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A simple formula for predicting settling velocity of sediment particles

Song Zhiyao*[1, 2, 3], Wu Tingting[1, 2], Xu Fumin[2, 3], Li Ruijie[2, 3]

1. State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing 210098, P. R. China
2. College of Ocean, Hohai University, Nanjing 210098, P. R. China
3. Key Laboratory of Coastal Disaster and Defence, Ministry of Education, Hohai University, Nanjing 210098, P. R. China

Abstract: Based on the general relationship described by Cheng between the drag coefficient and the Reynolds number of a particle, a new relationship between the Reynolds number and a dimensionless particle parameter is proposed. Using a trial-and-error procedure to minimize errors, the coefficients were determined and a formula was developed for predicting the settling velocity of natural sediment particles. This formula has higher prediction accuracy than other published formulas and it is applicable to all Reynolds numbers less than 2×10^5.

Key words: *settling velocity*; *spherical particle*; *sediment particle*; *sediment transport*; *trial-and-error method*

1 Introduction

The settling velocity of sediment particles, also called the terminal or fall velocity, is one of the key variables in the study of sediment transport and is important in understanding suspension, deposition, mixing and exchange processes. Nevertheless, it is still difficult to accurately predict the settling velocity, even for a single spherical particle. For its engineering application, the settling velocity for single particles, W_s , has been extensively studied. The drag coefficient, C_d, is inversely proportional to the particle Reynolds number, which is defined as $Re = W_s d/\nu$ (where Re is the Reynolds number, d is the particle diameter and ν is the fluid kinematic viscosity), when Re < 1 (Stokes flow). Later studies (Dallavalle 1948; Schlichting 1979) showed that C_d approaches a constant if $Re > 10^5$ (turbulent flow). If the effective weight is considered equivalent to the Newtonian expression of drag resistance, C_d can be defined as follows:

$$C_d = \frac{4}{3}\frac{\Delta g d}{W_s^2} \tag{1}$$

where $\Delta = \rho_s/\rho - 1$ (ρ_s and ρ represent the density of particles and the density of the fluid, respectively) , and g is the gravitational acceleration. There are two asymptotic equations of the settling velocity depending on the particle Reynolds number, i.e., $C_d = A/Re$ when Re < 1(Stokes flow) and $C_d = B$ when $10^5 < Re < 2\times10^5$ (turbulent flow), where A and B

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*Corresponding author (e-mail: *zysong@hhu.edu.cn*)
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are constants. We can form the following relationship for Stokes flow conditions:

$$W_s = \frac{4}{3A}\frac{\Delta g d^2}{\nu} \tag{2}$$

Likewise, the turbulent flow settling velocity can be expressed as

$$W_s = \sqrt{\frac{4}{3B}\Delta g d} \tag{3}$$

Most of the existing quasi-theoretical or semi-empirical formulas are based on these two asymptotic solutions, Eq. (2) and Eq. (3) (McGauhey 1956; Zanke 1977; Concha and Almendra 1979; Turton and Clark 1987; Zhang 1989; Julien 1995; Soulsby 1997; Cheng 1997; Ahrens 2000; Guo 2002; Jimenez and Madsen 2003; Brown and Lawler 2003; She et al. 2005; Camenen 2007).

The object of this paper is to establish a new relationship between the Reynolds number and a dimensionless particle parameter, and present a simple formula for predicting the settling velocity of natural sediment particles that is applicable to a wide range of Reynolds numbers from Stokes flow to turbulent regime. The precision of the present formula is tested by its comparison with other published formulas.

2 Development of an expression for the settling velocity

Cheng (1997) pointed out that the relationship between C_d and Re takes a similar form in many semi-empirical equations for the drag coefficient. Its basis is the asymptotic behavior of both the settling velocity and Re. This recognition leads to an expression for W_s that utilizes a dimensionless particle parameter. The process was described by Camenen (2007), and is outlined below.

C_d and Re are related such that

$$C_d = \left[\left(\frac{A}{Re}\right)^{1/n} + B^{1/n}\right]^n \tag{4}$$

where n is an exponent. The dimensionless particle diameter is defined as

$$d_* = \left(\frac{\Delta g}{\nu^2}\right)^{1/3} d \tag{5}$$

This definition is incorporated into Eq. (1), the relationship between drag resistance and dimensionless particle diameter can be expressed as:

$$C_d = \frac{4}{3}\frac{d_*^3}{Re^2} \tag{6}$$

Finally, the settling velocity can be calculated when Eq. (4) and Eq. (6) are combined:

$$W_s = \frac{\nu}{d}\left[\sqrt{\frac{1}{4}\left(\frac{A}{B}\right)^{2/n} + \left(\frac{4}{3}\frac{d_*^3}{B}\right)^{1/n}} - \frac{1}{2}\left(\frac{A}{B}\right)^{1/n}\right]^n \tag{7}$$

Various values for A, B and n that have been used by different authors for spherical particles and natural sediment particles are listed in Table 1.

Table 1 Calibration coefficients for Eq. (7)

Author(s)	Material	*A*	*B*	*n*
Dallavalle (1948)	Spherical particles	24.0	0.40	2.0
Concha and Almendra (1979)	Spherical particles	30.6	0.37	2.0
Zigrang and Sylvester (1981)	Spherical particles	23.2	0.40	2.0
Brown and Lawler (2003)	Spherical particles	21.4	0.36	2.0
Zanke (1977)	Natural sediment particles	26.7	1.33	1.0
Zhang (1989)	Natural sediment particles	34.1	1.22	1.0
Julien (1995)	Natural sediment particles	24.0	1.50	1.0
Soulsby (1997)	Natural sediment particles	26.4	1.27	1.0
Cheng (1997)	Natural sediment particles	32.0	1.00	1.5
Camenen (2007)	Natural sediment particles	24.0–66.0	0.40–7.00	0.8–2.0

Camenen (2007) proposed three equations, one for each of the coefficients in Eq. (7), that took into account the shape and roundness of the particles. This method allows Eq. (7) to produce better results for particles of different shapes, sizes and densities, but it is slightly difficult to put into practice; especially when n = 1.5, its results are not as accurate as those of the formula proposed by Cheng (1997). Therefore, it is necessary to work out another formula for predicting the settling velocity of natural sediment that is both simple and highly accurate.

For this reason, Re in Eq. (4) is derived by combing Eq. (2) and Eq. (5) , such that

$$Re = \frac{4d_*^3}{3A} \tag{8}$$

Eq. (4) can then be rewritten as

$$C_\mathrm{d} = \left[\left(\frac{\sqrt{3}A}{2d_*^{3/2}} \right)^{2/n} + B^{1/n} \right]^n \tag{9}$$

Finally, we obtain the following solution for the settling velocity, which is similar to Eq. (7) and applicable to all Reynolds numbers less than 2×10^5:

$$W_\mathrm{s} = \frac{\nu}{d} d_*^3 \left[\left(\frac{3A}{4} \right)^{2/n} + \left(\frac{3B}{4} d_*^3 \right)^{1/n} \right]^{-n/2} \tag{10}$$

Values from previous studies for A, B and n in Eq. (10) are listed in Table 2.

Jimenez and Madsen (2003) proposed that coefficients A and B depend on characteristic values of the particle roundness factor, but did not give the relative equations for them, so it is inconvenient to apply their method in engineering. Furthermore, of the above authors, only Guo (2002) and Jimenez and Madsen (2003) have taken into account the natural sediment particles.

In fact, coefficients *A*, *B* and *n* in Eq. (10) are not constants; they are complicated functions of the shape and roundness of the natural particles, as noted by Camenen (2007) about their theoretical explanation, this makes the formula difficult to use in many sediment engineering problems.

Table 2 Calibration coefficients for Eq. (10)

Author(s)	Material	*A*	*B*	*n*
McGauhey (1956)	Spherical particles	24.0	0.69	2.00
Swanson (1967)	Spherical particles	26.2	1.19	2.00
Swanson (1975)	Spherical particles	26.2	1.91	2.00
Turton and Clark (1987)	Spherical particles	24.0	0.43	2.43
Guo (2002)	Spherical particles	24.0	4/9	2.00
Brown and Lawler (2003)	Spherical particles	24.0	0.44	2.22
Jimenez and Madsen (2003)	Spherical particles	24.6	0.83	2.00
Guo (2002)	Natural sediment particles	32.0	1.00	2.00
Jimenez and Madsen (2003)	Natural sediment particles	25.5–33.4	0.90–1.92	2.00

In view of the potential engineering applications, it is necessary to seek a more accurate formula than Eq. (10) for utilizing constant coefficients *A*, *B* and *n*. Using a trial-and-error procedure to minimize the error between predicted data calculated with Eq. (10) and experimental data provided by Engelund and Hansen (1972) and Cheng (1997), we can derive the coefficients, i.e., *A*=32.2, *B*=1.17, and *n*=1.75, and obtain the following formula:

$$W_s = \frac{\nu}{d} d_*^3 \left[38.1 + 0.93 d_*^{12/7}\right]^{-7/8} \tag{11}$$

3 Formula comparison

In this section, Eq. (11) was compared with other published formulas, and the result shows that Eq. (11) has a higher degree of accuracy in predicting the settling velocity of natural sediment particles.

Different scholars have proposed many settling velocity formulas for both spherical and non-spherical particles. The following formulas for natural sediment particles were chosen for comparison.

From Zhu and Cheng (1993):

$$W_s = \frac{\nu}{d} d_*^3 \left[\frac{1}{\sqrt{144\cos^6\alpha + \left(4.5\cos^3\alpha + 0.9\sin^2\alpha\right) d_*^3} + 12\cos^3\alpha}\right] \tag{12}$$

where $\alpha = 0$ for $d_* \leqslant 1$ and $\alpha = \pi\left[2 + 2.5(\log d_*)^{-3}\right]^{-1}$ for $d_* > 1$.

From Cheng (1997):

$$W_s = \frac{\nu}{d}\left(\sqrt{25+1.2d_*^2} - 5\right)^{1.5} \tag{13}$$

From Ahrens (2000):

$$W_s = \frac{\nu}{d} d_*^{1.5}\left(C_1 d_*^{1.5} + C_2\right) \tag{14}$$

where

$$C_1 = 0.055\tanh\left[12 d_*^{-1.77}\exp\left(-0.0004 d_*^3\right)\right] \quad \text{and}$$

$$C_2 = 1.06\tanh\left[0.016 d_*^{1.5}\exp\left(-120/d_*^3\right)\right]$$

From Guo (2002):

$$W_s = \frac{\nu}{d} d_*^3\left[24 + \sqrt{3}/2\, d_*^{3/2}\right]^{-1} \tag{15}$$

From She et al. (2005):

$$W_s = 1.05\frac{\nu}{d} d_*^{1.5}\left[1 - \exp(-0.315 d_*^{0.765})\right]^{2.2} \tag{16}$$

Table 3 presents a comparison of calculated settling velocities using formulas (11) to (16) with the experimental data from Engelund and Hansen (1972) and Cheng (1997). The accuracy of each formula is determined by the average value of the relative error, defined as

$$E = \frac{1}{N}\sum_{i=1}^{N}\left|W_{sc}/W_{se} - 1\right| \times 100\% \tag{17}$$

and the coefficient of variation, defined as

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left|W_{se}/W_{sc} - 1\right|^2} \times 100\% \tag{18}$$

where W_{sc} stands for the value calculated from the equations, W_{se} for the experimental value and N for the number of data points (She et al. 2005).

Table 3 Accuracy of fit of formulas against experimental data

Equation	E (%)	σ (%)
(11)	6.36	9.10
(12)	7.02	11.30
(13)	6.96	10.96
(14)	16.93	16.84
(15)	6.87	10.56
(16)	16.34	16.49

Table 3 and Figure 1 demonstrate that: (1) The new formula (11) is significantly more accurate than those of Ahrens (2000) and She et al. (2005), and slightly more accurate than those of Zhu and Cheng (1993), Cheng (1997) and Guo (2002), in addition to being simpler than all except that of Guo (2002); (2) The Ahrens (2000) and She et al. (2005) formulas overestimate the settling velocity of medium and fine grains.

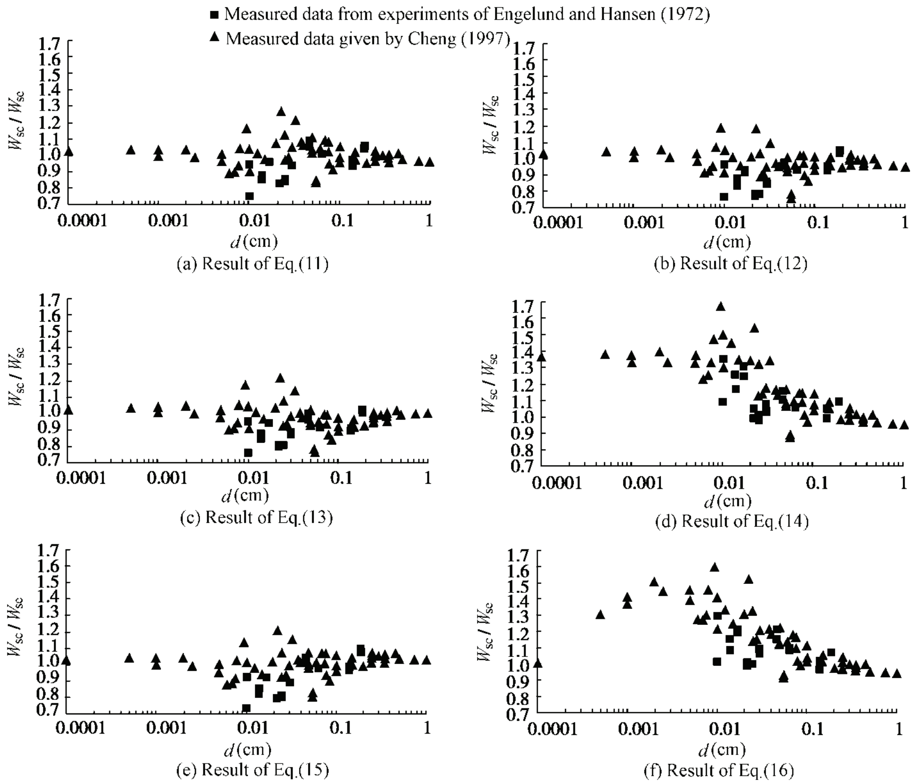

Figure 1 Ratio of predicted to measured settling velocities for each formula

4 Concluding remarks

(1) The results of this study expand the existing formulas for calculating the settling velocity of natural sediment particles. The results also verify the relationships derived by McGauhey (1956), Swanson (1967, 1975), Turton and Clark (1987), Guo (2002), Brown and Lawler (2003), and Jimenez and Madsen (2003) that express the settling velocity (or particle Reynolds number) of spherical particles or natural sediment particles in terms of the dimensionless grain size in a simpler form with three coefficients.

(2) The three coefficients in the proposed Eq. (10) can be determined by a trial-and-error procedure in order to minimize errors. The Eq. (11) obtained has a higher degree of prediction accuracy than other published formulas and it is suitable for engineering application.

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