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ORIGINAL ARTICLE

Probabilistic Fuzzy Goal Programming Problems Involving Pareto Distribution: Some Additive Approaches

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Abstract In many real-life decision making problems, probabilistic fuzzy goal programming problems are used where some of the input parameters of the problem are considered as random variables with fuzzy aspiration levels. In the present paper, a linearly constrained probabilistic fuzzy goal programming programming problem is presented where the right hand side parameters in some constraints follows Pareto distribution with known mean and variance. Also the aspiration levels are considered as fuzzy. Further, simple, weighted, and preemptive additive approaches are discussed for probabilistic fuzzy goal programming model. These additive approaches are employed to aggregating the membership values and form crisp equivalent deterministic models. The resulting models are then solved by using standard linear mathematical programming techniques. The developed methodology and solution procedures are illustrated with a numerical example.

Keywords Goal programming · Fuzzy goal programming · Probabilistic fuzzy goal programming · Pareto distribution

1. Introduction

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Linear programming (LP) has been used as an important technique to solve many complex real-life decision making problems such as, finance, transportation, railways, engineering, economics, agriculture, telecommunications, production planning and scheduling, mining industries, energy sectors, military etc. LP problems involve the optimization of a linear objective function, subject to some linear equality and inequality constraints involving some continuous/discrete non-negative decision variables. It was first introduced by Dantzig [7] and applied initially to solve complex planning problems in the operations of second world war. In most of the engineering and management science decision making problems that often contain multiple and conflicting objectives. Such types of problems can generally be modeled as multi-objective optimization problems. Various methodology has been developed to solve multi-objective optimization problems from which goal programming (GP) is an important technique for solving multi-objective programming problem. GP was originally proposed by Charnes and Cooper [2, 3], and further development carried out by Lee [16], Ignizio [10], and Romero [21], among others [4, 8, 30]. It has been applied to many areas such as accounting, agriculture, economics, engineering, transportation, finance, government, international context, and marketing. GP is an important technique for decision-makers (DMs) to consider simultaneously several objectives in finding a set of desirable solutions. Tamiz et al. [28] have proposed an overviews of the current state-of-the-art in GP in which the DMs set some acceptable aspiration levels for their goals, say $g_k (k = 1, 2, \cdots, K)$, and try to achieve these goals as closely as possible. The purpose of GP is to minimize the deviations between the achievement of goals, say $f_k(X), X = (x_1, x_2, \cdots, x_n)$, and these acceptable aspiration levels, $g_k (k = 1, 2, \cdots, K)$. Therefore, GP can be expressed as follows:

GP Model:
A general mathematical model of GP can be stated as:

$$
\begin{aligned}
\text{Min} \quad & \sum_{k=1}^{K} |f_k(X) - g_k| \\
\text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, 2, \cdots, m, \\
& x_j \geq 0 \qquad\quad \text{for } j = 1, 2, \cdots, n,
\end{aligned}
\tag{1}
$$

where K is the total number of goals, a_{ij} is the coefficient of constraint matrix, b_i is the right hand side constraint coefficient, $f_k(X)$ is the k-th objective/goal, and g_k is the aspiration level of the k-th goal.

The variants of GP such as lexicographic GP, weighted GP, and MINMAX GP are also applied in many real life decision making problems [27]. Now, the above model of GP can be rewritten as:

$$
\begin{aligned}
\text{Min} \quad & \sum_{k=1}^{K} (\rho_k + \eta_k) \\
\text{s.t.} \quad & f_k(X) + \eta_k - \rho_k = g_k, k = 1, 2, \cdots, K, \\
& \sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1, 2, \cdots, m, \\
& \rho_k, \eta_k, x_j \geq 0 \qquad \text{for } k = 1, 2, \cdots, K, j = 1, 2, \cdots, n,
\end{aligned}
\tag{2}
$$

where $\rho_k = \max(0, f_k(x) - g_k)$ and $\eta_k = \max(0, g_k - f_k(x))$ are respectively, over- and under-achievement of the kth goal; other variables are defined as above.

In the above GP model formulation, goals are precisely defined. That is, the formulation assumes that the decision-maker (DM) is able to determine goal values accurately for their problems. In reality, many imprecise aspiration levels may exist in managerial decision-making problems such as "somewhat larger than", "substantially lesser than", or "around" the vague goal g_k due to DMs ambiguous understanding of their nature. Thus, the DM may find some difficulties to state precisely exact aspiration levels to the goals for their problems. In doing so, if the imprecise aspiration level is introduced to each objective, then the problem is turned into fuzzy GP (FGP). The FGP was first considered by Narasimhan [20] with a preference-based membership function. Many achievements have been reported in the literature such as preemptive fuzzy goal programming [31], weighted max-min model with concave and quasi-concave piecewise membership function [17], interactive FGP [24], weight additive model [29], binary FGP model [5], linear programming with multiple fuzzy goals model [9], FGP on mini-max approach [32] and some case studies [1, 15]. Zangiabadi and Maleki [34] proposed an application of fuzzy goal programming to the linear multiobjective transportation problem. Biswas and Modak [35] presented a fuzzy goal programming technique to solve multiobjective mathematical programming problems in a stochastic environment where the right sided parameters associated with the system constraints are exponentially distributed fuzzy random variables. Lotfi and Ghaderi [36] developed a fuzzy mixed integer linear goal programming problem by considering several objectives with lower relative importance for the short-term unit commitment in the deregulated hybrid markets. Since every real-life problem consists of more than one conflicting objective functions. So the FGP model can be stated as follows:

FGP Model:

A general mathematical model of FGP can be stated as:
Find $X = (x_1, x_2, \cdots, x_n)$ to optimize the following fuzzy goals:

$$\begin{array}{ll} & f_k(X) \succeq g_k, \quad k = 1, 2, \cdots, K_0 \\ & f_k(X) \preceq g_k, \quad k = K_0 + 1, 2, \cdots, K_1 \\ & f_k(X) \cong g_k, \quad k = K_1 + 1, 2, \cdots, K_2 \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1, 2, \cdots, m, \\ & x_j \ge 0 \qquad \text{for } j = 1, 2, \cdots, n, \end{array} \tag{3}$$

where X is n-dimensional decision vector, the symbol '$\succeq$' (the type of fuzzy-max) referring to that $f_k(X)$ should be approximately greater than or equal to the aspiration level g_k signifies that DM is satisfied even if less than g_k up to a certain limit, the symbol '$\preceq$' (the type of fuzzy-min) referring to that $f_k(X)$ should be approximately less than or equal to the aspiration level g_k signifies that the DM is satisfied even if greater than g_k up to a certain tolerance limit, the symbol '$\cong$' (the type of fuzzy-equal) referring to that $f_k(X)$ should be in the vicinity of the aspiration g_k signifies that the DM is satisfied even if greater than (or less than) g_k up to a certain limit, $f_k(X)$ is the k-th fuzzy goal constraints, K_0 is the number of fuzzy-max goal constraints, $K_1 - K_0$ is the number of fuzzy-min constraints, $K_2 - K_1$ is the number of fuzzy-equal constraints; other variables are defined as in GP.

While dealing with real-world problems, the DM often faces problems of optimizing more than one conflicting objectives at a time without knowing the values of some or all parameters of the mathematical programming models, such types of problems comes under stochastic programming [14]. Charnes and Cooper [2, 3] first introduced chance constrained programming model which is also known as probabilistic programming. They suggested three models that have different objective functions and probabilistic types of constraints as:

- E-model which maximizes the expected value of the objective function,
- V-model which minimizes the generalized mean square of the objective function,
- P-model which maximizes the probability of exceeding an aspiration level, i.e., the goal of the objective function.

When the constraint coefficients of the FGP problem contains some random variables with known probability distributions, i.e., coefficient matrix or right hand side coefficients may be consider as random in nature, is known as probabilistic FGP (PFGP). Some literature have been reported such as chance-constrained fuzzy goal programming [18, 19], stochastic FGP model [11, 12]. So the PFGP model can be stated as follows:

PFGP Model:

A mathematical model of PFGP can be stated as:
Find $X = (x_1, x_2, \cdots, x_n)$ to optimize the following fuzzy goals:

$$\begin{array}{lll} & f_k(X) \geq g_k, & k = 1, 2, \cdots, K_0 \\ & f_k(X) \leq g_k, & k = K_0 + 1, 2, \cdots, K_1 \\ & f_k(X) \cong g_k, & k = K_1 + 1, 2, \cdots, K_2 \\ \text{s.t.} & \Pr(\sum_{j=1}^{n} a_{ij} x_j \leq b_i) \geq 1 - \gamma_i, & i = 1, 2, \cdots, m, \\ & x_j \geq 0 \text{ for} & j = 1, 2, \cdots, n, \end{array} \tag{4}$$

where $0 < \gamma_i < 1, i = 1, 2, \cdots, m$ are given constants. It is assume that $a_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$ are deterministic, and $b_i, i = 1, 2, \cdots, m$ are considered as random variables with known probability distribution; other variables are defined as in FGP.

In this paper, some additive approaches namely, simple, weighted, and preemptive priority have been proposed to PFGP by employing the usual addition as an operator to aggregate the membership functions of the fuzzy goals. Also, considering the right hand side constraint coefficient (i.e., b_i) as Pareto random variable with known mean and variance and chance-constrained program is used to convert the probabilistic constraints to its deterministic form.

2. Probabilistic Linearly Constrained FGP with Pareto Distribution (PD)

Let us assume that $b_i, i = 1, 2, \cdots, m$ are independent random variables following Pareto distribution (PD) [13]. The probability density function (pdf) of the i-th random variables b_i is given as:

$$f(b_i) = \begin{cases} \dfrac{p_i(q_i)^{p_i}}{b_i^{(p_i+1)}}, & \text{if } b_i > q_i, p_i > 0, q_i > 0, \\ 0, & \text{if } b_i \leq q_i. \end{cases} \tag{5}$$

The mean $E(b_i)$ and variance $Var(b_i)$ are given as:

$$E(b_i) = \frac{p_i q_i}{p_i - 1}, p_i > 1, i = 1, 2, \cdots, m \tag{6}$$

and

$$Var(b_i) = \frac{p_i q_i^2}{(p_i - 1)^2(p_i - 2)}, p_i > 2, i = 1, 2, \cdots, m. \tag{7}$$

The parameters, i.e., shape parameters $p_i, i = 1, 2, \cdots, m$ and scale parameters $q_i, i = 1, 2, \cdots, m$, are calculated from (6) and (7).

Now using Eq. (5) and the i-th probability constraint from Eq.(4), obtain the following as:

$$\int_{y_i}^{\infty} f(b_i) db_i \geq 1 - \gamma_i, i = 1, 2, \cdots, m, \tag{8}$$

where $y_i = \sum\limits_{j=1}^{n} a_{ij} x_j$ and $y_i \geq 0$.

Now Eq.(8) can be written as

$$\int_{y_i}^{\infty} \frac{p_i(q_i)^{p_i}}{b_i^{(p_i+1)}} db_i \geq 1 - \gamma_i, i = 1, 2, \cdots, m. \tag{9}$$

For the case when $y_i \geq q_i$, the above integration can be calculated as:

$$\left(\frac{q_i}{y_i}\right)^{p_i} \geq 1 - \gamma_i, i = 1, 2, \cdots, m, \tag{10}$$

i.e.,

$$y_i \leq \frac{q_i}{(1 - \gamma_i)^{\frac{1}{p_i}}}, i = 1, 2, \cdots, m, \tag{11}$$

i.e.,

$$\sum_{j=1}^{n} a_{ij}x_j \leq \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, i = 1, 2, \cdots, m. \tag{12}$$

For the case when $y_i \leq q_i$, the above integration becomes zero.

Thus, the equivalent FGP problem of the PFGP problem (4) can be stated as:

Find $X = (x_1, x_2, \cdots, x_n)$ to optimize the following fuzzy goals:

$$\begin{array}{lll} & f_k(X) \geq g_k, & k = 1, 2, \cdots, K_0 \\ & f_k(X) \leq g_k, & k = K_0 + 1, 2, \cdots, K_1 \\ & f_k(X) \cong g_k, & k = K_1 + 1, 2, \cdots, K_2 \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij}x_j \leq \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, & i = 1, 2, \cdots, m, \\ & x_j \geq 0, & \text{for} j = 1, 2, \cdots, n. \end{array} \tag{13}$$

3. Solution Approaches to PFGP problem with Pareto Distribution and Their Deterministic Models

3.1. Simple Additive Approach to PFGP problem with Pareto Distribution

Consider PFGP problem as follows:

Find $X = (x_1, x_2, \cdots, x_n)$ to optimize the following fuzzy goals:

$$\begin{array}{lll} & f_k(X) \geq g_k, & k = 1, 2, \cdots, K_0 \\ & f_k(X) \leq g_k, & k = K_0 + 1, 2, \cdots, K_1 \\ & f_k(X) \cong g_k, & k = K_1 + 1, 2, \cdots, K_2 \\ \text{s.t.} & \Pr(\sum_{j=1}^{n} a_{ij}x_j \leq b_i) \geq 1 - \gamma_i, & i = 1, 2, \cdots, m, \\ & x_j \geq 0, & \text{for } j = 1, 2, \cdots, n, \end{array} \tag{14}$$

where assume that the right-hand side variables $b_i, i = 1, 2, \cdots, m$ are Pareto random variables with known mean and variance. The parameters, i.e., shape parameters $p_i, i = 1, 2, \cdots, m$ and scale parameters $q_i, i = 1, 2, \cdots, m$, are calculated from mean and variance and all other variables defined are the same as in previous Section 2.

Now, the linear membership functions $\mu_k(f_k(X))$ for the k-th fuzzy goals are defined according to Zimmermann [33] as follows:

For the fuzzy goal of type $f_k(X) \geq g_k$, i.e., fuzzy-max, the membership function is defined as:

$$\mu_k(f_k(X)) = \begin{cases} 1, & \text{if } f_k(X) \geq g_k, \\ \dfrac{f_k(X) - L_k}{g_k - L_k}, & \text{if } L_k \leq f_k(X) \leq g_k, \; k = 1, 2, 3, \cdots, K_0, \\ 0, & \text{if } f_k(X) \leq L_k, \end{cases} \tag{15}$$

where L_k is the lower tolerance limit for the k-th fuzzy goal $f_k(X)$.

In case of the fuzzy goal of type $f_k(X) \leq g_k$, i.e., fuzzy-min, the linear membership function is defined as

$$\mu_k(f_k(X)) = \begin{cases} 1, & \text{if } f_k(X) \leq g_k, \\ \dfrac{U_k - f_k(X)}{U_k - g_k}, & \text{if } g_k \leq f_k(X) \leq U_k, \quad k = K_0 + 1, 2, 3, \cdots, K_1, \\ 0, & \text{if } f_k(X) \geq U_k, \end{cases} \tag{16}$$

where U_k is the upper tolerance limit.

In case of the fuzzy goal of type $f_k(x) \cong g_k$, i.e., fuzzy-equal, the membership function is defined as:

$$\mu_k(f_k(X)) = \begin{cases} 0, & \text{if } f_k(X) \leq L_k, \\ \dfrac{f_k(X) - L_k}{g_k - L_k}, & \text{if } L_k \leq f_k(X) \leq g_k, \quad k = K_1 + 1, 2, 3, \cdots, K_2, \\ \dfrac{U_k - f_k(X)}{U_k - g_k}, & \text{if } g_k \leq f_k(X) \leq U_k, \quad k = K_1 + 1, 2, 3, \cdots, K_2, \\ 0, & \text{if } f_k(X) \geq U_k. \end{cases} \tag{17}$$

Now, if the fuzzy goal of type fuzzy-max, then the simple additive model of the PFGP problem from (14) is formulated by using the membership function (15) together as follows:

$$\begin{aligned} \text{Max} \quad & D(\mu) = \sum_{k=1}^{K_0} \mu_k(f_k(X)) \\ \text{s.t.} \quad & \mu_k(f_k(X)) = \frac{f_k(X) - L_k}{g_k - L_k}, && k = 1, 2, \cdots, K_0, \\ & \Pr\Big(\sum_{j=1}^{n} a_{ij} x_j \leq b_i \Big) \geq 1 - \gamma_i, && i = 1, 2, \cdots, m, \\ & \mu_k(f_k(X)) \leq 1, && k = 1, 2, \cdots, K_0, \\ & x_j \geq 0 && \text{for } j = 1, 2, \cdots, n, \end{aligned} \tag{18}$$

where $D(\mu)$ is called a fuzzy achievement function or fuzzy decision function. This is a single objective optimization problem which can be solved by using a suitable classical technique. Since the goals are fuzzy, unlike conventional GP (minimizing the deviations) the fuzzy decision function consisting of $\mu_k(f_k(X))$'s is to be maximized here.

Similarly, if the fuzzy goal of type fuzzy-min, then the simple additive model of the PFGP problem from (14) is formulated by using the membership function (16) together as follows:

$$\begin{array}{lll} \text{Max} & D(\mu) = \sum\limits_{k=K_0+1}^{K_1} \mu_k(f_k(X)) & \\ \text{s.t.} & \mu_k(f_k(X)) = \dfrac{U_k - f_k(X)}{U_k - g_k}, & k = K_0+1, 2, \cdots, K_1, \\ & \Pr(\sum\limits_{j=1}^{n} a_{ij}x_j \le b_i) \ge 1-\gamma_i, i = 1,2,\cdots,m, & \\ & \mu_k(f_k(X)) \le 1, & k = K_0+1, 2, \cdots, K_1, \\ & x_j \ge 0 & \text{for } j = 1,2,\cdots,n. \end{array} \tag{19}$$

But for the fuzzy goal of fuzzy-equal type, difficulty is faced to define k-th membership function with two linear functions. If membership functions are considered as the part that the degree of membership is increasing from 0 to 1 and decreasing from 1 to 0, it will be possible to deal with the difficulty. In this case, the problem is transformed into two sub-problems. In other words, it is tried to determine that X^M (solution vector) belongs to the intervals $[L_k, g_k]$ and $[g_k, U_k]$. Thus the sub-problems are formed for the k-th fuzzy goal as follows:

First sub-problem is stated as:

$$\begin{array}{lll} \text{Max} & D(\mu) = \sum\limits_{k=K_1+1}^{K_2} \mu_k(f_k(X)) & \\ \text{s.t.} & \mu_k(f_k(X)) = \dfrac{f_k(X) - L_k}{g_k - L_k}, & k = K_1+1, 2, 3, \cdots, K_2, \\ & L_k \le f_k(X) \le g_k, & k = K_1+1, 2, 3, \cdots, K_2, \\ & \Pr(\sum\limits_{j=1}^{n} a_{ij}x_j \le b_i) \ge 1-\gamma_i, & i = 1,2,\cdots,m, \\ & \mu_k(f_k(X)) \le 1, & k = K_1+1, 2, \cdots, K_2, \\ & x_j \ge 0 & \text{for } j = 1,2,\cdots,n. \end{array} \tag{20}$$

Similarly, second sub-problem can be written as:

$$\begin{array}{lll} \text{Max} & D(\mu) = \sum\limits_{k=K_1+1}^{K_2} \mu_k(f_k(X)) & \\ \text{s.t.} & \mu_k(f_k(X)) = \dfrac{U_k - f_k(X)}{U_k - g_k}, & k = K_1+1, 2, 3, \cdots, K_2, \\ & g_k \le f_k(X) \le U_k, & k = K_1+1, 2, 3, \cdots, K_2, \\ & \Pr(\sum\limits_{j=1}^{n} a_{ij}x_j \le b_i) \ge 1-\gamma_i, & i = 1,2,\cdots,m, \\ & \mu_k(f_k(X)) \le 1, & k = K_1+1, 2, \cdots, K_2, \\ & x_j \ge 0 & \text{for } j = 1,2,\cdots,n. \end{array} \tag{21}$$

Since X^M vector satisfies $L_k \le f_k(X) \le g_k, k = K_1+1, 2, 3, \cdots, K_2$ inequality for any fuzzy goal, it can also satisfy $g_k \le f_k(X) \le U_k, k = K_1+1, 2, 3, \cdots, K_2$ inequality for another fuzzy goal. For this reason, the sub-problems are combined as [20]:

$$\begin{aligned}
\text{Max}\quad & D(\mu)=\sum_{k=K_1+1}^{K_2}\mu_k(f_k(X)) \\
\text{s.t.}\quad & \mu_k(f_k(X))=\frac{f_k(X)-L_k}{g_k-L_k}, && k=K_1+1,2,3,\cdots,K_2,\\
& L_k\le f_k(X)\le g_k, && k=K_1+1,2,3,\cdots,K_2,\\
& \mu_k(f_k(X))=\frac{U_k-f_k(X)}{U_k-g_k}, && k=K_1+1,2,3,\cdots,K_2,\\
& g_k\le f_k(X)\le U_k, && k=K_1+1,2,3,\cdots,K_2,\\
& \Pr\Big(\sum_{j=1}^{n}a_{ij}x_j\le b_i\Big)\ge 1-\gamma_i, i=1,2,\cdots,m,\\
& \mu_k(f_k(X))\le 1, && k=K_1+1,2,\cdots,K_2,\\
& x_j\ge 0 && \text{for } j=1,2,\cdots,n.
\end{aligned}\tag{22}$$

Thus, using Eq.(12) the equivalent linear models of PFGP can be expressed as follows:

For the fuzzy goal type fuzzy-max, the equivalent linear model of PFGP can be penned as:

$$\begin{aligned}
\text{Max}\quad & D(\mu)=\sum_{k=1}^{K_0}\mu_k(f_k(X)) \\
\text{s.t.}\quad & \mu_k(f_k(X))=\frac{f_k(X)-L_k}{g_k-L_k}, && k=1,2,\cdots,K_0,\\
& \sum_{j=1}^{n}a_{ij}x_j\le\frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i=1,2,\cdots,m,\\
& \mu_k(f_k(X))\le 1, && k=1,2,\cdots,K_0,\\
& x_j\ge 0 && \text{for } j=1,2,\cdots,n.
\end{aligned}\tag{23}$$

Similarly, for the fuzzy goal type fuzzy-min , the equivalent linear model of PFGP can be written as:

$$\begin{aligned}
\text{Max}\quad & D(\mu)=\sum_{k=K_0+1}^{K_1}\mu_k(f_k(X)) \\
\text{s.t.}\quad & \mu_k(f_k(X))=\frac{U_k-f_k(X)}{U_k-g_k}, && k=K_0+1,2,\cdots,K_1,\\
& \sum_{j=1}^{n}a_{ij}x_j\le\frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i=1,2,\cdots,m,\\
& \mu_k(f_k(X))\le 1, && k=K_0+1,2,\cdots,K_1,\\
& x_j\ge 0 && \text{for } j=1,2,\cdots,n
\end{aligned}\tag{24}$$

and for the fuzzy-equal type of fuzzy goal, the equivalent linear model of PFGP can be formulated as:

$$\begin{array}{lll}
\text{Max} & D(\mu)=\sum\limits_{k=K_1+1}^{K_2}\mu_k(f_k(X)) & \\
\text{s.t.} & \mu_k(f_k(X))=\dfrac{f_k(X)-L_k}{g_k-L_k}, & k=K_1+1,2,3,\cdots,K_2,\\
& L_k\le f_k(X)\le g_k, & k=K_1+1,2,3,\cdots,K_2,\\
& \mu_k(f_k(X))=\dfrac{U_k-f_k(X)}{U_k-g_k}, & k=K_1+1,2,3,\cdots,K_2,\\
& g_k\le f_k(X)\le U_k, & k=K_1+1,2,3,\cdots,K_2,\\
& \sum\limits_{j=1}^{n}a_{ij}x_j\le\dfrac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, & i=1,2,\cdots,m,\\
& \mu_k(f_k(X))\le 1, & k=K_1+1,2,3,\cdots,K_2,\\
& x_j\ge 0 & \text{for } j=1,2,\cdots,n.
\end{array} \tag{25}$$

The above obtained deterministic models can be solved by using any linear programming techniques or can be solved by using LINGO 11.0 [26] optimization tool.

3.2. Weighted Additive Approach to PFGP Problem with Pareto Distribution

The weighted additive approach is widely used in GP and multi-objective optimization techniques to reflect the relative importance of the goals/objectives. In this approach, the DM assigns differential weights as coefficients of the individual terms in a simple additive fuzzy achievement function to reflect their relative importance, i.e., the objective function is formulated by multiplying each membership of the fuzzy goal with a suitable weight and then adding them together. This leads to the following equivalent models with respect to different fuzzy goals similar to Subsection 3.1 as:

For the fuzzy goal type fuzzy-max, the equivalent linear model of PFGP can be penned as:

$$\begin{array}{lll}
\text{Max} & D(\mu)=\sum\limits_{k=1}^{K_0}w_k\mu_k(f_k(X)) & \\
\text{s.t.} & \mu_k(f_k(X))=\dfrac{f_k(X)-L_k}{g_k-L_k}, & k=1,2,\cdots,K_0,\\
& \sum\limits_{j=1}^{n}a_{ij}x_j\le\dfrac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, & i=1,2,\cdots,m,\\
& \mu_k(f_k(X))\le 1, & k=1,2,\cdots,K_0,\\
& x_j\ge 0 & \text{for } j=1,2,\cdots,n,
\end{array} \tag{26}$$

where w_k is the relative weight of the k-th fuzzy goal.

Similarly, for the fuzzy goal type fuzzy-min, the equivalent linear model of PFGP can be written as:

$$\begin{array}{lll}
\text{Max} & D(\mu)=\sum\limits_{k=K_0+1}^{K_1}w_k\mu_k(f_k(X)) & \\
\text{s.t.} & \mu_k(f_k(X))=\dfrac{U_k-f_k(X)}{U_k-g_k}, & k=K_0+1,2,\cdots,K_1,
\end{array}$$

$$\begin{aligned}
&\sum_{j=1}^{n} a_{ij}x_j \le \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i = 1, 2, \cdots, m, \\
&\mu_k(f_k(X)) \le 1, && k = K_0 + 1, 2, \cdots, K_1, \\
&x_j \ge 0 && \text{for } j = 1, 2, \cdots, n,
\end{aligned} \tag{27}$$

where w_k is the relative weight of the k-th fuzzy goal.

Again, for the fuzzy-equal type of fuzzy goal, the equivalent linear model of PFGP can be formulated as:

$$\begin{aligned}
\text{Max} \quad & D(\mu) = \sum_{k=K_1+1}^{K_2} w_k\mu_k(f_k(X)) \\
\text{s.t.} \quad & \mu_k(f_k(X)) = \frac{f_k(X) - L_k}{g_k - L_k}, && k = K_1 + 1, 2, 3, \cdots, K_2, \\
& L_k \le f_k(X) \le g_k, && k = K_1 + 1, 2, 3, \cdots, K_2, \\
& \mu_k(f_k(X)) = \frac{U_k - f_k(X)}{U_k - g_k}, && k = K_1 + 1, 2, 3, \cdots, K_2, \\
& g_k \le f_k(X) \le U_k, && k = K_1 + 1, 2, 3, \cdots, K_2, \\
& \sum_{j=1}^{n} a_{ij}x_j \le \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i = 1, 2, \cdots, m, \\
& \mu_k(f_k(X)) \le 1, && k = K_1 + 1, 2, 3, \cdots, K_2, \\
& x_j \ge 0 && \text{for } j = 1, 2, \cdots, n,
\end{aligned} \tag{28}$$

where w_k is the relative weight of the k-th fuzzy goal.

The above obtained deterministic models can be solved by any linear programming techniques or can be solved by LINGO 11.0 [26] optimization tool.

DMs are facing many problems to assign weights according to the relative importance of the goals exactly. The phrase 'relative importance' is a fuzzy concept whose various levels can be stated only imprecisely. However, in the literature there are some good approaches to assess these weights, such as eigenvector method of Saaty [23], and the weighted least squares method of Chu et al. [6]. These can be used to suitably specify the weights. In this paper, eigenvector method of Satty is used to calculate the weight for a membership function of the fuzzy goals.

3.3. Preemptive Priority Additive Approach to PFGP Problem with Pareto Distribution

In many decision making problems, the goals are not commensurable (i.e., not in the same measurable unit). Further, sometimes the goals are such that unless a particular goal or a subset of goals is achieved, the other goals should not be considered. In such situations, the weighting scheme of the previous section is not an appropriate method. The preemptive priority structure may be written as $P_r >>> P_{r+1}$ which means that the goals in the r-th priority level have higher priority than the goals in the $(r + l)$-th priority level, i.e., however large N (a number) may be, P_r cannot be equal to NP_{r+1} [25]. For the present investigation the problem is subdivided into R subproblems, where R is the number of priority levels. In the first subproblem, the fuzzy goals belonging to the first priority level have only been considered and solved by using a simple additive model as described in Subsection 3.1. But at other priority levels, the membership values achieved earlier for higher priority levels are imposed as additional constraints. In general, the r-th subproblem for fuzzy goal type fuzzy-max can be written as:

$$\begin{aligned}
\text{Max} \quad & D(\mu) = \sum_{k=1}^{K_0} P_r\mu_k(f_k(X)), && r = 1,2,\cdots,R \\
\text{s.t.} \quad & \mu_k(f_k(X)) = \frac{U_k - f_k(X)}{U_k - g_k}, && k = 1,2,\cdots,K_0, \\
& \sum_{j=1}^{n} a_{ij}x_j \leq \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i = 1,2,\cdots,m, \\
& P_r\mu_k(f_k(X)) = P_r\mu_k^*(f_k(X)), && k = 1,2,\cdots,K_0, r = 1,2,\cdots,R-1, \\
& \mu_k(f_k(X)) \leq 1, && k = 1,2,\cdots,K_0, \\
& x_j \geq 0 && \text{for } j = 1,2,\cdots,n,
\end{aligned} \tag{29}$$

where $P_r\mu_k(f_k(X))$ represents the membership functions of the goals in the r-th priority level, and $P_r\mu_k^*(f_k(X))$ is the achieved membership value in the r-th priority level.

Similarly, the r-th subproblems for fuzzy goal types fuzzy-min and fuzzy-equal can be written as:

$$\begin{aligned}
\text{Max} \quad & D(\mu) = \sum_{k=K_0+1}^{K_1} P_r\mu_k(f_k(X)), && r = 1,2,\cdots,R \\
\text{s.t.} \quad & \mu_k(f_k(X)) = \frac{f_k(X) - L_k}{g_k - L_k}, && k = K_0+1,2,\cdots,K_1, \\
& \sum_{j=1}^{n} a_{ij}x_j \leq \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i = 1,2,\cdots,m, \\
& P_r\mu_k(f_k(X)) = P_r\mu_k^*(f_k(X)), && k = K_0+1,2,\cdots,K_1, r = 1,2,\cdots,R-1, \\
& \mu_k(f_k(X)) \leq 1, && k = K_0+1,2,\cdots,K_1, \\
& x_j \geq 0 && \text{for } j = 1,2,\cdots,n
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
\text{Max} \quad & D(\mu) = \sum_{k=K_1+1}^{K_2} P_r\mu_k(f_k(X)), && r = 1,2,\cdots,R \\
\text{s.t.} \quad & \mu_k(f_k(X)) = \frac{f_k(X) - L_k}{g_k - L_k}, && k = K_1+1,2,3,\cdots,K_2, \\
& L_k \leq f_k(X) \leq g_k, && k = K_1+1,2,3,\cdots,K_2, \\
& \mu_k(f_k(X)) = \frac{U_k - f_k(X)}{U_k - g_k}, && k = K_1+1,2,3,\cdots,K_2, \\
& g_k \leq f_k(X) \leq U_k, && k = K_1+1,2,3,\cdots,K_2, \\
& \sum_{j=1}^{n} a_{ij}x_j \leq \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}}, && i = 1,2,\cdots,m, \\
& P_r\mu_k(f_k(X)) = P_r\mu_k^*(f_k(X)), && k = K_1+1,2,\cdots,K_2, r = 1,2,\cdots,R-1, \\
& \mu_k(f_k(X)) \leq 1, && k = K_1+1,2,3,\cdots,K_2, \\
& x_j \geq 0 && \text{for } j = 1,2,\cdots,n.
\end{aligned} \tag{31}$$

4. Numerical Example

A manufacturing company produces three different products namely, Product 1, Product 2 and Product 3. Management of the company divides the remaining capacity to three products. Table 1 gives the capacity of machines that restricts the amounts of output and assumes that the total required weekly machine hours considered as Pareto random variable for the production of a product. According to the information taken from the sales department, sale potential of Product 1 and Product 2 are bigger than the production amount and the sale potential for Product 3 is approximately greater than or equal to 40 units per week. The unit profit of Product 1 is 30 USD, the unit profit of Product 2 is 12 USD and the unit profit of Product 3 is 11 USD. The company wants to earn approximately greater than or equal to 1800 USD profit per week.

Table 1: Machine hours required for production of products and the capacity of machines.

	Required unit time (per hour)			
Machine type	Product 1	Product 2	Product 3	Weekly machine time (hour) approximately
Rubbing	9	3	5	570
Polishing	5	4	0	360
Finishing	3	0	2	150

By using the above information in Table 1, determine yearly production plan and the profit amount of the company. Suppose that the amount of Product 1 will be produced by x_1 variable, the amount of Product 2 by x_2 and Product 3 by x_3. So the problem can be formulated as multi-objective probabilistic linearly constraint fuzzy goal programming model as follows:

Find $X = (x_1, x_2, x_3)$ to optimize the following fuzzy goals:

$$\begin{aligned} & f_1(X) = 30x_1 + 12x_2 + 11x_3 \gtrsim 1800 \\ & f_2(X) = x_3 \gtrsim 40 \\ \text{s.t.}\quad & \Pr(9x_1 + 3x_2 + 5x_3 \leq b_1) \geq 1 - \gamma_1, \\ & \Pr(5x_1 + 4x_2 \leq b_2) \geq 1 - \gamma_2, \\ & \Pr(3x_1 + 2x_3 \leq b_3) \geq 1 - \gamma_3, \\ & x_j \geq 0 \text{ for } j = 1, 2, 3, \end{aligned} \tag{32}$$

where $\gamma_1 = 0.05, \gamma_2 = 0.08, \gamma_3 = 0.1$ are the specified probability levels decided by company management. Also, company management assume that $a_{ij}, i = 1, 2, 3, j = 1, 2, 3$ is deterministic in the model and right-hand side variables $b_i, i = 1, 2, 3$ are Pareto random variables with known means $E(b_1) = 600$, $E(b_2) = 400$, $E(b_3) = 200$ respectively and variances $Var(b_1) = 1000$, $Var(b_2) = 2000$, $Var^2(b_3) = 5000$ respectively. The parameters are calculated using Eqs. (6) and (7) as follows:

shape parameters $p_1 = 20, p_2 = 10, p_3 = 4$
and
scale parameters $q_1 = 570, q_2 = 360, q_3 = 150$.
Suppose that tolerance amounts for profit and sale goals are determined by manager of the company as respectively 200 USD and 20 unit Product 3, i.e., only lower tolerance amounts for profit and sale goals are determined by manager of the company as respectively 1600 USD and 20 units of Product 3. Under these circumstances, fuzzy-max type of linear membership functions $\mu_1(f_1(X))$ and $\mu_2(f_2(X))$ are defined for profit goal and sale goal respectively according to Zimmermann [33] as follows:

$$\mu_1(f_1(X)) = \begin{cases} 1, & \text{if } 30x_1 + 12x_2 + 11x_3 \geq 1800, \\ \dfrac{30x_1 + 12x_2 + 11x_3 - 1600}{200}, & \text{if } 1600 \leq 30x_1 + 12x_2 + 11x_3 \leq 1800, \\ 0, & \text{if } 30x_1 + 12x_2 + 11x_3 \leq 1600 \end{cases} \tag{33}$$

and

$$\mu_2(f_2(X)) = \begin{cases} 1, & \text{if } x_3 \geq 40, \\ \dfrac{x_3 - 20}{20}, & \text{if } 20 \leq x_3 \leq 40, \\ 0, & \text{if } x_3 \leq 20. \end{cases} \tag{34}$$

Using simple additive approach along with Eqs. (12), (33), and (34), the deterministic equivalent linear mathematical model of PFGP can be formulated as:

$$\begin{aligned} \text{Max} \quad & D(\mu) = \mu_1(f_1(X)) + \mu_2(f_2(X)) \\ \text{s.t.} \quad & \mu_1(f_1(X)) = \frac{30x_1 + 12x_2 + 11x_3 - 1600}{200}, \\ & \mu_2(f_2(X)) = \frac{x_3 - 20}{20}, \\ & 9x_1 + 3x_2 + 5x_3 \leq 575.75, \\ & 5x_1 + 4x_2 \leq 363.63, \\ & 3x_1 + 2x_3 \leq 154.63, \\ & \mu_1(f_1(X)) \leq 1, \\ & \mu_2(f_2(X)) \leq 1, \\ & \mu_1(f_1(X)) \geq 0, \mu_2(f_2(X)) \geq 0, x_j \geq 0 \text{ for } j = 1, 2, 3. \end{aligned} \tag{35}$$

The above deterministic linear model can be solved by using LINGO 11.0 software tool and an obtained optimal solution are as follows:

$$x_1 = 23.83,\ x_2 = 53.75,\ x_3 = 40.00$$

with achieved goal values as:

$$g_1 = 1799.9,\ g_2 = 40$$

and the membership values are as:

$$\mu_1(f_1(X)) = 1.00, \mu_2(f_2(X)) = 1.00.$$

Similarly, suppose the relative weights of the two objectives, given by the company management, are $w_1 = 0.8, w_2 = 0.2$. Then using weighted additive approach along with Eqs.(12), (33), and (34), the deterministic equivalent linear mathematical model of PFGP can be obtained as:

$$\begin{aligned}
\text{Max}\quad & D(\mu) = 0.8\mu_1(f_1(X)) + 0.2\mu_2(f_2(X)) \\
\text{s.t.}\quad & \mu_1(f_1(X)) = \frac{30x_1 + 12x_2 + 11x_3 - 1600}{200}, \\
& \mu_2(f_2(X)) = \frac{x_3 - 20}{20}, \\
& 9x_1 + 3x_2 + 5x_3 \le 575.75, \\
& 5x_1 + 4x_2 \le 363.63, \\
& 3x_1 + 2x_3 \le 154.63, \\
& \mu_1(f_1(X)) \le 1, \\
& \mu_2(f_2(X)) \le 1, \\
& \mu_1(f_1(X)) \ge 0,\ \mu_2(f_2(X)) \ge 0, x_j \ge 0 \text{ for } j = 1, 2, 3.
\end{aligned} \tag{36}$$

The above deterministic linear model can be solved by using LINGO 11.0 software tool and obtained optimal solution are as follows:

$$x_1 = 23.83,\ x_2 = 53.75,\ x_3 = 40.00$$

with achieved goal values as:

$$g_1 = 1799.9,\ g_2 = 40.00$$

and the membership values are as:

$$\mu_1(f_1(X)) = 1.00,\ \mu_2(f_2(X)) = 1.00.$$

Further, in case of preemptive priority company management has consider two priority levels:

Priority 1: profit goal;

Priority 2: sale goal.

For this case, the main problem is subdivided into two subproblems which are formulated as:

First subproblem:

$$\begin{aligned}
\text{Max}\quad & D(\mu) = \mu_1(f_1(X)) \\
\text{s.t.}\quad & \mu_1(f_1(X)) = \frac{30x_1 + 12x_2 + 11x_3 - 1600}{200}, \\
& 9x_1 + 3x_2 + 5x_3 \le 575.75, \\
& 5x_1 + 4x_2 \le 363.63, \\
& 3x_1 + 2x_3 \le 154.63, \\
& \mu_1(f_1(X)) \le 1, \\
& \mu_1(f_1(X)) \ge 0, x_j \ge 0 \text{ for } j = 1, 2, 3.
\end{aligned} \tag{37}$$

The first subproblem is solved by using LINGO 11.0 and the optimal solution is obtained as:

$$x_1 = 51.54,\ x_2 = 21.142,\ x_3 = 0.00$$

with achieved goal values as:

$$g_1 = 1799.99,\ g_2 = 0.00$$

and the membership values are as:

$$\mu_1(f_1(X)) = 1.00.$$

Second or final subproblem:

$$\begin{aligned}
\text{Max} \quad & D(\mu) = \mu_2(f_2(X)) \\
\text{s.t.} \quad & \mu_1(f_1(X)) = \frac{30x_1 + 12x_2 + 11x_3 - 1600}{200}, \\
& \mu_2(f_2(X)) = \frac{x_3 - 20}{20}, \\
& 9x_1 + 3x_2 + 5x_3 \leq 575.75, \\
& 5x_1 + 4x_2 \leq 363.63, \\
& 3x_1 + 2x_3 \leq 154.63, \\
& \mu_1(f_1(X)) = 1, \\
& \mu_2(f_2(X)) \leq 1, \\
& \mu_1(f_1(X)) \geq 0, \mu_2(f_2(X)) \geq 0, x_j \geq 0 \text{ for } j = 1, 2, 3.
\end{aligned} \tag{38}$$

The second subproblem is solved by using LINGO 11.0 and the optimal solution is obtained as:

$$x_1 = 23.83,\ x_2 = 53.75,\ x_3 = 40.00$$

with achieved goal values as:

$$g_1 = 1799.9,\ g_2 = 40.00$$

and the membership values are as:

$$\mu_1(f_1(X)) = 1.00,\ \mu_2(f_2(X)) = 1.00.$$

5. Conclusion

In this paper, some additive approaches for solving PFGP problems are presented where the right-hand side parameter follows Pareto distribution. The crisp deterministic models are established for PFGP problems and solved by using suitable mathematical programming techniques. Chance-constraint program is used to handled the randomness in the problem formulation and fuzziness is handled by using linear membership functions due to Zimmermann [33]. Hannan [9] assigns aspiration values for the membership functions of the fuzzy goals (which restricts the membership function from full achievement, i.e., unity) and uses the additive property to aggregate the membership functions to be minimized. It is difficult to assign the aspiration values for the membership functions when the corresponding goals themselves are fuzzy. The differential weights (cardinal weights) for differing fuzzy goals within the same priority level can also be used as in conventional GP. In Subsection 3.3, while employing preemptive priority in PFGP, the achievement values (membership values) of the higher priority goals are preserved when considering the lower priority levels. This contradicts to preserving the achieved goal values (aspiration values) as in conventional GP. If the coefficients $a_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$ are Pareto random variables, then the deterministic models turn out to be complicated. In such cases, heuristic method like genetic algorithm may be used to solve these problems directly without finding their deterministic models.

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