The Analysis on the Processing Dexterity of a 3-TPT Parallel Machine Tool

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Abstract

This paper took a 3-TPT Parallel Machine Tool (PMT) as the objective of research. First, the kinematics equation and the Jacobian matrix were obtained according to the kinematics analysis of PMT and the condition number of Jacobian matrix was obtained further. Then, the analysis on the processing dexterity of PKM was conducted in terms of the condition number of Jacobian matrix as measurable index. Therefore, its distribution characteristics within the workspace were extracted. The result shows that this PMT is in effective processing dexterity. The conclusions based on the analysis have an important theoretical significance to the structural design, workspace design and inspection as well as kinematics simulation of this PMT.

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1. Introduction

Parallel Machine Tool (PMT) has attracted people’s extensive attention from the beginning of its appearance. With in-depth of the study and improvement of the related technologies, the precision requirements for the operation of PMT are also getting higher and higher. Therefore, processing dexterity analysis as one of the most important indices to measure the movement performance of PMT has a more practical significance [1, 2]. This paper took a 3-TPT PMT as the objective of research, which mainly analyzed the measurable index for the processing dexterity of PMT and its distribution characteristics within the workspace.

2. The Structure of Parallel Machine Tool

The parallel mechanism of 3-TPT PMT mainly consists of the movable platform, the fixed platform, the parallelogram mechanism, the driving rod groups and the end actuating mechanism several parts [3]. The structure of this 3-TPT PMT is shown in Fig 1(a), three groups of driving rods are installed in parallel between the fixed platform and the movable platform, and both ends of the driving rod are jointed.
to the fixed platform and the movable platform separately by Hooker joint. The position of movable platform can be adjusted by changing every length of driving rod. In addition, the parallelogram mechanism has been added between the fixed platform and the movable platform, so the movable platform can be restrained by the parallelogram mechanism to move translationally relative to the fixed platform.

3. The Kinematics Analysis and Workspace

The kinematics analysis is the necessary condition for the overall design of PMT, and the basis of the dynamics analysis, precision analysis and other researches of PMT [4]. Here, kinematics equation is established by analyzing the kinematics of PMT.

3.1. The establishment of kinematics equation

Fig 1(b) shows the sketch of parallel mechanism. The positions of the joint points of the fixed platform and the movable platform are arranged into an equilateral triangle distribution, and make two triangles corresponding sides parallel. The fixed platform coordinate system $O-XYZ$ is established on the fixed platform of parallel mechanism, the origin $O$ is in the centre of the triangle of fixed platform; Likewise, the movable platform coordinate system $P-x'y'z'$ is established on the movable platform, the origin $P$ is in the centre of the triangle of movable platform, every coordinate axis respectively paralleled with coordinate axis of the fixed platform coordinate system and in the same direction.

![Fig. 1. (a) Structure of 3-TPT PMT; (b) Sketch of 3-TPT parallel mechanism](image-url)
Take \( l_i \) (\( i=1,2,3 \)) as the length of driving rod. Then \( b_i' \) is the coordinate vector of the joint point of movable platform in the movable platform coordinate system, \( b_i \) is the coordinate vector of the joint point of movable platform in the fixed platform coordinate system, and \( B_i \) is the coordinate vector of the joint point of fixed platform in the fixed platform coordinate system. Take \( r_p \) as the coordinate vector of the movable platform coordinate origin in the fixed platform coordinate system, is to be \((x, y, z)\). Because of the restraint function of the parallelogram mechanism, the movable platform produces only the translatory motion relative to the fixed platform, and then the inverse kinematics equation of parallel mechanism in the PMT can be obtained as follow

\[
\begin{align*}
(l_1 &= |b_1 - B_1| = |r_p + R \cdot b_1' - B_1| = \sqrt{(x + \frac{c}{2\sqrt{3}})^2 + (y + \frac{c}{2})^2 + z^2} \\
(l_2 &= |b_2 - B_2| = |r_p + R \cdot b_2' - B_2| = \sqrt{(x + \frac{c}{2\sqrt{3}})^2 + (y - \frac{c}{2})^2 + z^2} \\
(l_3 &= |b_3 - B_3| = |r_p + R \cdot b_3' - B_3| = \sqrt{(x - \frac{c}{\sqrt{3}})^2 + y^2 + z^2} \\
\end{align*}
\]

Where, \( R \) is the transformation matrix of the movable platform coordinate system relative to the fixed platform coordinate system; \( c \) is the side difference of joint point triangle in the fixed platform and movable platform.

3.2. The Jacobian matrix

The Jacobian matrix is an important parameter for PMT. It refers to the generalized drive ratio of the movement velocity transmission from joint-space to operation-space, which is the basis of research about PMT on kinematics, statics, dynamics and error analysis etc [4, 5].

Then the Jacobian matrix of parallel mechanism can be obtained from the kinematics equation,

\[
J = \begin{bmatrix}
\frac{l_1}{\sqrt{3}c} & \frac{l_2}{\sqrt{3}c} & \frac{l_3}{\sqrt{3}c} \\
\frac{1}{c} & -\frac{1}{2c} & 0 \\
2\left(l_1^2 + l_2^2 - 2l_1^2 + c^2\right)l_1 \sqrt{6cd} & 2\left(l_2^2 + l_3^2 - 2l_2^2 + c^2\right)l_2 \sqrt{6cd} & 2\left(l_3^2 + l_3^2 - 2l_3^2 + c^2\right)l_3 \sqrt{6cd}
\end{bmatrix}
\]

Where,

\[
d = \sqrt{l_1^2 + l_2^2 + l_3^2 + c^2} - 3\left(l_1^4 + l_2^4 + l_3^4 + c^4\right)
\]

3.3. The workspace of Parallel Machine Tool

The workspace means the maximal range of the movable platform coordinate origin movement in the space when the PMT to work normally [6]. According to the pose characteristic of movable platform in working, the workspace can be divided into the accessible workspace and flexible workspace [7]. The workspace described in this paper all mean accessible workspace. In considering of the case that the changes of driving rod length and Hooker joint corner can influence the workspace, the shape of machine workspace can be obtained as shown in Fig 2 according to the actual design size of PMT. It can be seen
from the figure that the shape of PMT workspace is irregular and the whole workspace is continuous and no empty hole. The actual work-area of PMT is limited in this space.

Fig. 2. The workspace of 3-TPT PMT

4. The Processing Dexterity of Parallel Machine Tool

4.1. The condition number of Jacobian matrix

The processing dexterity of PMT usually refers to the movement capacity of PMT along the designated direction in the current pose [8]. Since the determinant value of the matrix can not actually represent the stability of matrix inversion operations, there exist some defects to use the operational degree to analyze the dexterity of PMT. The condition number of Jacobian matrix may represent the stability of matrix inversion operation quantitatively, therefore it is more reasonable to use that to measure the processing dexterity of PMT. The expression of the condition number of Jacobian matrix can be defined as:

\[ \text{cond} = C(J) = \| J \| \| J^{-1} \| \]  

(3)

Where, \( \| J \| \) represents the norm of Jacobian matrix.

The range of value for the condition number of Jacobian matrix is within \( 1 \leq \text{cond} \leq \infty \). When the condition number is 1, the Jacobian matrix should be an orthogonal matrix, the mechanism possesses the best movement transmission performance, and the position of the mechanism at the time is called kinematics isotropy. When condition number is infinity, the Jacobian matrix should be a singular matrix and the mechanism is in a special position. When the design to PMT mechanism is conducted, the condition number of its Jacobian matrix should be made the best of the smallest value within the operation space.

4.2. The analysis on the processing dexterity

A cubic area of 600×600×400(mm) within the workspace of PMT can be taken as its actual workspace to study the condition number, and the detailed range for each of the coordinates were defined respectively as follows: \(-300 \leq x \leq 300\), \(-300 \leq y \leq 300\), \(800 \leq z \leq 1200\). The processing dexterity of PMT was analyzed within the given workspace from the values taken by the condition number.

The variations of the condition number of Jacobian matrix within the workspace are shown in Fig 3. It can be seen from Figure that the transformation range of the condition number is \(2.5 \leq \text{cond} \leq 5.5\), the condition number changes continuously within the selected workspace and there is no mutation, so it means the processing dexterity of this PMT is very good. Moreover it can be found that the variation of condition number is symmetrical about the XZ plane (namely \(Y=0\)) of fixed platform coordinate system. This character is coincides with the fact this PKM is symmetrical in the structure, which also shows the
result of the analysis is correct. In addition, the value of condition number is relatively small in the \(XZ\) plane of symmetry, and the condition number increases gradually with the deviating from symmetrical position. The value of condition number becomes smaller with the decreasing of the coordinates along \(X\) axis and \(Z\) axis directions and gradually becomes larger with the increasing of their coordinates.

Fig. 3. The change of condition number in the workspace

5. Conclusion

From the results of analysis on the condition number of PMT, the following conclusions can be reached on the processing dexterity of PMT within the selected workspace:

- The variation range of the condition number of PMT is \(2.5 \leq \text{cond} \leq 5.5\). The condition number varies continuously within the whole workspace.
- The variation of condition number is symmetrical about the \(XZ\) plane (namely \(Y=0\)) of fixed platform coordinate system in the PMT.
- In the \(XZ\) plane of symmetry, the value of condition number of PMT becomes relatively smaller with the decreasing of the coordinates along \(X\) axis and \(Z\) axis directions. For this, the processing dexterity of PMT so to becomes relatively best.

The conclusions based on the analysis to the processing dexterity of PMT have an important theoretical significance to the structural design, actual workspace design and inspection as well as kinematics simulation of this PMT.

References