Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 645 (2007) 412–421
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Energy losses in the black disc regime and correlation effects in the STAR forward pion production in dAu collisions

Leonid Frankfurt [a], M. Strikman [b,*]

[a] Department of Physics, Tel Aviv University, Israel
[b] Department of Physics, Pennsylvania State University, University Park, PA 16802, USA

Received 21 June 2006; received in revised form 4 December 2006; accepted 2 January 2007

Available online 17 January 2007

Editor: J.-P. Blaizot

Abstract

We argue that in the small x processes, in the black disc QCD regime (BDR) a very forward parton propagating through the nuclear matter should loose a significant and increasing with energy and atomic number fraction of its initial energy as a result of dominance of inelastic interactions, causality and energy–momentum conservation. We evaluate these energy losses and find them to lead to the significant suppression of the forward jet production in the central NA collisions at collider energies with a moderate suppression of recoiling jet at central rapidities. We confront our expectations with the recent RHIC data of the STAR Collaboration on the probability, P, for emission of at least one fast hadron at a central rapidity in association with production of a very forward high p_t neutral pion in pp and dAu collisions. We calculate the A-dependence of P, and find that the data imply a strong suppression of leading pion production at central impact parameters. We also conclude that production of recoil jets in the hard subprocess is not suppressed providing further evidence for the dominance of peripheral collisions. Both features of the data are consistent with the onset of BDR. We suggest new phenomena and new observables to investigate BDR at RHIC and LHC.

1. Introduction

It is well understood now that one of distinctive properties of hard processes in pQCD is the fast increase with energy of cross sections of hard inelastic processes and their significant value. Thus the interactions of the partons produced in the sufficiently small x hard processes should be highly inelastic. Dominance of inelastic processes leads to the specific pattern of energy losses for a parton propagating through the nuclear medium which is the main subject of this Letter. Really in the elastic rescatterings which dominate in the large x processes energetic parton looses a finite energy [1] while propagating a distance L: $\Delta E \approx 0.02\ \text{GeV}\, L^2/\text{fm}^2$. The analysis of Ref. [2] of the πA Drell–Yan pair production indicate that the data are consistent with the rate of energy loss by a quark of Ref. [1] and correspond to a energy loss $\leqslant 4$ GeV for quark of energy ~ 200 GeV propagating through a center of a heavy nucleus. In contrast in the deep inelastic processes for example DIS off a proton the fraction of initial photon energy lost by incident parton is $\sim 10\%$ within DGLAP approximations, cf. discussion in Section 2. Numbers are probably similar within the NLO BFKL approximation corresponding to the rapidity interval between the leading particle and next rung in the ladder of about two. (It is equal to zero within the LO BFKL approximation which systematically neglects the loss of energy by energetic particles.)

In the black disc regime the contrast between the different patterns of energy losses becomes dramatic. A parton with energy E propagating sufficiently large distance L through the nuclear media should loose energy:

$$\Delta E = cE(L/3\ \text{fm}) \tag{1}$$

with $c \approx 0.1$ in small x processes. This energy loss exceeds by orders of magnitude the losses in the large x regime.

* Corresponding author.
E-mail addresses: frankfur@tauphy.tau.ac.il (L. Frankfurt), strikman@phys.psu.edu (M. Strikman).

0370-2693
doi:10.1016/j.physletb.2007.01.007

Another subtle effect characteristic for a quantum field theory has been found long before the advent of QCD: eikonal interactions of energetic particle are canceled out as the consequence of causality [3,4]. This cancellation including additional suppression of eikonal diagrams due to energy–momentum conservation is valid for the exchanges by pQCD ladders with vacuum quantum numbers in the crossed channel [5]. The cancellation of the contribution of eikonal diagrams has been demonstrated also for the exchanges by color octet ladders as the consequence of bootstrip condition for the reggeized gluon [6]. Thus sufficiently energetic parton may experience only one inelastic collision. To produce n inelastic collisions wave function of energetic parton should develop component containing at least n constituents [5]. This effect leads to the additional depletion of the spectrum of leading partons in the kinematics close to BDR where inelastic interactions of the energetic parton is important part of unitarization of amplitudes of hard processes.

Since the number of inelastic collisions is controlled by the number of scattering centres at given impact parameter the effect of the suppression of the yield of leading partons should be largest at the central impact parameters. We evaluate energy losses of leading parton in small x regime of QCD and show that blackening of pQCD interaction leads to dominance of peripheral collisions in the production of the leading hadrons/jets in high energy hadron–nucleus interactions and to a significant, increasing with energy and atomic number loss of finite fraction of leading parton energy in the central collisions. Inclusive cross section is $\propto A^{1/3}$ deep in the BDR region with suppression of the recoil jets depending on x of jet. One of characteristic features of BDR regime is that there is no suppression of recoil jet in the peripheral collisions. At moderately small x which are reached at RHIC, suppression of recoil jet should depend on its rapidity and be maximal if both jets carry a significant fraction of the projectile energy. We will show that this prediction is supported by the recent RHIC data on leading hadron production in dA collisions.

It is instructive to compare the kinematics of partons involved in the production of leading hadrons at RHIC with that for small x phenomena at HERA. Taking for example the STAR highest rapidity ($y = 4$) and $\langle p_T \rangle = 1.3$ GeV/c bin [7] we find that $x_N \geqslant 0.7$ for the incoming parton. Hence, minimal x_g resolved by such a parton are $\sim 4p_T^2/(x_N s_{NN}) \sim (2\text{–}3) \times 10^{-4}$. This is very close to the kinematics reached at HERA. The analyses of the HERA data within the dipole model approximation show that the partial amplitude for the quark interaction reaches at HERA strength up to 1/2 of the maximal strength, see review in Ref. [8]. In the case of heavy nuclei one gets an enhancement factor $\sim 0.5A^{1/3}$ so the quark interaction with heavy nuclei should be close to BDR for $p_t^2 \leqslant 1.5$ GeV^2 and $x_{\text{projectile}} \sim 0.5$. In the LHC kinematics BDR will cover much larger p_t^2 range, see for example Fig. 17 in Ref. [8].

First evidence for suppression of the forward spectra in the deuteron–gold collisions in the kinematics rather close to the BDR was reported by the BRAHMS [9], and further studied by PHENIX [10], and STAR [7]. High p_t spectra of h^- at $2 \leqslant y \leqslant 3.2$ are suppressed by a factor [9]

$$R^{h^-} = \frac{d\sigma^{d+A\to h^-+X}}{dy\, d^2 p_T} \Big/ 2A \frac{d\sigma^{p+p\to h^-+X}}{dy\, d^2 p_T}, \quad (2)$$

which is ≈ 0.8 for $y = 3.2$, $p_t = 2$ GeV/c. Since in the kinematics of the experiment $\sigma(pp \to h^- + X)/\sigma(pp \to h^+ + X) \leqslant 2$, the π^- yield produced by the proton projectile relative to that for the deuteron projectile (per nucleon) is substantially smaller. As a result in the case of the π^0 production which is produced with equal strength by protons and neutrons one expects a bigger suppression. For example $R^{\pi^0}_{d\text{Au}}(y = 3.2,\ p_t = 2\ \text{GeV}/c) \approx 0.55$ [11]. This suppression factor is significantly larger than expected suppression due to the leading twist nuclear shadowing. Suppression was observed in the kinematics where the hadron production in pp collisions is in a reasonable agreement with the recent pQCD calculations based on the NLO DGLAP approximation [12]. Very recently STAR [7] has reported new results for the π^0 ratios for $y \sim 4$ and $p_t \leqslant 2.0$ GeV. They observed a larger suppression $R^{\pi^0}_{d\text{Au}} \sim 1/3$, which is consistent with a linear extrapolation of $R^{h^-}_{d\text{Au}}$ to $y = 4$ taking into account the 2/3 factor due to the isospin effects [11].

The STAR experiment also reported the first observation of the correlations between the forward π^0 production with the production of the hadrons at the central rapidities $|\eta_h| \leqslant 0.75$. Such correlations provide a new information about the mechanism of the suppression of the inclusive spectrum.

The Letter is organized as follows. In Section 2 we evaluate energy losses of leading partons of the proton propagating through the nuclear medium in the kinematics of the onset of the BDR and find them to be > 10% for central proton-heavy ion collisions in the RHIC kinematics. In Section 3 we discuss expectations of the QCD BDR for the spectra of the leading particles. In Section 4 to disentangle interplay of soft and hard QCD phenomena we evaluate correlations between forward and central hadron production using information obtained in the BRAHMS experiment [9] on the dependence of the central multiplicity on the number of the wounded nucleons. We calculate the dependence of correlation parameters studied by STAR, on the number of wounded nucleons and find that the data require this number to be ~ 3, which is significantly smaller than the number of wounded nucleons for central impact parameters ~ 13 strongly suggesting dominance of the peripheral collisions in π^0 production. We also want to stress that the Letter considers the yield of partons with transverse momenta $\leqslant$ than that typical for the BDR. At the same time pion production with transverse momenta significantly larger than that typical for BDR should be dominated by the scattering at central impact parameters. For example, color glass condensate (CGC) inspired models predict for this case enhancement of production at central impact parameters by the factor of $\approx A^{1/6}$ [13]. In Section 5 we perform a detailed analysis of the correlation observables within peripheral models of pion production constrained to reproduce the inclusive data. We reproduce the observed values of the correlation parameters and find that the suppression of the correlation parameter related to production of recoil jets observed by STAR [7] is due to soft interactions and does not indicate suppression of the pQCD mechanism of the production of the recoil jets. Thus RHIC data are consistent with the pattern expected energy losses in central collisions, cf. Ref. [11]. In Section 6 we suggest several new observables which could allow to diminish model dependence of comparison between the hard components of the interaction in pp and dAu cases, quantitative study of the suppression on the number of wounded nucleons, which also will provide a probe of the color transparency effects as well as effects of large gluon fields.

2. Energy losses of forward parton in the vicinity of black disk regime

Energy losses for the parton propagation through the nucleus medium are dominated in moderate x processes by its elastic rescatterings off the constituents of the media due to the Coulomb gluon exchange. Therefore they depend weakly on energy and proportional to the square of distance propagated by the parton [1].

However, the amplitude with color octet quantum numbers decreases with energy due to the gluon reggeization in pQCD as [14,15]:

$$A_g \propto \alpha_s^2 s^{\beta(t)}\bigl(i + \tan\bigl(\pi\beta(t)/2\bigr)\bigr), \tag{3}$$

where $\beta(t)$ is the gluon Regge trajectory with $\beta(t=0) < 1$. Infrared divergences of $\beta(t)$ are regulated by hadron wave functions. At the same time the amplitude due to exchange by a ladder with the vacuum quantum numbers in the crossed channel rapidly grows with energy:

$$A \propto \alpha_s^2 s^{(1+\lambda(t))}\bigl(i + \tan\bigl((\pi/2)\lambda(t)\bigr)\bigr), \tag{4}$$

where $\lambda(t=0) \approx 0.2$. (For the simplicity we restrict ourselves here by the phenomenological fit to the theoretical formulae and to the HERA data on structure functions of a proton.) Hence such amplitudes (modeled at moderately small x as the two gluon exchange ladder) fastly exceed single gluon exchange term and at larger energies achieve maximum values permitted by probability conservation.

Thus dominance of elastic collisions breaks down at high energies leading to the regime where incoherent processes and incoherent energy losses dominate leading to the loss of finite fraction of initial energy of a parton, cf. Ref. [16]. This is the major difference from moderate x processes considered in [1] where coherent energy losses seems to dominate. Consequently, single inelastic collision of the parton produced in a hard high energy NN collision off another nucleon is described by the imaginary part of the two gluon ladder with the vacuum quantum numbers. By definition, the inelastic cross section is calculable in terms of the probability of inelastic interaction, $P_{\rm inel}(b)$ of a parton with a target at a given impact parameter b [17]:

$$\sigma_{\rm inel} = \int d^2b\, P_{\rm inel}\bigl(b, s, Q^2\bigr). \tag{5}$$

Since $\sigma_{\rm inel}$ is calculable in QCD [18] above equation helps to calculate $P_{\rm inel}(b, s, Q^2)$. The probability of inelastic interaction of a quark is cf. [8,19]:

$$P_{\rm inel}\bigl(b, x, Q^2\bigr) = \frac{\pi^2}{3}\alpha_s\bigl(k_t^2\bigr)\frac{\Lambda}{k_t^2} x G_A\bigl(x, Q^2, b\bigr), \tag{6}$$

where $x \approx 4k_t^2/s_{qN}$, $Q^2 \approx 4k_t^2$, $\Lambda \sim 2$ (for the gluon case $P_{\rm inel}(b)$ is 9/4 times larger). We use gluon density of the nucleus in impact parameter space, $G_A(x, Q^2, b)$ ($\int d^2b\, G_A(x, Q^2, b) = G_A(x, Q^2)$). Above equation for the probability of inelastic interaction is valid only for the onset of BDR when $P_{\rm inel}(b, s, Q^2) < 1$ (which is the unitarity limit for $P_{\rm inel}(b, s, Q^2)$).

If $P_{\rm inel}(b, x, Q^2)$ as given by Eq. (6) approaches one or exceeds one it means that average number of inelastic interactions, $N(b)$ becomes larger than one. Denoting as $G_{\rm cr}(x, Q^2, b)$ for which $P_{\rm inel}(b)$ reaches one we can evaluate $N(b, x, Q^2)$ as

$$N\bigl(b, x, Q^2\bigr) = G_A\bigl(x, Q^2, b\bigr)/G_{\rm cr}\bigl(x, Q^2, b\bigr). \tag{7}$$

As soon as $P_{\rm inel}$ becomes close to one, we can easily evaluate lower boundary for the energy losses arising from the single inelastic interaction of a parton. This boundary follows from the general properties of the parton ladder. Really, the loss of finite fraction of incident parton energy $-\epsilon$ arises from the processes of parton fragmentation into mass M which does not increase with energy. For binary collision $M^2 = \frac{k_t^2}{\epsilon(1-\epsilon)}$. For the contribution of small $\epsilon \leqslant 1/4$

$$\epsilon \approx k_t^2/M^2. \tag{8}$$

Here k_t is transverse momentum of incident parton after inelastic collision. The spectrum over the masses in the single ladder approximation (NLO DGLAP and BFKL approximations) is as follows

$$d\sigma \propto \int dM^2/M^2\bigl(s/M^2\bigr)^\lambda \theta\bigl(M^2 - 4k_t^2\bigr), \tag{9}$$

where we accounted for the high energy behavior of the two gluon ladder amplitude Eq. (4). We effectively take into account the energy–momentum conservation i.e. NLO effects. Consequently the *average* energy loss (for the contribution of relatively small energy losses ($\epsilon \leqslant \gamma \sim 1/4$) where approximation of Eq. (8) is valid):

$$\epsilon_N \equiv \langle\epsilon\rangle = \frac{\int_0^\gamma \epsilon\, d\epsilon/\epsilon^{1-\lambda}}{\int_0^\gamma d\epsilon/\epsilon^{1-\lambda}} = \gamma\frac{\lambda}{1-\lambda}. \tag{10}$$

For the realistic case $\gamma = 1/4$, $\lambda = 0.2$ this calculation gives the fractional energy loss of 6%. This is lower limit since we neglect here a significant contribution of larger ϵ (it will be calculated elsewhere).

In the kinematics of onset of BDR effective number of inelastic interactions becomes significantly larger than 1 so in the evaluation of fractional energy loss one should multiply evaluated above energy loss by the factor: $N(b)$. In addition one should account for the phenomenon specific for a quantum field theory in small x regime. The sum of Feynman diagrams which leads to eikonal contribution at moderately small x is canceled out at large energies as the consequence of the causality i.e. analytic properties of amplitudes and their decrease with the parton virtuality, cf. Refs. [3,4] and for the generalization to QCD [5,6]) and/or energy–momentum conservation, cf. [5,8]. At central impact parameters absorption at high energies is due to $N(b) > 1$ inelastic collisions (interaction with several ladders). The energy of initial parton is shared before collisions at least between N constituents in the wave function of the incident parton to satisfy causality and energy–momentum conservation. This quantum field theory effect which is absent in the framework of eikonal approximation can be interpreted as an additional energy loss [8]:

$$\epsilon_A(b) \approx N(b)\epsilon_N. \tag{11}$$

Here ϵ_N is the energy lost due to exchange by one ladder—Eq. (10). Above we do not subtract scattering off nucleon since our interest in the Letter is in energy losses specific for nuclear processes in the regime when interaction with a single nucleon is still far from the BDR. If collision energies are far from BDR, the energy losses estimated above should be multiplied by small probability of secondary interactions. Inclusion of enhanced "pomeron" diagrams will not change our conclusions based on the necessity to account for the energy–momentum conservation law.

Yields of leading hadrons carrying fraction of projectile momentum $\geqslant x_F$ are rapidly decreasing with x_N as $\propto (1 - x_F)^n$. For pion production $n \sim 5$–6. Obviously for large x_F average values of x for progenitor parton are even larger, leading to strong amplification of the suppression due to the energy losses. The spectrum of leading pions is given in pQCD by the convolution of the quark structure function, $\propto (1 - x)^n$, $n \sim 3.5$ and the fragmentation function $\propto (1 - z)^m$, $m \sim 1.5$–2 leading to a very steep dependence on x_F, $\propto (1 - x_F)^{n+m+1}$. As a result for the STAR kinematics $x \sim 0.7$ and $z \sim 0.8$ correspondingly energy losses of 10% lead to a suppression roughly by a factor $[(0.9 - x_F)/(1 - x_F)]^6$. For $x_F = 1/2$ this corresponds to suppression by a factor of four. In particular, introducing the energy loss of $\sim 6\%$ in the NLO calculation of the pion production is sufficient [11] to reproduce the suppression observed by BRAHMS [9]. Similar estimate shows that average losses of ~ 8–10% reproduce the suppression of the inclusive yield observed by STAR [7]. This value is of the same magnitude as the above estimate. Also, Eq. (11) leads to much stronger suppression for production at central impact parameters than in peripheral collisions.

In the kinematics of LHC the same $k_t(\mathrm{BDR})$ would be reached at x_N which are smaller by a factor $s_{\mathrm{RHIC}}/s_{\mathrm{LHC}} \sim 10^{-3}$, while for the same x_N one expects much larger values of $k_t(\mathrm{BDR})$ (see e.g. Fig. 17 in [8]). Thus in the kinematics of LHC the regime of large energy losses should extend to smaller x_N.

There are two effects associated with the interaction of partons in the BDR—one is an increase of the transverse momenta of the partons and another is the loss of the fraction of the longitudinal momentum [16]. The net result is that distribution of the leading hadrons should drop much stronger with x_F than in the CGC models [20] where only k_t broadening, change of the resolution scale and suppression of coalescence of partons in the final state but not the absorption and related energy losses were taken into account. At the same time, the k_t distribution for fixed x_F should be broader. Note here that the leading particle yield due to the scattering with $k_t \gg k_{\mathrm{BDR}}$ is not suppressed and may give a significant contribution at smaller k_t via fragmentation processes.

This discussion shows that selection in the final state of the leading hadron ($x_F \geqslant 0.3$–0.5 at RHIC) with moderately large k_t should strongly enhance the relative contribution of the peripheral collisions where BDR effects are much smaller. We will demonstrate below that these expectations are consistent with the STAR data.

At extremely high energies where kinematics of the BDR will be achieved for a broad range of the projectile's parton light-cone fractions and virtualities, QCD predicts dominance of scattering off the nuclear edge leading to:

$$\frac{d\sigma^{p+A\to\pi+X}}{dx_N\,dp_t^2}\Big/\frac{d\sigma^{p+p\to\pi+X}}{dx_N\,dp_t^2} \propto A^{1/3}, \tag{12}$$

for a large enough x_N and a wide range of p_t. With increase of incident energy the range of p_t for fixed x_N would increase. Also the suppression for a given p_t would be extended to smaller x_N.

3. Interaction of leading partons with opaque nuclear medium

At high energies leading partons with light cone momentum x_N, p_t are formed before nucleus and can be considered as plane wave if

$$(x_N s/m_N)\big(1/M^2\big) \gg 2R_A. \tag{13}$$

Here M is the mass of parton pair (and bremsstrahlung gluon) produced in the hard collision. If sufficiently small x are resolved, the BDR regime would be reached:

$$4p_t^2/x_N s \leqslant x(\mathrm{BDR}). \tag{14}$$

In the BDR interaction at impact parameters $b \leqslant R_A$ is strongly absorptive as the medium is opaque. As a result, interaction of leading parton lead to a hole of radius R_A in the wave function describing incident parton. Correspondingly, propagation of parton at large impact parameters leads to elastic scattering—an analogue of the Fraunhofer diffraction of light off the black screen. However since the parton belongs to a nucleon, the diffraction for impact parameters larger than $R_A + r_{\mathrm{str}}$ (where r_{str} is the radius of the strong interaction) will lead to the proton in the final state—elastic pA scattering. Only for impact parameters $R_A + r_{\mathrm{str}} > b > R_A$ the parton may survive to emerge in the final state and fragment into the leading hadron. Cross section of such diffraction is $2\pi R_A r_{\mathrm{str}}$. Another contribution is due to the propagation of the parton through the media. This contribution is suppressed due to fractional energy losses which increase with the increase of energy, leading to gradual decrease of the relative contribution of the inelastic mechanism (see discussion in Section 5).

Thus we predict that in the kinematics when BDR is achieved in pA but not in pN scattering, the hadron inclusive cross section should be given by the sum of two terms—scattering from the nucleus edge which has the same momentum dependence as the elementary cross section and scattering off the opaque media which occurs with large energy losses:

$$\frac{d\sigma(d+A\to h+X)/dx_h\,d^2p_t}{d\sigma(d+p\to h+X)/dx_h\,d^2p_t}=c_1A^{1/3}+c_2(A)A^{2/3}. \quad (15)$$

The coefficient c_1 is essentially given by the geometry of the nucleus edge—cross section for a projectile nucleon to be involved in an inelastic interaction with a single nucleon of the target. Coefficient $c_2(A)$ includes a factor due to large energy losses and hence it decreases with increase of the incident energy for fixed x_h, p_t. Deep in the BDR the factor $c_2(A)$ would be small enough, so that the periphery term would dominate.

It is worth to compare outlined pattern of interaction in the BDR with the expectations of the CGC models for small x hard processes in the kinematics where transverse momenta of partons significantly larger than that characteristic for BDR. These models employ the LO BFKL approximation with saturation model [21] used as initial condition of evolution in $\ln(x_0/x)$, see [22] and references therein. In these models the dependence on atomic number is hidden in the "saturation scale" and in the blackness of interaction at this scale. In this model partons interact with maximal strength at small impact parameters without significant loss of energy. Note that leading parton looses significant fraction of incident energy in the NLO BFKL approximation but not in LO BFKL [23]. As a result the cross section is dominated by the scattering at small impact parameters and depends on A at energies of RHIC approximately as $A^{5/6}$ [13]. Also, the process which dominates in this model at central impact parameters is the scattering off the mean field leading (in difference from BDR where DGLAP approximation dominates in the peripheral processes in the kinematics of RHIC) to events without balancing jets. With increase of jet transverse momenta interaction becomes less opaque, leading to a graduate decrease of the probability of inelastic collisions and hence to the dominance of the volume term.

A natural way to distinguish between these possibilities is to study correlations between production of forward high p_t hadrons and production of hadrons at central rapidities. First such study was undertaken by the STAR experiment [7].

4. Hadron production in soft nucleon–nucleus interactions at central rapidities

The STAR experiment reported correlations between the leading pion trigger and central leading charged hadron production. The procedure picks a midrapidity track with $|\eta_h|\leqslant 0.75$ with the highest $p_T\geqslant 0.5$ GeV$/c$ and computes the azimuthal angle difference $\Delta\phi=\phi_{\pi^0}-\phi_{\mathrm{LCP}}$ for each event. This provides a coincidence probability $f(\Delta\phi)$. It is fitted as a sum of two terms—a background term, $B/2\pi$, which is independent of $\Delta\phi$ and the correlation term $S(\Delta\phi)$ which is peaked at $\Delta\phi=\pi$. By construction,

$$\int_0^{2\pi} f(\Delta\phi)\,d\Delta\phi=B+\int_0^{2\pi} S(\Delta\phi)\,d\Delta\phi\equiv B+S\leqslant 1. \quad (16)$$

We will argue below that the A-dependence of B and S is sensitive to dependence of the leading pion production on the centrality of the collision.

The low p_t (soft) particle contribution which is uncorrelated in ϕ with the trigger originates both from the collisions of the second nucleon of the deuteron with the nucleus and from interactions with nucleon involved in the hard collision with several nucleons. This contribution should grow with A since the low p_t hadron multiplicity for $y\sim 0$ increases with A. To make quantitative estimate of this contribution we will make an approximation that the rate of these soft processes is weakly correlated with production of the forward pion provided *we compare the processes at the same impact parameter*. This natural assumption is valid in a wide range of models including CGC models. It is consistent also with the information provided by STAR on the weak dependence of the central multiplicity on x_F of the trigger pion, and lack of long range rapidity correlations for low p_t processes which was observed in many studies of hadron–hadron collisions. Note that at the LHC energies one would have to correct this approximation for the correlation of soft and hard interactions in the elementary interactions due to more localized transverse distribution of the valence partons, see discussion in Ref. [8].

Based on generic geometric considerations one expects that the multiplicity should be a function of the number of nucleons on the projectile nucleon impact parameter. Within this approximation to estimate effects of soft production on the correlation observables we can use information on the impact parameter dependence of the hadron multiplicity which is available from several dA RHIC experiments.

Using the BRAHMS data [9] we find that R^h for the STAR cuts can be roughly described by a simple parametrization

$$R^h=\left(\frac{N_{\mathrm{coll}}}{2}\right)^{-r}, \quad (17)$$

with $r\sim 0.2$. Here the factor of two in the denominator takes into account that each of the nucleons of the deuteron experiences, on average, equal numbers of collisions. For example, for an average number of collisions $N_{\mathrm{coll}}\approx 7.2$, Eq. (17) gives $R^h=0.77$ while the BRAHMS data reports $R^h=0.7$–0.75.

Note in passing, that the Gribov–Glauber approximation for the hadron–nucleus scattering combined with AGK cutting rules [24] which neglects energy conservation leads to $R^h=1$. If one takes into account energy conservation—the split of the energy between N_{coll}, and the increase of the central multiplicity with energy $\propto s^{0.2}$ one roughly reproduces Eq. (17).

First we want to find out what information about centrality of the interactions leading to production of the leading pion is contained in the A-dependence of B, the probability that a fast hadron within the experimental cuts does not belong to the recoil jet. Obviously, with an increase in the number of nucleons in the nucleus involved in the interactions practically all events would contain at least one particle in the cuts of STAR leading to B very close to one even if the elementary hard interaction is not affected by the nuclear environment. Using Eq. (17) we can express B for collisions with n nucleons, B_n through characteristics measured for pp collisions.

Let us denote the probabilities to produce a hadron within the central cuts of STAR due to soft and hard interactions in pp collisions by p_B and p_S respectively. Since the p_T cut of STAR is rather high (comparable to the momentum of the leading hadron in the recoiling jet for the trigger jet with $\langle p_T\rangle \sim 1.3$ GeV$/c$), we will assume that in the pp events where both soft and hard mechanisms resulted in the production of a hadron (hadrons) within the STAR cuts there is an equal probability for the fastest hadron to belong to either the soft or hard component (this is essentially an assumption of a reasonably quick convergence of the integrals over p_T for $p_{T\,\min} = 0.5$ GeV$/c$).[1] Within this assumption, the probability to produce no fast hadrons is $(1 - p_B)(1 - p_S)$; the probability to produce a fast hadron from the background and not from hard process is $p_B(1 - p_S)$; probability to produce a fast hadron in hard process and not in the background is $p_S(1 - p_B)$, and $p_S p_B$ is the probability to produce two fast hadrons—one in the background and one in the hard process. Since the last outcome contributes equally to B_{pp} and S_{pp} we have

$$B_{pp} = p_B(1 - p_S/2), \qquad S_{pp} = p_S(1 - p_B/2). \tag{18}$$

Since S_{pp} is small, then to a very good approximation the solution of Eq. (18) is $p_B = B_{pp}(1 + S_{pp}/(2 - B - S))$, $p_S = S_{pp}(1 + B_{pp}/(2 - B - S))$. Hence p_B is slightly larger than B, while for p_S a relative correction is significantly larger.

We can now calculate the probability that no hadrons will be produced in the inelastic collision of a nucleon with m nucleons of the nucleus:

$$(1 - B - S)_{m\,\text{collisions}} = (1 - p_B)^m(1 - p_S). \tag{19}$$

Using STAR data for $S + B$ we find $m = 2.8$. It is easy to check that, due to $p_S \ll 1$, this estimate of m is insensitive to the presence of two contributions to the multiplicity.

The same picture allows one to estimate the value of S for dAu collisions. Qualitatively, we expect that S should drop as more hadrons are produced in soft collisions and the chance for the fastest hadron to be attributed to the recoiling jet becomes smaller. In the case of an inelastic collision of a nucleon with N nucleons of the nucleus, the probability that in exactly m soft interactions a fast hadron would be produced, and that also a fast nucleon would be produced in a hard collisions is $p_S C_N^m p_B(1 - p_B)^{N-m}$. For these events there is $\approx 1/(1 + m)$ chance that the fastest hadron would belong to the hard subprocess. Summing over m we obtain

$$S_{N\,\text{collisions}} = p_S \sum_{m=0}^{m=N} \frac{C_N^m(1 - p_B)^{N-m} p_B^m}{(m + 1)}. \tag{20}$$

Taking $N \sim 3$ we find $S(d\text{Au}) \approx 0.1$ which agrees well with the data. Thus we conclude that the increase of the associated soft multiplicity explains the reduction of S observed in the data without invoking any suppression of the recoil hadron production on the level of the hard subprocess.

We have checked that accounting for the decrease of the soft multiplicity per N_{coll} leads to a small increase in our estimate (see also below).

One can see from these equations that if the contribution of the central impact parameters ($N_{\text{coll}} \sim 13$) were dominating in the π^0 production like in the CGC models one would obtain $(1 - B - S)$, $S \ll 0.01$ which is in a qualitative contradiction with the data. One would reach this conclusion even in the case of color transparency for the interaction of the nucleon involved in the hard interaction as the second nucleon would still experience ~ 6.5 interactions. To compare predictions of the CGC models with data one should trigger for events at central impact parameters and look for suppression of recoil jets in such collisions using the method described in Section 6.

Note also that a simple test of the relative importance of the central and peripheral mechanisms of the pion production is the ratio of the total hadron multiplicity in the events with the pion trigger and in the minimal bias events. We expect this ratio to be

$$(N_{\text{trigger}}/N_{\text{min.bias}})^{0.8} \approx (3/7.2)^{0.8} \approx 1/2.$$

At the same time in the CGC model this ratio should be larger than one, since the relative contribution of the central impact parameters is enhanced as compared to the minimal bias sample by a factor $A^{1/6} \sim 2.4$ [13]. Unfortunately, information about this ratio was not released so far by the STAR Collaboration.

5. The distribution over the number of collisions

In the previous section we calculated B and S for a fixed number of collisions. In a more realistic calculation we need to take into account distribution over the number of collisions. The important constraint here is that the suppression factor, $R_{d\text{Au}}$, for inclusive π^0 production is $R_{d\text{Au}} \sim 0.3$. This requires that at least nucleons with the impact parameter $b \geqslant b_{\min}$ satisfying condition

$$\int d^2b\, T_A(b)\theta(b - b_{\min}) = R_{d\text{Au}}, \tag{21}$$

should contribute to the inclusive pion yield. Here $T_A(b)$ is the conventional optical density which is expressed through the nuclear matter density $\rho_A(r)$, $\int d^3r\, \rho_A(r) = 1$ as

$$T_A(b) = \int_{-\infty}^{+\infty} dz\, \rho_A\big(\sqrt{b^2 + z^2}\big). \tag{22}$$

Condition of Eq. (21) corresponds to $b \geqslant 5$ fm for $R_{d\text{Au}} = 0.3$. For collisions with $b \sim 5$ fm the average number of collisions is already larger than 3. However, the presence of the more peripheral collisions still may lead to an average number of collisions close to 3.

To simplify the discussion we will consider the case of pA scattering and later on correct for the presence of the second

[1] Hereafter we are making an implicit assumption that one can neglect production of two hadrons from the soft or hard pp interactions within the experimental cuts. In the case of soft interactions this is justified both by small overall multiplicity and presence of short-range negative correlations in rapidities. In the case of hard process this is justified by a relatively small value of the p_T of the trigger. Obviously one can improve this procedure by using information from the STAR experiment which is not available yet.

nucleon in the projectile. In the probabilistic/geometrical picture one can rewrite the inelastic cross section $\sigma_{\rm in}^{pA}$ as a sum of cross sections with exactly m inelastic interactions [25]:

$$\sigma_{\rm in}^{pA} = \sum_{m=1}^{m=A} \sigma_m,$$
$$\sigma_m = \frac{A!}{(A-n)!n!} \times \int d^2b \left(T_A(b)\sigma_{\rm in}^{NN}\right)^m \left(1 - T_A(b)\sigma_{\rm in}^{NN}\right)^{A-m}. \tag{23}$$

These partial cross sections satisfy the sum rule $\sum_{m=1}^{m=A} m\sigma_m = A\sigma_{\rm in}^{NN}$. As a result, if the emission of the particles in each inelastic interaction is the same as in the NN collisions, all shadowing corrections are canceled reflecting the AGK cancellation [24].

To model distribution over the number of soft interactions, we need to introduce a suppression factor $SF(b)$ which is a function of the nuclear density per unit area at a given optical density which is given by $T(b)$.

Using this model we can calculate the A-dependence of the quantities measured by STAR taking into account the distribution over the number of the collisions:

$$(1-B-S) = \frac{\sum_{m=1}^{m=A} (1-B-S)_{m\,\rm collisions} \int \sigma_m(b) SF(b)\, d^2b}{\sigma_{\rm in}^{NN} R_{d\rm Au}^{\pi^0}}, \tag{24}$$

where $\sigma_m(b)$ is the integrand of Eq. (23) and $(1-B-S)_{m\,\rm collisions}$ is given by Eq. (19).

Similarly, to calculate $S(p\rm Au)$ we can combine Eqs. (20), (23) to find

$$S = \frac{\sum_{m=1}^{m=A} S_{m\,\rm collisions} \int \sigma_m(b) SF(b)\, d^2b}{\sigma_{\rm in}^{NN} R_{d\rm Au}^{\pi^0}}. \tag{25}$$

For a numerical study we choose two models inspired by the energy loss estimate of Section 2 for interaction near BDR and the regime of absorption deep in the BDR (cf. Eq. (15))[2]:

$$SF^{\{1\}}(b) = \left(1 + a_1 T_A(b)\right)^{-1},$$
$$SF^{\{2\}}(b) = \left(1 + a_2 T_A(b)\right)^{-2}, \tag{26}$$

with the parameter $a_1 = 2.5$, $a_2 = 1.63$ fixed by the condition $\int T_A(b) SF^i(b)\, d^2b = R_{d\rm Au}^{\pi^0} = 0.286$ as measured by STAR for the higher p_T correlation bin corresponding to averaging over $30 < E_\pi < 55$ GeV [7]. We found that the two models of suppression give very similar results for the observables measured experimentally with the second model for $SF(b)$ giving slightly larger values of S and $(1-B-S)$ since it yields a stronger suppression of scattering at the central impact parameters (the suppression factor is ~ 5.4 in the first model and ~ 20 in the second model). The numerical values are $S = 0.068$ and 0.075; $(1-B-S) = 0.070$ and 0.086. If we try to model the decrease of the multiplicity per wounded nucleon in line with the BRAHMS data we naturally find an increase of S, $(1-B-S)$: $S = 0.085$ and 0.090; $(1-B-S) = 0.11$ and 0.12.

However, in the actual experiment the dAu interaction was studied. In this case, the average number of collisions is about a factor 1.4–1.5 higher due to the interaction of the second nucleon. We can make a rough estimate of this effect by substituting collisions in pA scattering when the proton experiences N inelastic interactions by a superposition with equal probabilities of N and $2N$ inelastic collisions. Clearly, a more detailed modeling of dAu interactions is necessary—we will address it elsewhere. We find, when we account for the energy splitting $S = 0.067$ and 0.072; $(1-B-S) = 0.066$ and 0.079. These numbers should be compared with $(1-B-S) = 0.1$, $S = 0.093 \pm 0.04$ measured by STAR for the higher p_T bin which we analyze here. This suggests that, with inclusion of the second nucleon interaction in the Gribov–Glauber model, one gets a somewhat larger suppression of the jets than reported experimentally[3] and a smaller probability not to observe any hadrons than the one observed experimentally leaves room for effects due possible deviations from the geometrical picture. One effect of such kind which was suggested in [26] is the presence of the color fluctuations the projectile nucleon. Selection of large x_F in the nucleon may select fluctuations with smaller interaction cross section and lower the number of the interactions. However, the observables used in the analysis are not very sensitive to the distribution over the number of interactions *as long as a peripheral model of π^0 production is used.*

To summarize, the results of our analysis of the data at higher p_T, the data are consistent with no suppression of the balancing hadron production for the trigger with $\langle p_T\rangle \sim 1.3$ GeV/c.[4] Lack of the suppression of the pQCD mechanism for $\langle x_A\rangle \sim 0.01$ which dominates in the correlation measurements of the STAR puts an upper limit on the x range where coherent effects may suppress the pQCD contribution. Since the analysis of [11] find that the pQCD contribution is dominated by $x_A \geqslant 0.01$, we can conclude that the main contribution both to inclusive and the correlated cross section originates from pQCD hard collisions at large impact parameters.

The observation of the recoil jets in the pp case with a strength compatible with pQCD calculations suggests that the mechanism for pion production in the STAR kinematics is predominantly perturbative so that it is legitimate to discuss the propagation of a parton through the nucleus leading to pion production. To ensure a suppression of the pion yield at central impact parameters for the discussed kinematics one needs a mechanism which is related to the propagation of the projectile parton which is generating a pion in a hard interaction with the $x \sim 0.01$ parton. For example, the rate of suppression

2 Use of two models allows us to test weak sensitivity of our conclusions to a choice of the specific model for dependence of suppression on the nuclear thickness.

3 If some of the pions were due to a production mechanism without a recoil jet, S would decrease increasing discrepancy with the data.

4 In the case of the lower p_T trigger data set, our estimate for the dAu scattering gives $(1-B-S) = 0.060$–0.072 and $S \sim 0.045$ which is a bit above the reported value of $S = 0.020 \pm 0.013$. However application of hard scattering picture in any case rather problematic for p_T(trigger) ~ 1 GeV/c.

observed by BRAHMS would require fractional energy losses $\sim 3\%$ both in the initial and final state [11]. Similar losses would produce a suppression of the pion yield in STAR kinematics comparable with the inclusive data. Modeling performed above using Eq. (26) indicates that for the central impact parameters the fractional energy losses should be at least a factor of 1.5 larger. Note here that such losses are sufficient only because the kinematics of the elementary process is close to the limit of the phase space. At the same time, this estimate assumes that fluctuations in the energy losses should not be large. For example, processes with energy losses comparable to the initial energy (like in the case of high energy electron propagation through the media) would not generate necessary suppression provided overall losses are of the order of few percent. Note also that the second jet in the STAR kinematics has much smaller longitudinal momentum and hence is far from the BDR. Therefore in the STAR kinematics one does not expect the suppression of the correlation with production of the second jet. However a strong suppression is expected for production of two balancing forward jets since both of them are interacting in the BDR.

Hence the data are qualitatively consistent with the scenario described in the introduction that leading partons of the projectile (with $x \approx 0.7$) interact at central impact parameters with the small x nuclear gluon fields with the strength close to the BDR.

6. Suggestions for future measurements to reveal onset of BDR

We have seen that the quantities used in the STAR analysis involve an interplay of the hard and soft interactions. Here we want to suggest a few other observables which allow either to suppress this interplay or to optimize the sensitivity to the number of the collisions.

First let us discuss another procedure for studies of the modification of the characteristics of the hard collisions which is significantly less sensitive to the properties of the soft interactions. Let us consider the ratio of the double inclusive and single inclusive cross sections for production of a particle in forward and in central kinematics which are characterized by their rapidities and transverse momenta:

$$RR(y_f, |p_{tf}|, y_c, |p_{tc}|, \phi) = \frac{d\sigma(y_f, p_{tf}, y_c, p_{tc})}{dy_f\, dp_{tf}\, dy_c\, dp_{tc}} \Big/ \frac{d\sigma(y_f, p_{tf})}{dy_f\, dp_{tf}}, \tag{27}$$

where ϕ is the angle between p_{tf} and p_{tc}. We can now introduce

$$\Delta RR(y_f, |p_{tf}|, y_c, |p_{tc}|, \phi) = RR(y_f, |p_{tf}|, y_c, |p_{tc}|, \phi) - RR(y_f, |p_{tf}|, y_c, |p_{tc}|, -\phi). \tag{28}$$

Similar to the logic of the STAR analysis, we expect that only hard contributions to the central production depend on ϕ. Hence, in the case of inclusive quantities like ΔRR the soft interactions are canceled, while (as we have seen above) this is not the case for the quantities considered in [7].

Consequently, the ratio of $\Delta RR^{d\mathrm{Au}}$ and ΔRR^{pp} can be used to study how the p_T, ϕ, η dependences of the balancing jets depend on A (obviously one can consider the ratio of ΔRR integrated over all but one variable). The STAR analysis used a $p_T \geqslant 0.5$ GeV/c cutoff to enhance the hard contribution. In our procedure one is likely to be able to use a smaller cutoff, or no cutoff at all. It appears that already current statistics of STAR would allow at least some of these measurements. Note here that the nuclear shadowing effects are more important for the positive rapidities of the recoil jets. Hence, a study of η dependence in the kinematics studied by STAR could constrain the leading twist shadowing effects between $0.005 < x < 0.02$, albeit for rather large impact parameters where shadowing is smaller than on average. Note in passing that the current estimates of the suppression of the inclusive pion yield due to nuclear shadowing overestimate effect as they do not take into account that the process is dominated by the scattering at large impact parameters.

It is worth emphasizing here, that for a large range of impact parameters, one is likely to be in the regime too close to BDR to apply the leading twist approximation for nuclear shadowing. At the same time the important feature of the leading twist nuclear shadowing is likely to hold, namely that $R_g = G_A(x, Q^2)/AG_N(x, Q^2) < 1$ is achieved due to simultaneous interactions with $1/R_g$ nucleons, leading to an increase in hadron multiplicity at central rapidities and in the nuclear fragmentation region [27].

To study the dependence of pion production on the number of collisions, one needs to study the multiplicity distribution of the soft particle production at central rapidities. As a first step one would have to deconvolute the hard contribution which would be well constrained by a study of ΔRR (though this is actually a rather small correction, especially for large multiplicities). The tail of the distribution at large multiplicities will determine the relative contribution of the collisions with several nucleons—for the cuts of STAR the average multiplicity should grow $\approx N_{\mathrm{coll}}/2$.

This program would allow for a study of how the regime of large energy losses sets in as a function of the gluon/nucleon transverse density. Such a study would have important implications for LHC since in the large energy losses scenario, enhancement of the losses as compared to LT pQCD calculations is due to the proximity to the unitarity limit. Consequently, one would expect large energy losses for a much larger range of rapidities at LHC for the same parton virtualities.

Two complementary methods to obtain information about the centrality of dependence of the very forward pion production would be to use information from the zero degree calorimeters (ZDC) along the deuteron and gold beams. Measuring the number of neutron spectators produced in the fragmentation of the deuteron would be drastically different for the peripheral and central impact parameters scenarios—in the peripheral case one expects an increase of the spectator neutron multiplicity as compared to the minimal bias events as the neutron in peripheral interactions has a significant chance to survive (provided the pion was emitted in the interaction of the proton which occurs with 50% probability). We postpone a quantitative description of this significant effect for future publications. At the same time, a chance for a neutron to escape in the central collisions would be very small: $\exp(-\sigma_{\rm in}T(b\sim 0)) \ll 10^{-2}$.[5] The ZDC measuring production of neutrons from the nucleus fragmentation would observe a smaller than minimal bias multiplicity for the peripheral scenario and a larger multiplicity in the central collisions scenario. An important advantage of these observables is that they are practically insensitive to the issue of the split of the energy between soft interactions. Hence, one can reduce uncertainties in the extraction procedure by employing information about the production of neutrons in generic dAu collisions, in particular in collisions with production of soft hadrons at central rapidities. It appears likely that such studies would substantially improve the determination of the $T(b)$ dependence of the suppression factor.

Future experiments at RHIC would allow one to separate large energy losses and leading twist nuclear shadowing. One would have to measure ΔRR as function of the rapidity. The shadowing effects would lead to drop of $\Delta RR^{d\mathrm{Au}}(\eta)/\Delta RR^{pp}(\eta)$ for forward rapidities where $x_A \sim 10^{-3}$ dominates. Also, if one would be able to decrease x_N for fixed small x_A, one would enhance the shadowing effects as compared to BDR effects. In this limit one would observe an increase in the associated multiplicity at the central rapidities since for the nuclear shadowing mechanism, central impact parameters give a large relative contribution.

A color transparency effect would be manifested in a number of collisions with small $N_{\rm coll}$ significantly larger than that given by the Glauber model. Obviously use of the proton beams would nicely complement studies with the deuteron beams as one would be able to compare triggers for centrality solely based on the interactions of one nucleon and on the interactions of two nucleons.

After our first version of our analysis was completed PHENIX released the results of their analysis of the correlations [28] which are rather similar to the procedure we advocate. They study hadron correlations for smaller rapidities and higher p_t than those studied by STAR which is far from the BDR and find no suppression of the correlations.

7. Conclusions and open questions

We have demonstrated that partons with large transverse momenta corresponding to rather large virtualities for which the BDR is reached should lose a substantial fraction of their energy. In the case of inclusive production of very forward pions this leads to the dominance of the scattering at peripheral impact parameters. For partons with $x_N \geqslant 0.5$ the energy losses for central impact parameters should lead to suppression of the inclusive yield at least by a factor of five which corresponds to energy losses $\geqslant 10\%$. As a result BDR leads to an extension to a wider p_t range of the pattern of the strong suppression of the leading hadron production at small p_t observed in the central pA collisions.

With increase of energy from RHIC to LHC energy losses at large x_N should strongly increase, while substantial losses $\geqslant 10\%$ should persist for rapidities $|y| \geqslant 2$. It appears that this should lead to increase of the densities in the central collisions as compared to the current estimates. It will also lead to suppression of the production of the recoil jets at the rapidity intervals where no suppression is present at RHIC. In the forward direction we expect a significantly larger suppression than already large suppression found in [20] where fractional energy losses were neglected. Fractional energy losses result in modification of the form of the QCD factorization theorem at LHC energies. In particular they lead to suppression of Higgs/SUSY particle production in pp scattering by at least $[(1-x/(1+\epsilon))/(1-x)]^{10}$. Here $x \geqslant m_H/\sqrt{s}$ are the light-cone fractions carried by initial gluons which initiate production of the Higgs particle with accompanying bremsstrahlung, and $\epsilon \geqslant 0.1$ is the fractional energy loss. This corresponds to a suppression $\geqslant 10\%$ for $m_H = 140$ GeV.

Further studies of the proton/deuteron–nucleus interactions at the central impact parameters at RHIC and future experiments at LHC would provide important constrains on this important ingredient of high energy dynamics. Similar effects will be present in the central pp collisions at LHC. They would amplify the correlations between the hadron production in the fragmentation and central regions discussed in Ref. [8].

Acknowledgements

We would like to thank Yu. Dokshitzer, S. Heppelmann, D. Kharzeev, L. McLerran, G. Rakness, R. Venugopalan, I. Vitev and W. Vogelsang for useful discussions. The work was supported by the DOE and BSF grants.

[5] Since the deuteron is weakly bound system, there is a significant tail in the wave function at distances $\geqslant 4$ fm making selection of pure central collisions sample very difficult. This problem would be greatly alleviated if one would study A-dependence of production of spectators with transverse momenta $\geqslant 200$ MeV/c which selects configurations with transverse separations $\leqslant 2$ fm.

References

[1] R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne, D. Schiff, Nucl. Phys. B 483 (1997) 291.
[2] F. Arleo, Phys. Lett. B 532 (2002) 231;
M. Gyulassy, P. Levai, I. Vitev, Phys. Rev. Lett. 85 (2000) 5535.
[3] S. Mandelstam, Nuovo Cimento 30 (1963) 1148.
[4] V.N. Gribov, The Theory of Complex Angular Momenta: Gribov Lectures on Theoretical Physics, Cambridge Univ. Press, Cambridge, 2003.
[5] B. Blok, L. Frankfurt, hep-ph/0611062.
[6] J. Bartels, L. Lipatov, G. Vacca, Nucl. Phys. B 706 (2005) 391.
[7] STAR Collaboration, J. Adams, et al., nucl-ex/0602011.
[8] L. Frankfurt, M. Strikman, C. Weiss, Annu. Rev. Nucl. Part. Sci. 55 (2005) 403, hep-ph/0507286.
[9] I. Arsene, et al., Phys. Rev. Lett. 93 (2004) 242303;
I. Arsene, et al., Phys. Rev. Lett. 91 (2003) 072305.
[10] PHENIX Collaboration, S.S. Adler, et al., Phys. Rev. Lett. 94 (2005) 082302.
[11] V. Guzey, M. Strikman, W. Vogelsang, Phys. Lett. B 603 (2004) 173, nucl-ex/0602011.
[12] F. Aversa, P. Chiappetta, M. Greco, J.P. Guillet, Nucl. Phys. B 327 (1989) 105;
B. Jager, A. Schafer, M. Stratmann, W. Vogelsang, Phys. Rev. D 67 (2003) 054005;

D. de Florian, Phys. Rev. D 67 (2003) 054004.
[13] D. Kharzeev, E. Levin, L. McLerran, Phys. Lett. B 561 (2003) 93;
D. Kharzeev, Y.V. Kovchegov, K. Tuchin, Phys. Rev. D 68 (2003) 094013.
[14] L.L. Frankfurt, V.E. Sherman, Sov. J. Nucl. Phys. 23 (1976) 581.
[15] L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338.
[16] L. Frankfurt, V. Guzey, M. McDermott, M. Strikman, Phys. Rev. Lett. 87 (2001) 192301, hep-ph/0104154;
L. Frankfurt, M. Strikman, Phys. Rev. Lett. 91 (2003) 022301, nucl-ex/0212094.
[17] L. Landau, E. Lifshitz, Quantum Mechanics: Non-relativistic Theory, in: Course of Theoretical Physics, vol. 3, Pergamon, London, 1977.
[18] B. Blaettel, G. Baym, L.L. Frankfurt, H. Heiselberg, M. Strikman, Phys. Rev. D 47 (1993) 2761;
L. Frankfurt, G.A. Miller, M. Strikman, Phys. Lett. B 304 (1993) 1, hep-ph/9305228;
L. Frankfurt, A. Radyushkin, M. Strikman, Phys. Rev. D 55 (1997) 98, hep-ph/9610274.
[19] L. Frankfurt, M. Strikman, C. Weiss, Phys. Rev. D 69 (2004) 114010, hep-ph/0311231.
[20] A. Dumitru, L. Gerland, M. Strikman, Phys. Rev. Lett. 90 (2003) 092301;
A. Dumitru, L. Gerland, M. Strikman, Phys. Rev. Lett. 91 (2003) 259901, Erratum.
[21] K. Golec-Biernat, M. Wusthoff, Phys. Rev. D 59 (1999) 014017, hep-ph/9807513.
[22] A. Dumitru, A. Hayashigaki, J. Jalilian-Marian, Nucl. Phys. A 770 (2006) 57, hep-ph/0512129.
[23] M. Ciafaloni, G. Camici, Phys. Lett. B 430 (1998) 349;
M. Ciafaloni, D. Colferai, Phys. Lett. B 452 (1999) 372, hep-ph/9812366;
V.S. Fadin, L.N. Lipatov, Phys. Lett. B 429 (1998) 127, hep-ph/9802290.
[24] V.A. Abramovsky, V.N. Gribov, O.V. Kancheli, Yad. Fiz. 18 (1973) 595, Sov. J. Nucl. Phys. 18 (1974) 308.
[25] L. Bertocchi, D. Treleani, J. Phys. G 3 (1977) 147.
[26] L.L. Frankfurt, M.I. Strikman, Nucl. Phys. B 250 (1985) 143.
[27] L. Frankfurt, M. Strikman, Eur. Phys. J. A 5 (1999) 293, hep-ph/9812322.
[28] PHENIX Collaboration, S.S. Adler, et al., nucl-ex/0603017.