Research on multi-fidelity aerodynamic optimization methods

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Abstract Constructing high approximation accuracy surrogate model with lower computational cost has great engineering significance. In this paper, using co-Kriging method, an efficient multi-fidelity surrogate model is constructed based on two independent high and low fidelity samples. Co-Kriging method can use a greater quantity of low-fidelity information to enhance the accuracy of a surrogate of the high-fidelity model by modeling the correlation between high and low fidelity model, thus computational cost of building surrogate model can be greatly reduced. A wing-body problem is taken as an example to compare characteristics of co-Kriging multi-fidelity (CKMF) model with traditional Kriging based multi-fidelity (KMF) model. A sampling convergence of the CKMF model and the KMF model is conducted, and an appropriate sampling design is selected through the sampling convergence analysis. The results indicate that CKMF model has higher approximation accuracy with the same high-fidelity samples, and converges at less high-fidelity samples. A wing-body drag reduction optimization design using genetic algorithm is implemented. Satisfying design results are obtained, which validate the feasibility of CKMF model in engineering design.

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1. Introduction

Aerodynamic shape design faces more and more challenges because of increasing performance requirements of airplanes, and a large number of high-fidelity aerodynamic analysis methods have emerged to assist the detailed aircraft design. Higher fidelity analysis methods usually require more computation; although computing power has undergone tremendous changes in the past few decades, the high-fidelity analysis methods can still not be applied to optimization design process directly. Many studies have been carried out concerning the surrogate model approach to solve this problem. Surrogate model, the so called “model of models”, can express the relationship between design variables and performances more clearly with simple structure and high computational efficiency, and has been widely used in design space exploration and optimization design. Surrogate model’s approximation accuracy is directly related to the number of samples and the complexity of the real function. For multi-dimensional problems, a large number of samples are needed to obtain reasonable approximation accuracy. The computation of current surrogate models, such as
polynomial response surface, radial basis functions, neural networks and Kriging, etc. are still too large, so how to build a high-precision surrogate model with small computational cost has strong engineering significance.

Recent surrogate model researches mainly focus on using additional design information to enhance the prediction accuracy of surrogate model, and the additional design information been usually used including gradient information, information of other surrogate models and low-fidelity information, etc. Gradient information can effectively improve the predictive power of surrogate model. Liu effectively enhanced the predictability of Kriging model using gradient information; van Keulen and Vervenne have presented promising results for a gradient enhanced weighted least squares (WLS) method. Using information of other surrogate models is called integration model, also known as multi-layer model. To combine variety of surrogate models, methods of optimizing the weight coefficients of the model or using the mean sum of each surrogate model are commonly used. Surrogate model correction methods use a correction formulation to reduce the prediction error and correction methods are divided into zero, first, second and higher-order correction methods according to the use of correction functions. Zhang et al. have shown that second-order correction method has good applicability. The use of low-fidelity information is known as variable-fidelity model, also known as multi-fidelity model or variable-complexity model, which usually builds a relation model between low and high fidelity model to enhance the prediction ability of surrogate model. The applied research of multi-fidelity models has recently attracted wide attention because of its engineering applicability. Traditional multi-fidelity methods use surrogate of differences between high and low fidelity model as relation model, which need high, low fidelity model analysis on the same samples sites. For multi-dimensional problem, its computational cost is still very expensive for the ineffective usage of low-fidelity information. In this paper, based on two independent high, low fidelity samples (the high fidelity sample is much smaller), an effective surrogate of high-fidelity model is constructed using co-Kriging method. The co-Kriging multi-fidelity (CKMF) model can make full use of low-fidelity information, thus greatly reducing the computational cost of building surrogate model in premise of ensuring prediction accuracy. A wing-body problem is taken as an example, and the sampling convergence of CKMF model is compared to traditional Kriging based multi-fidelity (KMF) model. The results show that CKMF model has higher approximation accuracy when using the same high-fidelity samples, and converges at much smaller high-fidelity samples. Finally an appropriate sampling design is selected through the sampling convergence analysis and an effective CKMF model is built. The wing-body drag reduction optimization design using genetic algorithm is conducted using low-fidelity model, KMF model and CKMF model respectively. CKMF model obtains better design results with reduced computational cost, and the feasibility of the CKMF model in engineering design is validated by comparing the optimization results of different models.

2. Multi-fidelity aerodynamic analysis methods

Currently, lots of numerical aerodynamic analysis methods can be integrated in aerodynamic optimization design method, such as empirical methods, potential numerical methods, Euler numerical methods, Navier–Stokes numerical methods, etc. To construct a reasonable multi-fidelity aerodynamic analysis method, the chosen high-fidelity model must be as accurate as possible and can reflect all considered complex flow characteristics; the chosen low-fidelity model must reflect the basic flow characteristics and be as effective as possible. In this paper, to analyze the aerodynamic characteristics of wing-body, full potential coupled with boundary layer (BLFP) numerical method is chosen as low-fidelity model, which can reflect the basic flow characteristics but has a greater error, and the main advantage of the BLFP method is the small computational cost. The Reynolds-averaged Navier–Stokes (RANS) equations' numerical method is chosen as high-fidelity model. The RANS method is suitable for viscous flow and flow regimes with significant separation caused by viscosity, but its calculation method is relatively complex and a large amount of computation is needed.

Through the analysis of F6 wing-body's aerodynamic characteristics and comparison with experimental results, the characteristics of high and low fidelity aerodynamic analysis methods are verified. The RANS method uses a finite-volume cell-center-based parallel solver on the multi-block structure grid; the convective fluxes are discretized using Roe-FDS 2nd-order upwind total variation diminishing (TVD) scheme with the Harten’s entropy fix function; the viscous fluxes are discretized using the central difference; time integration to steady state is accomplished with the Lower–Upper Symmetric-Gauss–Seidel (LU-SGS) approximation factorization method, and the Menter’s κ-ω SST turbulence model is used. For convergence acceleration, the local time stepping and the multi-grid method with full approximation scheme are used. The full potential equation is solved using approximation factorization methods and viscous/inviscid iterated method is used to calculate viscosity. The aerodynamic calculation state of F6 wing-body is $Ma_{∞} = 0.7520$, $α = 0.49$, $Re = 3.0 \times 10^6$, and the number of multi-block structure grid (shown in Fig. 1) is 3.375 million.

Table 1 shows the comparison of calculated F6 wing-body's aerodynamic characteristics using the chosen high and low fidelity methods, in the table, $C_D$, $C_L$, and $C_M$ are the calculated drag coefficient, lift coefficient and pitching moment coefficient of the wing-body respectively. It can be seen that compared with experimental results, the results of RANS method are far more accurate than BLFP method. Nevertheless, solving RANS equations needs 5 h, and solving the full potential equations needs 4 s, only 1/4500 of the RANS method.

![Fig. 1 Grid of F6 wing-body for RANS simulation.](image)
The pressure coefficient $C_p$ distributions of wing sections are shown in Fig. 2. The pressure distributions of the RANS method fit the experimental results very well, but the results of BLFP method can only reflect the basic trend of flow characteristics and do not catch the exact position and strength of the wave. It can be seen that the computational efficiency of the chosen low and high fidelity model is aerodynamic analysis can be very different, and thus these two numerical methods of aerodynamic characteristics are suitable for multi-fidelity aerodynamic analysis.

3. Multi-fidelity surrogate model

Traditional multi-fidelity surrogate model uses surrogate of differences between high and low fidelity models to correct the low-fidelity model error and improve multi-fidelity model’s prediction accuracy. Its computational cost is still very expensive since the analysis of high and low fidelity model is carried on the same sample points, and its prediction accuracy mainly depends on the accuracy of low-fidelity model. In this paper, a greater quantity of cheap low-fidelity data coupled with a small amount of high-fidelity data are used to enhance the accuracy of surrogate model using co-Kriging method, and thus the efficiency of surrogate model is greatly improved.

Two independent sets of multi-fidelity data are selected using Latin hypercube method, where the high-fidelity model has $n_e$ samples and low-fidelity model has $n_i$ samples. Based on co-Kriging method, the formula used to approximate the high-fidelity model is as follows:

$$ Z_e(x) = \rho Z_d(x) + Z_h(x) $$

where $Z_e(x)$ denotes a Kriging model of the low-fidelity model and $Z_h(x)$ a Kriging model of the difference between low-fidelity model and high-fidelity model; $\rho$ is the scale factor between low-fidelity model and high-fidelity model. The co-variance between sample points can be described as

$$ \text{cov}(Z(x^0), Z(x^0)) = \sigma^2 R_{ij} $$

where $R_{ij}$ is the matrix of correlation between samples, which is determined by a spatial correlation function (SCF), $\sigma^2$ the model variance; $x^0$ and $x^0$ are the $i$th and $j$th samples.

$$ R_{ij} = \text{SCF}(x^0, x^0) = \prod_k \text{SCF}_k(x^0_k - x^0_k) $$

where $n_i$ is the number of design variables; $\theta_k$ and $p_k$ are the $k$th correlation parameters.

As with Kriging, the complete covariance matrix can be constructed as

$$ C = \begin{bmatrix} \sigma^2 R_e(X_e, X_e) & \rho \sigma^2 R_e(X_e, X_h) \\ \rho \sigma^2 R_e(X_h, X_e) & \rho^2 \sigma^2 R_e(X_h, X_h) + \sigma^2 R_d(X_e, X_h) \end{bmatrix} $$

The notation $R_e(X_e, X_h)$ denotes a matrix of correlations between the data $X_e$ and $X_h$, thus there are more correlation parameters ($\theta_e, \theta_h, p_e, p_h$ and the scaling parameter $\rho$) needed to be fitted by optimization process. As the low-fidelity dataset is independent of the high-fidelity dataset, we can find the approximate value of $\theta_e, p_e$ using the same way as Kriging does. In order to estimate $\theta_h, p_h$ and $\rho$, we first define

$$ d = y_e - \rho y_h(X_e) $$

where $y_h(X_e)$ denote the values of $y_e$ at locations common to those of $X_e$, then we can estimate $\theta_h, p_h$ and $\rho$ using the Kriging way. Then the co-Kriging prediction of the high-fidelity model is given by

$$ \hat{y}_e(x) = \hat{\mu} + \hat{\sigma}^2 C(y - \hat{\mu})^{-1} $$

where $\hat{\mu} = (f^T C^{-1} f)^{-1} f^T C^{-1} y$, $f$ is a column vector with dimension $n_e + n_i$, and

$$ C = \begin{bmatrix} \hat{\rho} \hat{\sigma}^2 R_e(X_e, x) \\ \hat{\rho} \hat{\sigma}^2 R_e(X_h, x) + \hat{\sigma}^2 R_d(X_e, x) \end{bmatrix} $$

The co-Kriging method can also give estimated mean square error (MSE) in prediction and is calculated as

$$ \hat{s}^2(x) = \hat{\rho}^2 \hat{\sigma}_e^2 + \hat{\sigma}_d^2 - \hat{\sigma}^2 C^{-1} + \frac{f^T f - \hat{\sigma}^2 C^{-1} f}{f^T f} $$

4. Characteristics verification of CKMF model

A wing-body example is used to demonstrate the characteristics of the co-Kriging multi-fidelity surrogate model. This wing-body configuration is an original design of commercial airplane in our research work; the fuselage and wing planform of the given wing-body (see Fig. 3) remain constant and the wing root, kink and wingtip section airfoils are parameterized using 12 variables Hicks-Henne method, and thus there are 36 variables in total ($n_i = 36$). The aerodynamic design condition of the wing-body is $Ma_{inf} = 0.785$, $\alpha = 2.4^\circ$, $Re = 25 \times 10^6$.

Aerodynamic analysis of the wing-body is conducted using the high-fidelity model of RANS method and the low-fidelity model of BLFP method, respectively. The number of multi-block structure grid for high-fidelity simulation is about 1.7 million and it takes 40 min to run a high-fidelity evaluation on a computer with Intel i7970 in parallel modes.

The most concerned characteristic of surrogate model is the prediction ability of true functions at non-sample locations. Mean relative square error (MRSE) and maximum relative error (MRE) of a separate validation dataset are chosen as criteria of surrogate model’s approximation accuracy.

$$ \text{MRSE} = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left( \frac{f(x) - \hat{f}(x)}{\hat{f}(x)} \right)^2 } $$
Fig. 2  Comparison of pressure coefficient distributions of wing sections.
MRE = \max_{b < c \in M} \left( \frac{f(x_i) - \hat{f}(x_i)}{\hat{f}(x_i)} \right) \tag{10}

where \( M \) is the number of validation samples.

Two independent samples are selected using Latin hypercube method. According to Ref. 20, the sample sizes must fulfill \( n_c \geq 10n_v + 1 \) and \( n_e \geq 3n_v + 1 \), therefore \( n_c = 400 \) and \( n_e = 160 \) samples are selected. The CKMF model is build based on the analysis of these sample datasets using low and high fidelity model respectively. The differences between high and low fidelity model are obtained by analysis of the \( n_e = 160 \) samples using both high and low fidelity model, and the KMF model is built using Kriging method based on the differences. Figs. 4 and 5 are the two models' high-fidelity sampling convergence of the lift coefficient and drag coefficient. The figures show that CKMF model gets its sampling convergence at 120 high-fidelity samples, but KMF model does not completely convergent even at 160 high-fidelity samples; meanwhile, whether the relative mean square error or the maximum relative error, the CKMF model all have higher prediction accuracy. As the CPU time of single low-fidelity analysis is only \( 1/4500 \) of one high-fidelity analysis, the computation of 400 low-fidelity samples can be basically negligible. The above analysis shows that CKMF model can get higher prediction accuracy with reduced computational cost of sample data and has greater engineering significance.

5. Optimization design and result analysis

Wing-body drag reduction optimization design is conducted, with low-fidelity model, the KMF model, and the CKMF model as aerodynamic analysis tools respectively, and genetic algorithm-based optimization process (see Fig. 6) is used. For efficiency consideration, we use genetic algorithm to optimize the correlation parameters of CKMF model at the constructing step; at the updating step, the validated points are added to the sample dataset and pattern search method is used to improve the constructed CKMF model’s accuracy. Surrogate model is updated every five generations, and best points are chosen to validate surrogate model’s accuracy until relative error is less than 3%.

The design state is consistent with the calculation state in Section 4. The population of genetic algorithm is 100 and the total evolution generation is 100. The mathematical model of the optimization design problem is illustrated as follows:

\[
\max \frac{1}{C_D + (C_L - 0.54)} \quad \text{s.t.} \quad \begin{cases} 
\ell_{\text{max root}} \geq 0.15 \\
\ell_{\text{max mid}} \geq 0.11 \\
\ell_{\text{max tip}} \geq 0.10 \\
|C_M| \leq 0.11
\end{cases}
\tag{11}
\]

where \( \ell_{\text{max root}}, \ell_{\text{max mid}} \) and \( \ell_{\text{max tip}} \) are the maximum thickness of the wing root, kink and tip airfoils. The setup of objective function is expecting lift coefficient close to 0.54 as much as possible while minimizing the drag coefficient, meanwhile, constraining the maximum thickness of control airfoils and pitching moment coefficient. The constraints can be satisfied by adding a penalty function to the resultant objective function.

Fig. 3 Wing-body’s original shape.

Fig. 4 Sampling convergence of lift coefficient.

Fig. 5 Sampling convergence of drag coefficient.

Fig. 6 Surrogate model-based optimization process.
If the constraints are $\Psi_i \geq D_i, i = 1, 2, \cdots, m$, the resultant objective function with constraints can be written as

$$ F = F \prod_{i=1}^{m} P_i $$

$$ P_i = \begin{cases} e^{4(D_i - \Psi_i)} & \Psi_i < D_i \\ 1 & \Psi_i \geq D_i \end{cases} \quad (12) $$

The optimized control airfoils all have smaller maximum thickness than the original airfoil but still fulfill the maximum thickness constrains as shown in Fig. 7. The comparison of the aerodynamic characteristics of the optimization design results are shown in Table 2. In the table, $L$ is the lift, $D$ is the drag. Comparing to the original shape, all optimized results get significant improvements of aerodynamic characteristics. Because the low-fidelity model has a large amount of analytical error, the optimization results based on low-fidelity model have a greater lift coefficient error with the expectation and the maximum drag coefficient, its pitching moment coefficient actually violate the maximum constraint.

The optimized results of the other two methods are better than the low-fidelity model-based optimization results to a great extent. Because CKMF model has the most accurate prediction, the lift coefficient of the CKMF model-based optimization results is the closest to the expectation, and the drag coefficient and pitching moment coefficient are the smallest. According to the computational cost, the KMF model requires more times of the surrogate model updating and adds more high-fidelity samples because of accuracy differences; the CKMF model-based optimization calls 165 high-fidelity evaluations (including the initial 120 samples), and the KMF model-based optimization calls 392 high-fidelity evaluations.

Table 2  Performance comparison for wing-body drag reduction optimization.

<table>
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<tr>
<th>Objective function</th>
<th>High-fidelity evaluation</th>
<th>$C_L$</th>
<th>$C_D$</th>
<th>$C_M$</th>
<th>$L/D$</th>
<th>$\Delta(L/D)(%)$</th>
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<tr>
<td>Origin</td>
<td>0</td>
<td>0.5400</td>
<td>0.0311</td>
<td>-0.108</td>
<td>17.36</td>
<td></td>
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<tr>
<td>Low-fidelity model</td>
<td>0</td>
<td>0.5389</td>
<td>0.0280</td>
<td>-0.111</td>
<td>19.25</td>
<td>10.9</td>
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<tr>
<td>KMF model</td>
<td>392</td>
<td>0.5394</td>
<td>0.0271</td>
<td>-0.108</td>
<td>19.89</td>
<td>14.6</td>
</tr>
<tr>
<td>CKMF model</td>
<td>165</td>
<td>0.5400</td>
<td>0.0267</td>
<td>-0.109</td>
<td>20.22</td>
<td>16.5</td>
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Fig. 7  Comparison of optimized control airfoils.
Fig. 8  Comparison of optimized chordwise pressure distributions.

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Fig. 8  Comparison of optimized chordwise pressure distributions.

including initial 200 samples). Fig. 8 shows a comparison of optimized chordwise pressure distributions, where the shock wave is remarkably reduced by the optimization process and CKMF model-based optimization gets the largest shock wave reduction. It can be seen that on the whole, the performance of CKMF model is superior to KMF model and engineering practicality of the CKMF model is validated by the optimization results.

6. Conclusions

In this paper, an efficient multi-fidelity surrogate model is built based on two independent high, low fidelity aerodynamic samples using co-Kriging method.

(1) Based on the analysis of aerodynamic characteristics calculation method, two suitable aerodynamic analysis methods are chosen for the multi-fidelity surrogate model.

(2) The sampling convergence analysis shows that CKMF model can achieve sampling convergence at fewer high-fidelity samples with the assistance of enough low-fidelity samples \( n_c \geq 10n_v + 1 \).

(3) Using the same high-fidelity samples, the prediction accuracy of CKMF model is remarkably higher than KMF model, and the computation of additional low-fidelity samples is basically negligible, only one fifteenth of a single high-fidelity evaluation.

(4) Finally, CKMF model is integrated with genetic algorithm to conduct wing-body drag reduction optimization. The optimized results are much better than low-fidelity model-based optimization and MKF model-based optimization, and the computational cost of CKMF model-based optimization is only half of MKF model-based optimization. It can be seen that CKMF model has a strong engineering practicality.

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References


Huang Likeng is a Ph.D. student at the School of Aeronautics, Northwestern Polytechnical University. He received his B.S. and M.S. degrees in aircraft design from Northwestern Polytechnical University in 2007 and 2010 respectively and then became a teacher there. His area of research includes aircraft design, optimization methods, surrogate models, as well as aerodynamics.

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