Screening and antiscreening in anisotropic QED and QCD plasmas

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Abstract

We use a transport-theory approach to construct the static propagator of a gauge boson in a plasma with a general axially- and reflection-symmetric momentum distribution. Non-zero magnetic screening is found if the distribution is anisotropic, confirming the results of a closed-time-path-integral approach. We find that the electric and magnetic screening effects depend on both the orientation of the momentum carried by the boson and the orientation of its polarization. In some orientations there can be antiscreening, reflecting the instabilities of such a medium. We present some fairly general conditions on the dependence of these effects on the anisotropy.

1. Introduction

One of the important problems in non-equilibrium QED and QCD is to study how such systems approach equilibrium. In principle one should solve the Schwinger–Dyson equation to do this at a purely quantum level, but it is impossible to do this without significant approximations [1]. For many practical purposes one can use transport equations to study equilibration of QED and QCD plasmas with particle collisions taken into account. For details we refer the reader to the extensive reviews in the literature [2,3]. However the scattering kernel in these transport equations can suffer from severe infrared divergences due to ultra-soft photons or gluons exchanged in t-channel.

It is well known that the collective modes in the QED plasma screen the long-ranged Coulomb force [4,5]. The screened potential then has a Yukawa form in the medium:

\[ V(r) \propto e^{-m_D r} \, \frac{e^{-m_D r}}{r}, \]

where \( m_D \) is the Debye screening mass. As a result the infrared behaviour of the exchanged gauge bosons is improved. This Debye screening effect can be generalized to non-equilibrium QCD systems [6] and used in the transport equations describing quark–gluon plasmas [7]. However for magnetic interactions the infrared problems...
persist. In the case of QED in equilibrium the magnetic screening mass vanishes not just at one-loop level but to all orders in perturbation theory [8]. It also has been argued that in QCD the magnetic screening mass should be of the order of $g^2 T$, and cannot be computed perturbatively [5].

The situation is quite different in non-equilibrium systems, at least if they are anisotropic. Recently the closed-time-path-integral formalism was used to derive the magnetic screening mass at one-loop level [9]. This showed that the magnetic screening mass in QED is non-zero if the plasma has an anisotropic single-particle momentum distribution, i.e., it depends on the direction of the momentum. This would suggest that there is no natural infrared problem in such a system. More recently, however, Romatschke and Strickland [10] have used the hard thermal loop (HTL) approach to show how non-zero magnetic screening effects can arise in anisotropic plasmas. Their work shows that there can be antiscreening, in the sense of negative screening masses, as well as screening. This feature of out-of-equilibrium systems has also been noted by Arnold et al. [11]. It reflects the instabilities of these systems which have been studied by Mrowczynski [12,13]. (For a review of these ideas, see Ref. [14].) The instability of these systems implies that, even though some components of the magnetic interactions are screened, a simple perturbative treatment is still not adequate.

It is well known that the polarization tensor of a gauge boson derived from the HTL approach at leading order can also be obtained from a transport equation [3,15]. The equivalence of the two approaches has also been generalized to the case of anisotropic but homogeneous systems [16]. In this Letter we use the transport-equation approach to derive the same expression for the magnetic screening mass as was found in [9]. The advantages of this approach are that the physical picture is more transparent and its classical character is emphasized. We present the general form of the propagator of a gauge boson in an anisotropic medium. This propagator is central to the scattering kernel of the transport equations [7]. The structure of this propagator agrees with that obtained by Romatschke and Strickland [10] who assumed a more restricted functional form for the anisotropic distribution of particles in the plasma. A minor technical difference from Ref. [10] is that we work in Lorentz gauge rather than the temporal axial gauge, but we have checked that our results are equivalent to theirs.

The propagator is more complicated than in the equilibrium case, since it depends on the orientations of the field momentum and polarization relative to the axes of the anisotropy of the system. As a result the screening or antiscreening of a transverse magnetic field can itself be very anisotropic. Under some fairly general assumptions about the form of the anisotropy, which are likely to cover most cases of physical interest, including those studied in Ref. [10], we are able to derive conditions on the appearance of screening or antiscreening in particular orientations.

The Letter is organized as follows. In Section 2 we present the derivation of the polarization tensors in terms of the non-equilibrium distribution function from transport equations. In Section 3 we discuss the screening of static fields and, in particular, the form of the static polarization tensor for systems with cylindrical symmetry. We summarize our results and their implications in Section 4.

2. Derivation of the polarization tensor from transport equations

The polarization tensor at one-loop level for a gauge boson in a plasma can be derived from classical transport equations [15] even if the system is out of equilibrium [16]. As a simple example, consider first the case of a gas of electrons interacting with a weak, spacetime dependent electromagnetic field. The single-particle distribution function $f(x, p, t)$ obeys the Boltzmann equation,

$$\frac{\partial f(x, p, t)}{\partial t} + v \cdot \nabla_x f(x, p, t) + \mathbf{F}(x, t) \cdot \nabla_p f(x, p, t) = 0,$$  \hspace{1cm} (2)

where $v = p/E(p)$ is the velocity of a particle with momentum $p$ and

$$\mathbf{F}(x, t) = e\left[\mathbf{E}(x, t) + v \times \mathbf{B}(x, t)\right],$$ \hspace{1cm} (3)

in terms of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$. 


We assume that on the distance- and time-scales of interest the plasma is close to homogeneous and so we can expand \( f(x, p) \) around an unperturbed distribution function \( f_0(p) \) as follows:

\[
f(x, p, t) = f_0(p) + f_1(x, p, t).
\]

This expansion is similar to the one which is usually used in the derivation of the transport coefficient except that \( f_0(p) \) does not need to be the equilibrium Fermi–Dirac distribution. The transport equation (2) to first order in \( f_1 \) and the electromagnetic fields is

\[
\frac{\partial f_1}{\partial t} + v \cdot \nabla f_1 = -\mathbf{F} \cdot \nabla f_0.
\]

Solving this for \( f_1 \) yields

\[
f_1(x, p, t) = -e \int_{-\infty}^{t} \frac{d^3 p}{(2\pi)^3} v_\mu f_0(p) \cdot \left[ \int_{-\infty}^{\infty} d\tau \left[ \mathbf{E}(t', x - v(t - t')) + v \times \mathbf{B}(t', x - v(t - t')) \right] \right]
\]

\[
= -e \int_{-\infty}^{t} \frac{d^3 p}{(2\pi)^3} v_\mu f_0(p) \cdot \left[ \int_{0}^{\infty} d\tau \left[ \mathbf{E}(t - \tau, x - \tau v) + v \times \mathbf{B}(t - \tau, x - \tau v) \right] \right].
\]

The induced current density is given by

\[
j^{\text{ind}}_{\mu}(x, t) = e \int \frac{d^3 p}{(2\pi)^3} v_\mu f_1(x, p, t),
\]

where \( v^\mu = p^\mu / E(p) = (1, \mathbf{v}) \). Using the above expression for \( f_1(x, p, t) \), we find that the induced current can be expressed in terms of the fields as

\[
j^{\text{ind}}_{\mu}(x) = -e^2 \int \frac{d^3 p}{(2\pi)^3} v_\mu \nabla_p f_0(p) \cdot \left[ \int_{0}^{\infty} d\tau \left[ \mathbf{E}(t - \tau, x - \tau v) + v \times \mathbf{B}(t - \tau, x - \tau v) \right] \right].
\]

Taking the Fourier transform of this we get

\[
j^{\text{ind}}_{\mu}(k) = -i\epsilon^2 \int \frac{d^3 p}{(2\pi)^3} v_\mu \frac{\nabla_p f_0(p)}{v \cdot k + i\epsilon} \cdot \left[ \mathbf{E}(k) + v \times \mathbf{B}(k) \right].
\]

To get this result we have regulated the \( \tau \) integral by inserting a factor of \( e^{-\epsilon \tau} \) with \( \epsilon \to 0^+ \) and we have used the fact that the Fourier transform of \( \int_{0}^{\infty} d\tau e^{\epsilon \tau} f(x - \tau v) \) is given by \( if(k)/(v \cdot k + i\epsilon) \). For isotropic systems, the gradient \( \nabla_p f_0(p) \) is proportional to \( p \). The second term in Eq. (9) vanishes in such cases and so there is no response of the plasma to a magnetic field.

The Fourier components of the fields can be expressed in terms of the electromagnetic potential \( A^\mu(k) \) as

\[
\mathbf{B}(k) = i\mathbf{k} \times \mathbf{A}(k),
\]

\[
\mathbf{E}(k) = -i\mathbf{k} A_0(k) + i\epsilon_0 \mathbf{A}(k).
\]

This allows us to express the current in the form

\[
j^{\text{ind}}_{\mu}(k) = -i\epsilon^2 \int \frac{d^3 p}{(2\pi)^3} v_\mu \frac{\nabla_p f_0(p)}{v \cdot k + i\epsilon} \cdot \left[ -i\mathbf{k} A_0(k) + i\epsilon_0 \mathbf{A}(k) + i (v \cdot \mathbf{A}(k)) \mathbf{k} - i(v \cdot \mathbf{k}) \mathbf{A}(k) \right]
\]

\[
= -e^2 \int \frac{d^3 p}{(2\pi)^3} v_\mu \left[ \frac{\mathbf{k} \cdot \nabla_p f_0}{v \cdot k + i\epsilon} v \cdot \mathbf{A}(k) - \nabla_p f_0 \cdot \mathbf{A}(k) \right].
\]
We can introduce a polarization tensor, defined in terms of the induced current by
\[ j^\text{ind}_\mu(k) = \Pi_{\mu\nu}(k)A^\nu(k). \] (12)
The polarization tensor obtained from the induced current is of course only the matter part, arising from the gas of electrons. The full tensor also includes a vacuum part which can be calculated using the standard field-theoretic methods. After some algebra we get from Eq. (11) the matter polarization tensor in the Lorentz gauge,
\[ \Pi^00(k) = -e^2\int \frac{d^3p}{(2\pi)^3} \left[ \frac{\mathbf{k} \cdot \nabla_p f_0(p)}{p_0 - \mathbf{k} \cdot \mathbf{v} + i\epsilon} \right], \]
\[ \Pi^0i(k) = -e^2\int \frac{d^3p}{(2\pi)^3} \left[ \frac{\mathbf{k} \cdot \nabla_p f_0(p)}{p_0 - \mathbf{k} \cdot \mathbf{v} + i\epsilon} \right] v_i, \]
\[ \Pi^{ij}(k) = e^2\int \frac{d^3p}{(2\pi)^3} \left[ \frac{f_0(p)}{E(p)}(\delta_{ij} - v_i v_j) - \frac{\mathbf{k} \cdot \nabla_p f_0(p)}{p_0 - \mathbf{k} \cdot \mathbf{v} + i\epsilon} \right]. \] (13)
If the particles are massless (or the typical energies are high enough that we may neglect their masses), the velocity \( \mathbf{v} \) may be replaced by the unit vector \( \hat{\mathbf{p}} \) and the energy \( E(p) \) by \( |p| \).
To generalize this result to QCD, one should replace the spacetime derivatives in Eq. (2) by covariant derivatives. However, so long as \( gA_\mu \) is small, we can neglect the higher-order terms that this introduces. We can then describe the QCD plasma by equations with the same form as Eq. (2). The resulting contributions to the gluon polarisation tensor have same forms as for the photon except for a trivial factor which counts the relevant degrees of freedom.

In QCD case one simply needs to replace \( f(p) \) in the expression (13) for the polarization tensor by a sum of quark, antiquark and gluon terms [8], \( 2N_f f_q(p) + f_q(p) + 2N_c f_g(p) \). The factor of 2 before \( N_f \) is due to the spin degree of freedom.

3. Screening of static fields in an anisotropic plasma

The long-distance behaviour of the static potential is governed by the low-momentum behaviour of the static photon propagator. In the static limit, \( p_0 = 0 \), the polarization tensor Eq. (13) depends the direction of \( \mathbf{k} \) only. Denoting the static tensor by \( \tilde{\Pi}^{\mu\nu}(\mathbf{k}) \), we find that its components take the forms
\[ \tilde{\Pi}^{00}(\mathbf{k}) = e^2\int \frac{d^3p}{(2\pi)^3} \left[ \frac{|\hat{\mathbf{k}} \cdot \nabla_p f_0(p)}{\hat{\mathbf{k}} \cdot \mathbf{p} - i\epsilon} \right], \]
\[ \tilde{\Pi}^{0i}(\mathbf{k}) = e^2\int \frac{d^3p}{(2\pi)^3} \left[ \frac{|\hat{\mathbf{k}} \cdot \nabla_p f_0(p)}{\hat{\mathbf{k}} \cdot \mathbf{p} - i\epsilon} \right] \hat{p}_i, \]
\[ \tilde{\Pi}^{ij}(\mathbf{k}) = e^2\int \frac{d^3p}{(2\pi)^3} \left[ \frac{|f_0(p)|}{|p|}(\delta_{ij} - \hat{\mathbf{p}}_i \hat{\mathbf{p}}_j) + |p| \frac{\hat{\mathbf{k}} \cdot \nabla_p f_0(p)}{\hat{\mathbf{k}} \cdot \mathbf{p} - i\epsilon} \hat{p}_i \hat{p}_j \right]. \] (14)
in a plasma of massless particles.
In the familiar case of isotropic matter, the static polarisation tensor takes the form [4,5]
\[ \tilde{\Pi}^{\mu\nu} = \tilde{\Pi}_L L^{\mu\nu} + \tilde{\Pi}_T T^{\mu\nu}, \] (15)
where, in the rest frame of the matter, \( L^{\mu\nu} = -\delta^{00}\delta_{\mu0} \) and \( T^{\mu\nu} = -g^{\mu\nu} + k^\mu k^\nu/k^2 - L^{\mu\nu} \). For comparison with the results for anisotropic matter, we note that the corresponding propagator can be written
\[ D^{00}(0, \mathbf{k}) = \frac{-1}{|\mathbf{k}|^2 + m_D^2}, \quad D^{ij}(0, \mathbf{k}) = \frac{1}{|\mathbf{k}|^2} T^{ij} + \frac{\delta^{ij}}{|\mathbf{k}|^2} \] (16)
where $\alpha$ is the gauge-fixing parameter ($\alpha = 1$ in Feynman gauge) and the Debye screening mass is, at the order to which we are working, $m_D^2 = - \text{Re}[\tilde{\Pi}_{00}] = \text{Re}[\tilde{\Pi}_L]$. Similarly, the magnetic screening mass is
\begin{equation}
\frac{m^2}{\kappa} = \frac{1}{2} T_{ij} \text{Re}[\tilde{\Pi}^{ij}] = \text{Re}[\tilde{\Pi}_T],
\end{equation}
but this vanishes in isotropic matter as discussed above.

In anisotropic matter, the polarization tensor cannot be reduced to just two screening masses since it contains, in general, off-diagonal elements and it depends on the direction of the momentum carried by the field and on the direction of polarization vector. To examine this in more detail we make some specific assumptions about the form of the momentum distribution $f_0(p)$, which we believe should be appropriate in the context of a relativistic heavy-ion collision. For matter near the collision axis, we assume that the system is symmetric about this axis. We therefore consider a cylindrically symmetric form for $f_0(p)$. Also, if we work in the local rest-frame of the matter, there is no net flow and we can assume that the distribution is symmetric under reflections,
\begin{equation}
f_0(-p) = f_0(p).
\end{equation}

In this case, as we shall see, the mixed spacetime components of the tensor are purely imaginary. This means that we can still define a Debye mass as above, although this mass now depends on direction. If we use Eq. (17) to define an averaged magnetic mass from the space components of the static tensor (14), then we find that it reproduces the result for the screening mass given in Ref. [9] using the more complicated closed-time-path-integral method,
\begin{equation}
m^2_{\kappa}(\kappa) = \frac{e^2}{2} \int \frac{d^3p}{(2\pi)^3} \left[ 1 + (\hat{p} \cdot \hat{k})^2 \right] f(p) + \left[ 1 - (\hat{p} \cdot \hat{k})^2 \right] \frac{\hat{k} \cdot \nabla_p f(p)}{\kappa \cdot \hat{p}}.
\end{equation}

Note that the derivation of (19) in [9] does not rely on the particular form of the distribution assumed here.

Without loss of generality, we can choose the $z$-axis along our axis of symmetry and the momentum of the field to lie in the $xz$-plane. We can then express $\kappa$ in the form
\begin{equation}
\kappa = k_\rho \hat{x} + k_z \hat{z},
\end{equation}
with $k_\rho, k_z \geq 0$. In terms of integrals over the longitudinal and radial components of $p$ and the angle $\phi$ between $p$ and $\kappa$ in the $xy$-plane, the components of the static polarization tensor can be written
\begin{align*}
\tilde{\Pi}^{00}(\kappa) &= \frac{e^2}{(2\pi)^3} \int \int \int \frac{d^3p}{0} d\phi \sqrt{p_\rho^2 + p_z^2} \left[ \frac{k_\rho \partial f_0}{\partial p_\rho} \cos \phi + \frac{k_z \partial f_0}{\partial p_z} \right] p_\rho k_\rho \cos \phi + p_z k_z - i\epsilon p_\rho \cos \phi, \\
\tilde{\Pi}^{0x}(\kappa) &= \frac{e^2}{(2\pi)^3} \int \int \int \frac{d^3p}{0} d\phi \left[ \frac{k_\rho \partial f_0}{\partial p_\rho} \cos \phi + \frac{k_z \partial f_0}{\partial p_z} \right] p_\rho k_\rho \cos \phi + p_z k_z - i\epsilon p_\rho \cos \phi, \\
\tilde{\Pi}^{0z}(\kappa) &= \frac{e^2}{(2\pi)^3} \int \int \int \frac{d^3p}{0} d\phi \left[ \frac{k_\rho \partial f_0}{\partial p_\rho} \cos \phi + \frac{k_z \partial f_0}{\partial p_z} \right] p_\rho k_\rho \cos \phi + p_z k_z - i\epsilon p_\rho \cos \phi, \\
\tilde{\Pi}^{xx}(\kappa) &= \frac{e^2}{(2\pi)^3} \int \int \int \frac{d^3p}{0} d\phi \left[ \frac{p_\rho^2 + p_z^2 \sin^2 \phi}{p_\rho^2 + p_z^2} \right] f_0 \left[ \frac{k_\rho \partial f_0}{\partial p_\rho} \cos \phi + \frac{k_z \partial f_0}{\partial p_z} \right] p_\rho k_\rho \cos \phi + p_z k_z - i\epsilon p_\rho \cos \phi.
\end{align*}
The expressions for the $0y$, $xy$, and $yz$ and so they integrate to zero. In general, with $\Pi_{zz}(\hat{k})$, $\Pi_{yy}(\hat{k})$, $\Pi_{xx}(\hat{k})$, carry out the integration over $\phi$ to get

\[
\Pi^{00} = -\frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} f_0 + \frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} \left[ p_\rho \frac{\partial f_0}{\partial p_z} - p_z \frac{\partial f_0}{\partial p_\rho} \right] + \frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} \left[ \frac{p_\rho}{k_z p_\rho^2 - k_\rho^2 p_\rho^2} \right] k_z p_z.
\]

\[
\Pi^{0\gamma} = -\gamma \Pi^{0z},
\]

\[
\Pi^{x\gamma} = \frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} \left[ p_\rho \frac{\partial f_0}{\partial p_z} - p_z \frac{\partial f_0}{\partial p_\rho} \right] + \frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} \left[ \frac{p_\rho}{k_z p_\rho^2 - k_\rho^2 p_\rho^2} \right] k_z p_z.
\]

\[
\Pi^{y\gamma} = \frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} \left[ p_\rho \frac{\partial f_0}{\partial p_z} - p_z \frac{\partial f_0}{\partial p_\rho} \right] - \frac{e^2}{\pi^2} \int_0^\infty dp_z \int_0^\infty dp_\rho \frac{p_\rho}{\sqrt{p_\rho^2 + p_z^2}} \left[ \frac{p_\rho}{k_z p_\rho^2 - k_\rho^2 p_\rho^2} \right] k_z p_z.
\]
corresponding static propagator for the gauge boson has the non-zero elements to imaginary. (This can be seen by making the change of variables \( p_z \) → \( k_z \). Note that if the distribution function satisfies the reflection condition (18) then the time–time and space–space components in the static polarization tensor are real and the mixed spacetime components elements are purely imaginary. (This can be seen by making the change of variables \( p_z \to -p_z, \phi \to \phi + \pi \) in (21).)

The elements of this tensor do not have definite signs and so there can be antiscreening in some directions as well as screening in others. Also, the appearance of a non-zero spacetime component means that in general the Debye screening of electric fields cannot be separated from the effects on magnetic fields. The structure of this tensor agrees with that obtained by Romatschke and Strickland [10] in the temporal axial gauge. However, the Lorentz gauge used here is more convenient for handling static electric fields.

For our chosen form of distribution, the spatial part of static polarization tensor is real and symmetric. Hence the tensor \( \Pi^{ij} \) has three real, orthogonal eigenvectors. The condition that the polarization tensor be transverse reduces to

\[
k_i \Pi^{ij} = 0, \tag{23}\]

in the static limit. This shows that \( \mathbf{k} \) is an eigenvector with zero eigenvalue. By inspection of Eq. (21) we see that the y-direction, perpendicular to the plane containing \( \mathbf{k} \) and the collision axis, is also an eigenvector. Hence, three unit eigenvectors for the static tensor are

\[
\hat{e}_1 = \hat{k}, \quad \hat{e}_2 = \hat{y}, \quad \hat{e}_3 = \hat{k} \times \hat{y}. \tag{24}\]

We denote the corresponding eigenvalues of the spatial part of the tensor by \( \Pi_1 = 0, \Pi_2 \) and \( \Pi_3 \). In the coordinate system defined by these axes the spatial part of the tensor is diagonal. Also we find that \( \Pi^{01} = \Pi^{02} = 0 \). The corresponding static propagator for the gauge boson has the non-zero elements

\[
\left( \begin{array}{c}
D^{00}(0, \mathbf{k}), \\
D^{03}(0, \mathbf{k}), \\
D^{33}(0, \mathbf{k})
\end{array} \right) = \frac{-1}{(|\mathbf{k}|^2 + \Pi_+)(|\mathbf{k}|^2 + \Pi_-)} \left( \begin{array}{c}
|\mathbf{k}|^2 + \Pi_3, \\
\Pi^{03}, \\
-|\mathbf{k}|^2 + \Pi^{00}
\end{array} \right),
\tag{25}\]

\[
D^{11}(0, \mathbf{k}) = \frac{1 + \alpha}{|\mathbf{k}|^2}, \quad D^{22}(0, \mathbf{k}) = \frac{1}{|\mathbf{k}|^2 + \Pi_2(\mathbf{k})}.
\]

Here we have introduced the two eigenvalues \( \Pi_{\pm} \) of the full tensor in the time-3 subspace. These are given by

\[
\Pi_{\pm} = -\frac{1}{2}(\Pi_{00} - \Pi_3) \pm \frac{1}{2}\sqrt{(\Pi_{00} - \Pi_3)^2 + 4|\Pi_{03}|^2}. \tag{26}\]

These results show that the screening or antiscreening of static electromagnetic fields in an anisotropic plasma can depend on both the orientation of \( \mathbf{k} \), the momentum carried by the field, and the orientation of the field itself. It is worth considering two special cases. First, when the momentum carried by the field is perpendicular to the
collision axis \((k_z = 0\) and hence \(\hat{e}_3 = \hat{k}\)) we have

\[
\Pi_1 = \Pi_2 = 0, \quad \Pi_3 = \Pi^{zz}, \quad \Pi_{03} = 0.
\]  

(27)

In this case fields in the \(y\)-direction are unscreened. In contrast when the momentum carried by the field lies along the axis \((k_\rho = 0\) and, hence, \(\hat{e}_3 = \hat{x}\)) we have

\[
\Pi_1 = 0, \quad \Pi_2 = \Pi_3 = \Pi^{xx}, \quad \Pi_{03} = 0.
\]  

(28)

In this case the screening masses for the transverse components of the field are degenerate. For both of these special directions of the momentum, there is no mixed time-3 component and so \(\Pi_{00}\) and \(\Pi_3\) are both eigenvalues.

In general there can be antiscreening as well as screening of electromagnetic fields in an anisotropic plasma. As discussed in Ref. [10], this reflects the fact that such a system is not in equilibrium and hence is unstable. The instability of a particular mode depends on the detailed form of the momentum distribution \(f_0(p)\). It is not possible to draw general conclusions about the pattern of screening masses except in cases where the combination of derivatives

\[
p_\rho \frac{\partial f_0}{\partial p_\rho} - p_\perp \frac{\partial f_0}{\partial p_\perp}
\]

(29)

has the same sign for all momenta. This covers many smooth anisotropic distributions, including the examples studied in Ref. [10]. A negative value for this combination corresponds to an oblate momentum distribution with \((p_\rho^2) < \frac{1}{4}(p_\perp^2)\), and a positive value to a prolate distribution. If the combination of derivatives has a definite sign, the integrals in \(\Pi_2 = \Pi^{xx}\) can be rewritten in a form which shows that they too have the same sign. For an oblate distribution we get \(\Pi_2 < 0\), indicating antiscreening while for a prolate one we get \(\Pi_2 > 0\) and screening.

In the oblate case we also find that \(\Pi_{00} - \Pi_3\) can be written in a form which shows that it is negative definite. This implies that \(\Pi_+ > 0\) and hence there is always screening for one component of the field. In addition, at \(k_\rho = 0\) we find that \(\Pi_{00} + \Pi_3\) is negative and \(\Pi_3\) is positive, and hence \(\Pi_- = \Pi_3\) is positive. Since at \(k_\rho = 0\) we have \(\Pi_- = \Pi_3 = \Pi_2 < 0\), this implies that \(\Pi_-\) must change from antiscreening to screening as the orientation of \(k\) moves away from the collision axis.

For prolate distributions, in contrast, the sign of \(\Pi_{00} - \Pi_3\) is not determined and so it is not possible to make such definite statements about \(\Pi_\pm\). Nonetheless the non-derivative term in \(\Pi_{00}\) (which is responsible for the usual Debye screening) is always negative and so \(\Pi_{00}\) and \(\Pi_{00} - \Pi_3\) remain negative unless the distribution is strongly prolate. Hence we expect the eigenvalue \(\Pi_+\) to correspond to screening for all except the most extreme prolate anisotropies. At \(k_\rho = 0\) we again find that \(\Pi_- = \Pi_3\), but now this must be negative. At \(k_\rho = 0\) we have \(\Pi_- = \Pi_3 = \Pi_2 > 0\) and so both \(\Pi_+\) and \(\Pi_-\) are positive for orientations of \(k\) close to the axes of distributions that are not too strongly prolate. However, as the deformation increases, \(\Pi_{00}\) becomes less negative and so the range of orientations for which \(\Pi_-\) is positive shrinks. For strong enough deformations, \(\Pi_-\) can become negative for all orientations.

The existence of at least one unstable component of the field, for some orientations of \(k\), agrees with the general results of Ref. [14]. All of the general features of \(\Pi_2\) and \(\Pi_{\pm}\) discussed above can be seen in the numerical examples studied in Section V of Ref. [10], and also in the analyses of ideally planar or linear distributions in Ref. [14].

4. Conclusion

In this Letter we have used a transport-theory approach to derive the polarization tensor of a gauge boson in a plasma which is out of thermal equilibrium. We find that the magnetic screening mass at lowest order is non-zero as long as the single-particle distribution function is anisotropic, in contrast to the more familiar case of a plasma in equilibrium. This confirms results previously found using a closed-time-path-integral approach.

The full propagator for static magnetic fields in such a medium has a complicated tensor structure and its eigenvalues need not be positive. There can thus be antiscreening rather than screening for some components of
the field. We have considered in detail the case of a plasma with a cylindrically symmetric momentum distribution. Such a distribution is expected to be relevant to relativistic heavy-ion collisions before thermal equilibrium has been reached, where the axis of collision can provide a symmetry axis for the anisotropy. In this case the spatial part of the static polarization tensor is real and has three orthogonal principal axes: \( \hat{k} \), lying along the direction the momentum carried by the gauge boson, \( \hat{y} \), orthogonal to the plane of \( \hat{k} \) and the collision axis, and \( \hat{k} \times \hat{y} \). The last two of these have non-zero screening in general. In addition there can be an imaginary off-diagonal component mixing the \( \hat{k} \times \hat{y} \) and time directions. In the special case where the momentum \( \hat{k} \) is perpendicular to the collision axis, there is no screening for the component in the \( \hat{y} \)-direction.

We have also been able to derive various conditions on the signs of two of the eigenvalues of the tensor, under the assumption that \( p_\rho \frac{\partial f_0}{\partial p_\rho} - p_z \frac{\partial f_0}{\partial p_z} \) has a definite sign. In particular we find that fields in the \( \hat{y} \)-direction are screened if the anisotropy is prolate, but antiscreened if it is oblate. We also find that there is one component which is always screened unless the distribution is extremely prolate.

These differences in the screening of interactions in different directions could have important effects in the equilibration of the matter produced in relativistic heavy-ion collisions. Although only the screening or antiscreening of static fields has been examined in detail here, it will also be very interesting to explore dynamical aspects, such as damping rates and unstable modes, in these anisotropic systems [10,12–14].

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