Effect of a DC Field on Temperature Distribution in a Thin FGM Metal Line Subjected to Distributed Local Heating Sources

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Abstract
The effect of a direct-current field on the temperature distribution in a thin, non-uniform functionally graded metal line subjected to distributed local heat sources is investigated. The material properties of the metal line are assumed to vary over the span following a linear functional relationship. Bump-like heat sources of different profiles are considered to simulate the condition of distributed local heating of the metal line. The governing differential equations associated with the electrical and thermal problems are derived in terms of variable thermal and electrical conductivity of the material. The solution of the coupled boundary-value problem is then obtained using a finite-difference computational scheme. The temperature distributions in the FGM line are determined for different environmental conditions as a function of intensity of the DC field.

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Keywords: Functionally-graded metal line; distributed heat source; temperature distribution; direct-current field.

1. Introduction
The information revolution and enabling era of ultra-large-scale integration (ULSI) have spawned an ever-increasing level of functional integration on chip, driving a need for more reliable circuit design and higher performance. The increasing density, performance and reliability requirement in circuit design has created significant process integration challenges for future interconnect systems. Several recent analysis [1-3] have highlighted interconnect performance issues for future design of circuit in microelectronic devices. Traditionally aluminum is being considered as one of the suitable candidates for designing metal lines in interconnects; etched

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aluminum alloy with damascene tungsten plugs is still the most predominant interconnect system. While copper conductors reduces Joule heating effects through improved reliability performance, integration of low-dielectrics with copper brings new integration concerns such as copper CMP (chemical mechanical polishing).

Copper metallization offers significant performance and reliability improvement but presents numerous integration and reliability challenges. The electro-migration behavior of copper also differs from aluminum in that surface diffusion tends to dominate over grain boundary diffusion, especially for narrow lines exhibiting bamboo grain structure [4]. This difference may be one reason preliminary data shows deterioration of copper reliability at smaller feature sizes [4]. Copper resistivity has also been projected to increase dramatically with smaller feature size due to electron scattering from grain boundaries and conductor walls [5]. Now-a-days, pure metals are of little use in engineering applications because of demand of conflicting property requirements and thus are being replaced by various kinds of advanced materials. Among the various combination processes of materials functionally graded material (FGM) are now treated as the most promising candidate for such materials. FGM belongs to a class of advanced material characterized by variation in properties as the dimension varies. The overall properties of FGM are unique, and are clearly different from any of the constituent materials that form it [6]. Accurate and reliable prediction of electro-thermal behaviour of conducting materials is of great importance for improved performance as well as integrity assessment of microelectronic devices. When an electrical conducting material is subjected to a current flow, Joule heating is induced, which eventually leads to generation of heat in the conductor and thus causes thermal stresses, which is considered to be one of the major reasons of metal line failure in electronic packaging. The problem of heat conduction in a wire under the influence of direct current flow has been explained theoretically by Carslaw and Jaeger [7]. Introducing a new Joule heating residue vector, heat conduction in symmetrical electro-thermal problems has been analyzed under the influence of direct current passing through symmetrical regions of the boundary [8]. The resulting temperature field of a 2D electro-thermal problem near the corner composed of two dissimilar materials in an angled metal line has been analyzed under a direct current flow [9]. Recently, electro-thermal responses of non-uniform functionally graded metal lines under a direct current field have been analyzed by Ghosh et al. [10]. The computational scheme has been extended to develop a simple procedure to determine the optimum material composition distribution of FGM metal lines under a dc field [11].

The present paper is on the analysis of the effect of an electrical field on the temperature distribution in a thin non-uniform FGM metal line subjected to distributed local heat sources. The electrical and thermal properties of the Cu-Al FGM line are assumed to vary over the line following a linear relationship. Different profiles of bump-like periodic heating sources are considered along with that of a uniform one. The numerical solutions of the present coupled multi-physics problem are obtained using a finite-difference computational algorithm. The temperature distributions in the FGM line are presented for different profiles of periodic heat sources as well as for different environmental conditions.

2. Mathematical modelling

2.1. Electrical problem

The differential equation that governs the distribution of electric potential in a non-uniform metal line with variable electrical resistivity, is

\[
\frac{d^2 \varphi}{dx^2} = -\left( \frac{d \rho(x)}{dx} J(x) + \rho(x) \frac{d J(x)}{dx} \right)
\]  

(1)
For uniform cross-sectional area and constant electrical resistivity, the derivatives in the right hand side of the Eq. (1) can be neglected. Equation (1) will then be reduced to the standard one dimensional Laplace equation. The end conditions of the metal line are simulated by the following relation of potential gradient:

\[
\frac{d \phi}{dx} = \pm \rho \left( \frac{I}{A} \right)
\]  

The negative sign of the equation (2) applies to the line end where current is being injected and the positive sign corresponds to the current outlet port.

2.2. Thermal problem

The general governing equation for steady state heat transfer in a metal line, the surface of which losses heat by convection to the surrounding atmosphere \((T_a)\) is

\[
\frac{1}{A(x)} \frac{\partial}{\partial x} \left[ A(x) k(x) \frac{\partial T}{\partial x} \right] - \frac{H C(x)}{A(x)} [T - T_a] + G(x) = 0
\]  

(3)

For the present electro-thermal problem, the heat generation rate per unit volume \((G)\) is related to Joule heating caused by the current flow and heat supplied by the external heat sources. For steady-state heat transfer in the metal line with variable thermal conductivity \(k(x)\), subjected to an electric field and distributed local heat sources, the governing equation becomes

\[
\frac{1}{A(x)} \frac{d}{dx} \left[ A(x) k(x) \frac{dT}{dx} \right] - \frac{H C(x)}{A(x)} [T - T_a] + \frac{1}{A(x) \rho(x)} \left( \frac{d\phi}{dx} \right)^2 + g_m(x) = 0
\]  

(4)

For the thermal problem, the temperatures at the two ends of the line are assumed to be known. It is mentioned that all possible physical conditions at the ends can readily be accommodated in the present program.

3. Statement of the thermal problem coupled with an electrical field

Figure 1 shows the analytical model of a variable cross-section FGM metal line with overall dimensions, \(L = 200\) mm, \(w_1 = 5\) mm, \(w_2 = 1\) mm, \(t = 100\) \(\mu\)m, which is subjected to a steady direct current field. The current flow is assumed to be, \(I = 2\) A. The FGM line is assumed to be composed of two metals (for example, Cu and Al), the composition of which varies linearly over the line span.

The entire metal line is assumed to be electrically insulated except for the two ends. For the solution of electrical problem, in addition to the given current densities at the two ends of the line, the zero potential condition is also satisfied at its mid-length position. For the thermal problem, the elevated temperature condition of the metal line was simulated by assigning a fixed temperature (313K) at two ends of the metal line. The surface of the line is assumed to transfer heat by convection to the surrounding environment which is kept at a temperature of 310K. The convection heat transfer co-efficient is assumed to be constant (10 Wm\(^{-2}\)K\(^{-1}\)) for the entire span of the FGM line.

The individual electrical resistivity and thermal conductivity of the two constituent metals (Cu and Al) assumed for the present analysis are listed in Table 1. The distribution of material composition of the FGM line is assumed to be
linear, as illustrated in Fig. 1; the corresponding variable material properties of the line are obtained following linear laws, which are depicted in Fig. 2.

Table 1: The assumed electrical resistivity and thermal conductivity of Copper and Aluminum at room temperature

<table>
<thead>
<tr>
<th>Metal</th>
<th>Electrical resistivity, $\rho$ (Ω-m)</th>
<th>Thermal conductivity, $k$ (Wm⁻¹K⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper (Cu)</td>
<td>$1.71 \times 10^{-8}$</td>
<td>400.35</td>
</tr>
<tr>
<td>Aluminum (Al)</td>
<td>$2.65 \times 10^{-8}$</td>
<td>238.97</td>
</tr>
</tbody>
</table>

The distributed local heat sources considered for the problem are assumed to be of three types, the profiles of which are, respectively, bump-like sinusoidal, bump-like rectangular and an average uniform one. The variations of the heat sources over the metal line are illustrated in Fig. 2(c).

4. Method of solution

The present steady-state heat conduction-convection boundary value problem has been solved numerically by using finite-difference technique. Both the governing differential equations associated with the electrical and thermal problems are discretized using the standard three-point central-difference scheme. The difference equations so developed for the electrical (Eq. (1)) and thermal problems (Eq. (4)) are, respectively, as follows:

\[
\phi_{i+1} - 2\phi_i + \phi_{i-1} = -\frac{h}{2} \left[ J_i \left( \rho_{i+1} - \rho_{i-1} \right) + \rho_i \left( J_{i+1} - J_{i-1} \right) \right] \\
\left[ 4k_i + k_iA_i^{-1}\left( A_{i+1} - A_{i-1} \right) + \left( k_{i+1} - k_{i-1} \right) \right]T_{i+1} - \left[ 8k_i + 4A_i^{-1}HC_ih^2 \right]T_i + \left[ 4k_i - k_iA_i^{-1}\left( A_{i+1} - A_{i-1} \right) \right]T_{i-1} \\
- \left( k_{i+1} - k_{i-1} \right)T_{i-1} = -4A_i^{-1}HC_ih^2T_{\infty} - \rho_i^{-1}A_i^{-1}\left( \phi_{i+1} - \phi_{i-1} \right)^2 - 4h^2g_{ext(i)}
\]

A MATLAB based computer code has been developed to solve the coupled problem. The resulting tri-diagonal systems of algebraic equations are solved for the nodal temperatures by the matrix decomposition method. For the calculation of secondary parameter of interest, namely, electrical heat generation, both the three-point forward and backward as well as central differencing schemes were adopted to keep the overall order of error the same ($O(h^2)$). A total of 1000 nodal points have been used to discretize the computational domain. The convergence as well as the stability of the numerical solution has however been verified by varying the nodal points from 10 to 3000.

5. Analysis of the thermal behavior

In this section, first, the thermal behavior of the FGM line is demonstrated together with those of the constituent metals, without the influence of any electrical field. Fig. 3 shows the distribution of temperature along the metal line for three different types of heating sources namely, sinusoidal, rectangular and uniform heat source under no current flow. For all three cases Al shows maximum temperature elevation whereas Cu shows the minimum. Temperature distribution of FGM metal line resides in between these two metal lines. Moreover the rise of temperature for sinusoidal heat source is found to be the minimum among three, which is due to the fact that overall heat generation...
for sinusoidal heat source is minimum among three as seen from figure 2(c). Finally, though the temperature profile is different, the maximum temperature rise for rectangular and uniform heating source is found to be same since the overall heat generation is equivalent for these two cases.

Fig. 3: Temperature distribution along the metal line for different heat sources (I = 0): (a) Sinusoidal; (b) Rectangular; (c) Uniform.

Fig. 4(a) shows the variation of electric potential along the axis of the FGM as well as the constituent metal lines. The potential distribution for the FGM metal line resides in between those of the parent metals, and maintains higher similarities with that of Cu for the starting section and Al for the end section. This is because the proportion of Cu is higher for the first half section and Al is higher for the last half section. Fig. 4(b) represents the volumetric heat generation for the FGM metal line subjected to dc current field under three different external heat sources. For all three cases the maximum heat generation is found near the mid-section of the metal line where the cross-sectional area in minimum.

The effect of electric field on the distribution of temperature over the FGM metal subjected to distributed local heating sources is shown in Fig. 5 for two different environmental conditions of the line, namely, bare and buried conditions. Both the conditions show a significant increase in temperature with the increase of current flow. It is also found that temperature rise under buried condition (Fig. 5(b)) is much higher compared to that of bare line (Fig. 5(a)). This is because of the fact that, in case of buried lines, no heat loss is allowed from the surfaces through convection to surroundings, thereby causing the overall state of temperature to assume a higher level compared to the bare lines.

Fig. 4: Distribution of (a) electric potential, (b) resulting heat generation along the metal lines for different heat sources (I = 2A).
6. Conclusions

Temperature distribution in a thin, non-uniform FGM metal line composed of two metals (Al and Cu) is investigated under the influence of a direct current field and distributed local heating sources. The distributions of material composition as well as the resulting material properties of the metal line are assumed to be linear functions of spatial coordinate. The distributions of electric potential as well as temperature along the FGM line are found to differ significantly from those of the individual constituent metal lines. The effect of electric field on the temperature distribution is also found to be quite significant even with the presence of local distributed heating sources. The results are claimed to be highly accurate and reliable, which are expected to provide a valuable design guide to functionally graded metal lines in modern electronic devices.

References