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Generating Pareto Surface for Multi Objective Integer Programming Problems with Stochastic Objective Coefficients

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Abstract

Stochastic multi objective programming problems commonly arise in complex systems such as portfolio analysis, medium- to long-term capacity planning and design applications under uncertainty. The identification of the candidate solution set is a main step in many applications which depends on the nature of uncertainty. This study presents a method to generate Pareto surface for multi-objective integer programs with stochastic coefficients in the objective functions based on minimum expectation and variance criteria. The objective function coefficients are represented through random discrete distributions. The methodology follows a two-phase approach where, in the first phase, the stochastic multiple objectives are converted into deterministic equivalents based on the minimum expectation and variance efficiency concepts. The second phase solves the deterministic multi objective problem, using a Pareto generation methodology which aims at generating the whole Pareto surface of multi objective integer programming problems. We present results of experimental study of applying the proposed method to an assignment problem with three objective functions.
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Keywords: Multi objective optimization, stochastic, Pareto surface, visualization.

1. Introduction

Multi-objective optimization has become an essential part of the design decisions in many industrial applications attributable to the increasing complexity of systems and the need for integrated decision making. In many applications, decision makers prefer evaluating a number of distinct solution alternatives before finalizing their decision. Ideally, the decision makers prefer access to the full Pareto front for a complete trade-off analysis. However, even in the case of multi objective integer programs with deterministic coefficients, generating the full Pareto surface, without resorting to complete enumeration, has been a challenge. Multi objective integer programming (MOIP) problems are even more difficult to tackle, due to the non-convex feasible solution space. Although stochastic multi objective programming (SMOP) problems are frequently encountered in practice, the literature on solution methodologies accounting for the stochasticity is still in nascency. The majority of the efforts have focused on developing custom methodologies for specific SMOP problems such as portfolio selection, capacity investment, reconfiguration of ring topologies or various combinatorial optimization problems.

The methods for solving SMOPs can be classified as exact and approximate (evolutionary or simulation based methods). Further, the exact methods can be classified into two groups (Caballero, Cerda, Munoz, & Rey, 2004) . The first group, *multi objective approach,* converts each stochastic objective into a deterministic equivalent and then solves a deterministic MOP problem. The second approach, *stochastic approach* first converts the SMOP into a single objective stochastic programming problem, then solves its deterministic equivalent. In this study, authors

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recommend using the stochastic approach for the achievement of efficient solutions rather than the multi-objective approach in the presence of stochastic dependencies among objectives. Abdelaziz (Abdelaziz, 1992) also classifies the stochastic MOIP (SMOIP) solution techniques according to the order by which the conversions are carried out, e.g. stochastic to deterministic and multi- to single-objective. In the conversion from stochastic to deterministic problem, various efficient solution concepts are employed: expected-value efficiency, minimum variance efficiency, or a combination of both such as the expected-value standard deviation efficiency, minimum-risk efficiency and efficiency with probabilities. The study of Cabellero et. al. (Caballero, Cerda, Munoz, & Rey, 2004) explores the relationships among these efficiency types under some assumptions. Unfortunately, their conclusions cannot be used in SMOIP since one of the assumptions of the study is the convexity of the feasible region. We refer the reader to the book by Stancu-Minasian (Stancu-Minasian, 1984) for a broader discussion of the SMOIP problems. Aouni et al. (Aouni & Torre, 2010) presents an approach to solve SMOP problems through goal programming. They used three types of deterministic conversion, namely optimization of expectation, standard deviation and both expectation and standard deviation of objectives. Abdelaziz et al.(Abdelaziz, Aouni, & Fayed, 2003) propose a chance constrained compromise programming model (CCCP) as a deterministic transformation to multi objective stochastic programming portfolio. CCCP is based on CP and chance constrained programming (CCP).

Another study to compute the dotted representation for the portfolio selection problem belongs to Qi et al. (Qi, Hirschberger, & Steuer., 2009). The authors study the Markowitz type problems, i.e. mean-variance problems with all linear constraints. The proposed approach utilizes the results of algorithms that can compute all hyperbolic segments of a Markowitz efficient frontier and places dots on the hyperbolic segments of the efficient frontier in a variety ways including equally spaced. The "ε −constraint" and "risk tolerance factor" methods are used for producing dotted representations of efficient frontiers. Both involve the repetitive optimization of a quadratic programming problem. Pena (Pena, Lara, & Castrodeza, 2009) worked on a multi objective stochastic programming for feed formation by introducing stochastic constraints in the single objective minimum cost model requiring fixing the level of probability desired for each one of the nutrients in advance. A chance constrained goal programming model has been designed by Bhattacharya (Bhattacharya, 2009), after considering the parameter corresponding to reach for different media as random variables in a advertising planning problem setting. Gabriel et al. (Gabriel, Shim, Llorca, & Milner, 2008), developed a heuristic for dynamic reconfiguration of ring topologies with stochastic load. The aim is to minimize both total cost total congestion with uncertain traffic load. The authors handled the problem by converting stochastic problem to a deterministic one. Capacity investment problem is also formulated as SMOIP, and studied by Claro and de Sousa (Claro & Sousa, 2010). They proposed a local search metaheuristic as the solution method.

SMOIP problems with stochastic objective function coefficients are the focus of this study. The coefficients of the objective function are discrete samples from arbitrary continuous distributions. Our goal is to identify the whole Pareto surface by considering both minimum expectancy and variance efficient solution concepts. Our solution approach converts the stochastic problem into its deterministic equivalent and then uses an exact algorithm to generate the Pareto surface of the deterministic equivalent. We apply this approach to solving multi-objective assignment problems with stochastic coefficients. We illustrate the results with graphical representation for different efficient solution concepts.

2. Stochastic models under study

The general formulation for a stochastic linear multi objective optimization problem (SMOP) in this study, is natural extension of MOP with probabilistic objective function coefficients. General form of SMOP is as follows:

$$\begin{gathered} \min \widetilde{z_1}\,(x, \tilde{c}) \\ \min \widetilde{z_2}\,(x, \tilde{c}) \\ \vdots \\ \min \widetilde{z_k}\,(x, \tilde{c}) \end{gathered} \tag{1}$$

$$s.t. \quad x \in X, \quad x \text{ integer}$$

where $\tilde{z_1}\ (x, \tilde{c})$= c($\omega$)$x$ and c(ω) being a stochastic parameter.

For this model the following notations and assumptions are employed:

- $x \in R^n$ is the vector of decision variables of the problem and $\tilde{c}$ is a random vector whose components are random variables, defined on the set E $\in R^s$. We assume given the family F of events (that is subsets of E and the distribution of probability P defined on F so that, for any subset of $E, A \subset E, A \in F$ the probability $P(A)$ is known. Also we assume that the distribution of probability P is independent of the decision variables, $x_1, \dots, x_n$.
- The functions, $\tilde{z_1}(x, \tilde{c})$, $\tilde{z_2}(x, \tilde{c})$,..., $\widetilde{z_k}(x, \tilde{c})$ are defined on $R \times E$.
- The set of feasible solutions $D \subset R^n$ is nonempty.

Let $\bar{z}_q(x)$ denote the expected value of the q^{th} objective function, and let $\sigma_q(x)$ be its standard deviation, $q \in \{1, \dots, k\}$. Let us assume that, for every q and for every feasible vector x of the SMOP problem, the standard deviation $\sigma_q(x)$ is finite.

The general solution approaches for SMOP problems with probabilistic objective function coefficients are divided into two main groups. First approach is to convert each stochastic multi objective function into its deterministic equivalent; this approach is called “Multi objective approach” in Cabellero et al. (Caballero, Cerda, Munoz, & Rey, 2004). The second approach is called “stochastic approach” which combines stochastic multi objectives into a single stochastic objective and then finds its deterministic equivalent. Cabellero et al. investigate whether these two approaches have a significant difference at producing the solutions where the feasible region of the problem is convex. In this study, we consider SMOIP with non-convex feasible space and independent random objective coefficients.

For the SMOP problems, there are five types of efficiency concepts that can be used to convert the stochastic function into its deterministic equivalent. These are as follows:

Expected value efficient solution, (White, 1982): $x \in X$ is an expected value efficient solution to the problem (1) if it is Pareto efficient to the problem

$$\min_{x \in X} \bar{z}_1(x), \bar{z}_2(x), \dots, \bar{z}_k(x)$$

where $\bar{z}_q(x)$ is the expected value of the random variable, $q \in \{1, \dots, k\}$.

Minimum variance efficient solution, (White, 1982): $x \in X$ is an expected value efficient solution to the problem (1), if it is Pareto efficient to the problem

$$\min_{x \in X} \sigma_1^2(x),\ \sigma_2^2(x), \dots, \sigma_k^2(x)$$

where $\sigma_q^2(x)$ is the variance of q^{th} objective function.

Weighted expected value & standard deviation efficient solution: $x \in X$ is weighted expected value & standard deviation efficient solution to the problem (1), if it is Pareto efficient to the problem

$$\min_{x \in X} 0.5\ \sigma_1^2(x) + 0.5\ \bar{z}_1(x),\ 0.5\ \sigma_2^2(x) + 0.5\ \bar{z}_2(x), \dots, 0.5\ \sigma_k^2(x) + 0.5\ \bar{z}_k(x)$$

which includes the expected value and the standard deviation of the problem's stochastic objective functions with equal importance.

3. Full Pareto Surface Generating Method

This algorithm is an exact algorithm proposed by Ozlen and Azizoglu (Ozlen & Azizoglu, 2009) which aims to obtain the whole Pareto surface of a multi objective integer problem with deterministic coefficients. It is a modified version of classical ε-constraint method, which searches within narrower efficiency ranges, jumping between non-dominated solutions, instead of incremental steps. One important difference from the original ε-constraint method is the structure of objective function. An important property of the algorithm all of the objective function coefficients should be integers; if this is not the case, then the coefficients can be converted to integers by proper scaling. The general formulation of the model and the algorithm can be found in (Ozlen & Azizoglu, 2009).

4. Sample problem

The Assignment Problem:
This problem is a famous problem that occurs in especially in most of the selection or decision occasions.

$$min \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{1ij} \, (x_{ij})$$

$$min \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{2ij} \, (x_{ij})$$

$$min \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{3ij} \, (x_{ij})$$

$$s.t. \qquad (3)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \;\; \forall j \; \in \{1, \dots, n\}$$
$$\sum_{ij=i1}^{n} x_{ij} = 1 \;\; \forall i \; \in \{1, \dots, m\}$$
$$x_{ij} \; \in \{0,1\}$$

where x_{ij} has $m \times n$ dimensions, with objective function costs c_{qij} , $q \in \{1, \dots, k\}$ For this study, the size of the decision variable matrix is 8×8 that is we have 64 decision number of variables. The objective coefficients are assumed to take three different values $\tilde{c}_{qij}^1, \tilde{c}_{qij}^2, \tilde{c}_{qij}^3,$ changing in the interval *(1 100)* with different probabilities, i.e. $p(\tilde{c}_{qij}^1), p\left(\tilde{c}_{qij}^2\right), p\left(\tilde{c}_{qij}^3\right)$. Three different sets of coefficients are generated randomly between ``1-100", for each objective coefficient; and corresponding probabilities are generated randomly between $(0,0.40)$ for first two levels of coefficients, then the probabilities for the last levels are calculated by subtracting their sum from the total probability, i.e. $p\left(\tilde{c}_{qij}^3\right) = 1 - \; p\left(\tilde{c}_{qij}^1\right) - p\left(\tilde{c}_{qij}^2\right)$.

The results are obtained using a computer with Intel Dual-Core processor with 2.66 GHz speed, and with 4 GB memory. The summary table of the results and depiction of two instances with the largest sizes on graph are presented in the next section.

Table 1: Comparison of Efficiency types applied to obtain the deterministic problem

	Type of efficiency applied		
	Minimum expectancy efficiency	Minimum variance efficiency	Weighted expectancy and standard deviation efficiency
Time to finish*	78,898	31,659	43,210
# of unique Pareto points	197	147	157

In Figure 1, Figure 2 and Figure 3, the probable outcome regions of the solutions are presented; one can conclude that these regions are affected to the extent one takes variance into account. In particular, the solutions obtained by minimum variance efficiency have the narrowest regions, while the solutions obtained as the result of minimum expectancy efficiency have the widest probable outcome region.

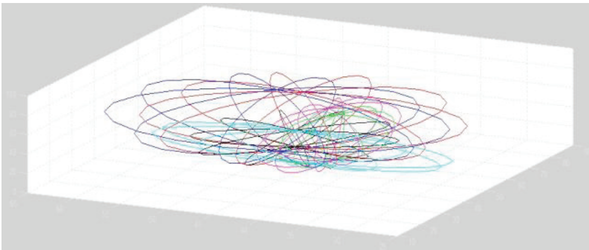

Figure 2: Regions for some selected solutions, obtained by applying minimum expectancy efficiency

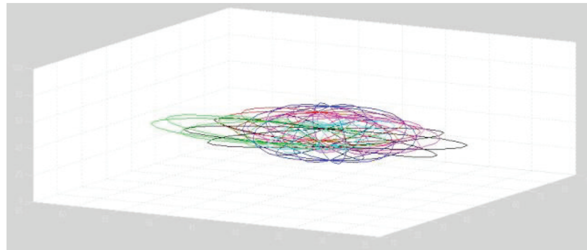

Figure 1: Regions for some selected solutions, obtained by applying minimum variance efficiency

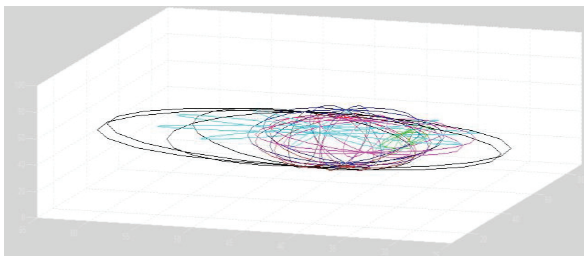

Figure 3: Regions for some selected solutions, obtained by applying weighted expectancy and variance efficiency

Similar results can be derived by observing the following figures Figure 4 and Figure 5, where Pareto optimal results obtained according to minimum expectancy concept are represented by red dot, " · " ; results obtained according to minimum variance concept are represented by magenta circle, " ° "; results obtained according to equally weighted expectancy and variance concepts are represented by green star, " * ".

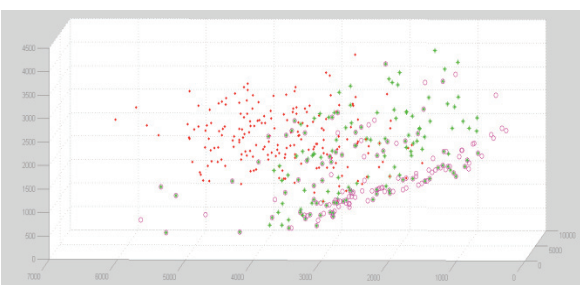

Figure 5: Comparison of variance values for each efficiency concept applied

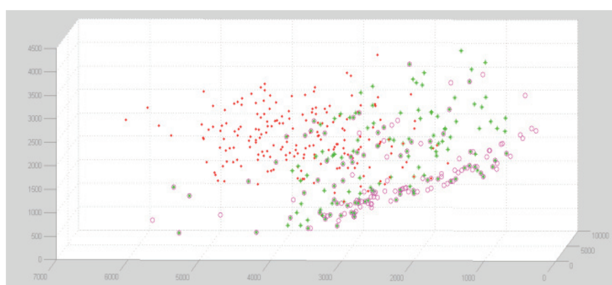

Figure 4: Comparison of expected values for each efficiency concept applied

5. Conclusion

Our results show that applying different efficiency concepts results in different distribution of the solutions in the objective space. Specifically, a decision maker wanting to reduce the probable region of a candidate solution needs to consider the variance. On the other hand, the decision maker may also want to minimize the expected outcome of candidate solutions. In such a situation, constructing objective functions which consider both expectation and variance can result in reduced regions of the probable outcome of candidate solutions which are good quality in the expected sense.

The CPU time requirement of an algorithm in generating the whole Pareto space grows exponentially with the size of the problem. Due to this computational complexity, determining a solution set which is guaranteed to represent the whole Pareto surface is more practical. This concept is defined as the ``cardinality measure" in the representativeness phenomenon of multi objective literature. Hence, a future extension for this study is to develop an algorithm which will generate an approximation set of the whole Pareto surface. The two requirements for such an algorithm are efficiency in terms of CPU time requirement and high degree of representativeness based on the cardinality measure. In addition to minimum expectancy and variance, minimum risk efficiency is also an important concept and will be investigated in the future extension of this study.

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