes obvious that this is mainly due to the factor $1/36$ in (23) resulting from the short-distance expansion of the charm loop in Fig. 1.

While the aim of our Letter was to calculate the soft gluon contributions to S^{SM} induced by the operator Q_2 and to check the estimate of Ref. [9] that it could induce a 10% effect, our result for $S^{\mathrm{SM,\ soft\ gluons}}$ has now become, due to the suppression factor $1/36$, that small that one may start to wonder about the size of other corrections. One source of such corrections, and actually the probably dominant one, are radiative corrections to the term in m_s/m_b which we estimate using the results of Ref. [21]. Another class of (soft gluon) corrections are diagrams with the same topology as Fig. 1, but a different operator. As long as there is a charm quark in the loop, these contributions are controlled by the matrix element $L - \tilde{L}$, but suppressed by small penguin Wilson-coefficients $C_{\mathrm{peng}} < 0.1$ and hence can be neglected. For light quarks in the loop, one cannot apply the short-distance expansion as done in this Letter, but has to follow a different approach. We will discuss this approach in a forthcoming paper on power-corrections to $B \to \rho\gamma$ [42]; the result is that the contribution of light quark loops is of approximately the same size as that of charm loops, so again these contributions are suppressed by small Wilson coefficients. A second, different topology is given by annihilation diagrams induced by the penguin operators $(\bar{s}b)_{V-A}(\bar{d}d)_{V\pm A}$. This contribution is enhanced by the fact that it is a tree diagram; it can be calculated using the results obtained in Ref. [43] for $B \to \gamma e\nu$ transitions. For the contribution with the largest Wilson-coefficient from the penguin operator Q_4, one has

$$a_{7R}^c \to a_{7R}^c + C_4\frac{Q_d}{Q_u}\frac{2\pi^2 f_{K^*} m_{K^*}}{m_B^2 T_1^{B\to K^*}(0)}\big(F_V(0) - F_A(0)\big),$$

where $Q_{u,d}$ are the electric charges of the corresponding quarks and $F_{V,A}$ are the form factors determining the $B \to \gamma$ transition. Using $F_V(0) - F_A(0) \approx 0.016$ [43] and $C_4(m_b) = 0.08$, the shift of a_{7R}^c turns out to be $\approx 0.3 \times 10^{-3}$ which is to be compared with the (dominant) m_s/m_b term $\approx 6 \times 10^{-3}$ and the term in C_2: $\approx 1 \times 10^{-3}$. Let us note in passing that $F_V(0) - F_A(0)$ is induced by long-distance photon emission and given in terms of three-particle Fock states of the photon [44], so also for this contribution the necessary spin-flip in the parton-level process $b \to s\gamma$ is induced by a higher Fock state, this time of the photon. One more possible topology are hard-spectator scattering diagrams involving the chromomagnetic dipole operator Q_8. Although we cannot give a firm estimate of this contribution to a_{7R}^c, we expect it to come mainly from long-distance photon emission governed by the same three-particle Fock state of the photon mentioned before and to contribute at the same level as the other terms discussed above.

Our final result for the CP asymmetry is the sum of (22) and (23):

$$S^{\mathrm{SM}} = S^{\mathrm{SM},s_R} + S^{\mathrm{SM,\ soft\ gluons}} = -0.022 \pm 0.012 \pm 0.01^{+0}_{-0.01}, \tag{25}$$

where the second uncertainty accounts for neglected contributions induced by penguin operators and the chromomagnetic dipole operator, and the third, asymmetric uncertainty is to account for neglected $O(\alpha_s)$ corrections to (22), which, based on the results of Ref. [21], we estimate to be negative and not to exceed 0.01. In principle these corrections can also be calculated in QCD factorisation, but this goes beyond the scope of this Letter.

To summarize, we have calculated the dominant contributions to the SM prediction for the time-dependent CP asymmetry S in $B^0 \to K^{*0}\gamma$. These come, on the one hand, from terms in m_s/m_b, and on the other hand from short-distance processes involving an additional gluon, see Fig. 1. We find that in contrast to recent suggestions that the latter be large, they are actually substantially smaller than the former. Additional hadronic corrections to our result are expected to be even smaller and due to radiative corrections, small Wilson coefficients and higher order terms in the heavy quark expansion. The most dominant correction is likely to be radiative corrections to (22), which have already been calculated in perturbative QCD and found to be ≈ -0.01. A confirmation of this result in QCD factorisation, if possible, would be welcome. Our result (25) confirms that the CP asymmetry is an excellent quasi null test of the SM in the sense of Ref. [11] and that any significant deviation of the experimental result from zero will provide a clean signal for new physics.

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