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A dark radiation era prior to the inflationary phase

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ABSTRACT

A cosmological model dominated at the beginning by a dark radiation followed by a period of inflation is presented. This model is based on a Randall–Sundrum II type brane-world. Current observational data are used to fix the parameters associated to the dark radiation.

1. Introduction

One interesting question to ask is: What was the previous period to the inflationary phase? or similarly, What was the initial condition that make inflation to occur? The obscureness of the physical mechanism underlying inflation results quite frustrating. Nowadays, the beginning and end of the inflationary period result, by themselves, theoretical challenges by themselves. In this respect, some researchers have tried to give answer to the above question. For instance, there have been attempts to find an initial state for the inflationary universe models from the quantum cosmology. Here, the so-called wave function of the universe [1] has been advised for doing this task. Furthermore, there have been attempts to address this question through the introduction of the "emerging universe". An emergent universe is a model universe in which there is no timelike singularity, having almost static behavior in the infinite past ($t \to -\infty$) [2]. It is assumed that the initial conditions are specified so as the static configuration represents the past eternal state of the universe, out of which the universe slowly evolves into an inflationary phase. Different schemes have been used for describing this sort of models. In Ref. [3] a brane world scenario was considered. Moreover, a Jordan–Brans–Dicke sort of theory has been also considered [4]. On the other hand, the pre-big-bang cosmology inspired by the superstring theories has been suggested as a possible implementation of the inflationary universe scenario [5]. However, the scenario validity as a viable inflationary model has been questioned on the grounds of its initial conditions [6]. We have not firm conclusions about this point at the moment.

On the other hand, the cyclic/oscillatory universes have been present in cosmology for a long time [7]. Here, the main attraction was that the initial conditions could in principle be avoided. However, these models show severe construction difficulties within the general relativity context. First of all, the number of bounces in the past are restricted to entropy constraints; nevertheless, the main difficulty is perhaps that any bounce would be singular, thus resulting in the breakdown of the general relativity. Recent developments on brane world scenarios have renewed interest on the cyclic/oscillatory universes [8]. The cyclic/oscillatory universe that ultimately undergoes inflationary expansion after a finite number of cycles has also been investigated [9]. However, a physical mechanism is missing for inducing the corresponding number of bounces. Some authors have considered the loop quantum cosmology in order to solve this problem [10]. Nevertheless, the oscillations in such models can be characterized by reformulating the semi-classical dynamics in terms of an effective phantom fluid.

The idea that our universe is confined to a brane in a higher dimensional bulk spacetime has been constantly under study in the last years [11]. One of the most studied model is the so-called Randall–Sundrum [12] (usually cited as RSII model), in which our universe lived in a brane embedded in an Anti-de Sitter (AdS) five-dimensional bulk spacetime. Soon after its appearance, a lot of work was made to built cosmological extensions of the RSII model [13]. One of the most relevant consequences of these studies was the modification of the Friedmann equation for energy densities of the order of the brane tension, and also the appearance of an additional term, usually called dark radiation, in addition [14]. This latter term, i.e. the dark radiation, results crucial, as its presence can spoil the big bang nucleosynthesis [15] or even modify the overall amplitude of the fluctuations spectrum associated to large

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scale structure formation processes. Thus, it seems natural to look for realizations of inflation where the dark radiation plays a role.

In this Letter, a model of the early universe is proposed, where the dark radiation is taken into account. In this context, the effects that the dark radiation might have previously to the inflationary period would be studied. Actually, this transition will be possible for acceptable values of the parameters that appear in the model.

The present Letter is organized as follows. The basic field equations are formulated in Section 2. Section 3 involves the dark radiation dominated era. The inflationary phase is described in Section 4, where some astronomical data are used in order to fix some parameters. Finally, the results of the Letter are summarized in Section 5.

2. The basic equations

The field equations are recasts in the following convenient form through a flat Friedmann–Robertson–Walker metric (FRW), in which the brane-bulk energy exchange is included (for detail see Ref. [16])

$$\dot{\rho}+3H(\rho+p)=-T(\rho), \tag{1}$$

$$\dot{\chi}+4H\chi=\left(\frac{\rho}{\sigma}+1\right)T(\rho), \tag{2}$$

and

$$H^2=2\gamma\rho\left(1+\frac{\rho}{2\sigma}\right)+2\gamma\chi+\Lambda^{(4)}. \tag{3}$$

Here, the constants σ, $\Lambda^{(4)}$ and $\gamma=4\pi G=4\pi/m_p^2$ are related to the brane tension, the effective cosmological constant on the brane and the Newton constant or equivalently, the Planck mass in 4-dimensions, respectively. On the other hand, $T(\rho)=2T_4^0$ is the discontinuity of the 04 component of the bulk energy momentum tensor and the matter is represented by a perfect fluid which is characterized by a pressure p and energy density ρ, which are related by a specific equation of state. The energy density χ represents the dark or the mirage radiation [17], and $H=\frac{\dot{a}}{a}$ the Hubble factor, with $a=a(t)$ the scale factor. Dots here represent derivative with respect to the cosmological time t.

It is interesting to note that if we assume some equation of state parameter ω, defined as $\omega\equiv\frac{p}{\rho}$ and vanishing cosmological constant, $\Lambda^{(4)}=0$, from the set of Eqs. (1)–(3), we get that

$$\rho^2-\left(\frac{1-3\omega}{1+3\omega}\right)\sigma\rho=-\left(\frac{1}{1+3\omega}\right)\frac{\sigma}{\gamma}\left(\dot{H}+2H^2\right), \tag{4}$$

which tells us how the matter energy density, ρ, cosmologically evolves for an arbitrary bulk energy momentum tensor, $T(\rho)$, i.e. we could get ρ as an explicit function of the cosmological time, t, after knowing the scale factor, $a(t)$. Here, we have assumed that ω is different to $-1/3$.

We could give an intuitive picture about our approach if we combine Eqs. (1) and (2) in such a way that we write down a single equation

$$\left[\dot{\rho}+3H(\rho+p)\right]\left(1+\frac{\rho}{\sigma}\right)=-(\dot{\chi}+4H\chi). \tag{5}$$

In this respect, our fundamental set of equations becomes Eqs. (3) and (5). We see that this set is equivalent to the set of Eqs. (8) and (9) of Ref. [18], if we take the curvature parameter $k=0$, and we identify χ with $\mathcal{M}$ (the "generalized comoving mass") via $\chi=\frac{12M^2}{\pi^2V}\frac{\mathcal{M}}{a^4}$ and $p_D=\frac{1}{3}\chi$ (see Ref. [18] for the meaning of the scripts that appear in these expressions). In this picture $\mathcal{M}$ gives information about the bulk-effect corrections on the brane, when the bulk spacetime contains an arbitrary matter configuration [19].

In this sense, following Ref. [18] our case would correspond to a configuration in which $p_D=\frac{1}{3}\chi$, thus representing a AdS-Vaidya bulk [20,21] with energy exchange between the brane and a radiation field in the bulk in the high-energy regime.[1]

Let us write down the field equations (1) and (2) in terms of the scale factor derivatives through the relation $dt=da/Ha$, represented by primes, so that these Equations take the form

$$\rho'+\frac{3}{a}(1+\omega)\rho=-\frac{T(\rho)}{Ha}, \tag{6}$$

and

$$\chi'+\frac{4}{a}\chi=\left(\frac{\rho}{\sigma}+1\right)\frac{T(\rho)}{Ha}, \tag{7}$$

respectively. These two latter equations along with the Friedmann equation (3), form the basic equations that we would like to take into account to describe a facet in which the beginning of the universe is dominated by the dark radiation energy. This situation will be processed in the following section.

3. General solutions and dark radiation dominated era

In order to find a solution to the previous set of field equations, let us begin supposing the following ansatz for the energy density, ρ, as a function of the scale factor a,

$$\rho(a)=\frac{B}{a^{3(1+\omega)}}+\frac{A}{a^{\alpha}}, \tag{8}$$

where A and B are two arbitrary constants. Note that the first term corresponds to the solution of the homogeneous Eq. (6), i.e. when $T(\rho)=0$. Therefore, the second term could be taken as a solution of the inhomogeneous part of this equation, with α an arbitrary constant.

Substituting Eq. (8) into (6) we find for the interaction term

$$T=\frac{HA}{a^{\alpha}}\left[\alpha-3(1+\omega)\right], \tag{9}$$

with $\alpha\neq 3(1+\omega)$.

Using Eqs. (8) and (9) into Eq. (7), we find for the dark radiation density, χ, the following solution

$$\chi(a)=\frac{\tilde{A}}{a^{2\alpha}}+\frac{\tilde{B}}{a^{\mu-1}}+\frac{\tilde{C}}{a^{\alpha}}+\frac{D}{a^4}, \tag{10}$$

where

$$\tilde{A}=\frac{A^2}{\sigma(4-2\alpha)}\left[\alpha-3(1+\omega)\right],\quad \alpha\neq 2, \tag{11}$$

$$\tilde{B}=\frac{AB}{\sigma(5-\mu)}\left[\alpha-3(1+\omega)\right],\quad \mu\neq 5, \tag{12}$$

$$\tilde{C}=\frac{A}{4-\alpha}\left[\alpha-3(1+\omega)\right],\quad \alpha\neq 4, \tag{13}$$

and D is an integration constant. Here, $\mu-1=3(1+\omega)+\alpha$. Note that if $T(\rho)=0$, i.e. $\alpha=3(1+\omega)$, then only the last term of Eq. (10) survives, and thus $\chi\sim a^{-4}$, which corresponds to the solution of the homogeneous equation for the dark radiation. In the following analysis, we will assume that $\alpha>3(1+\omega)$. As we will see in the next section, this assumption will simplify our analysis.

Regarding these solutions, i.e. Eqs. (8) and (10), let us assume that the universe is initially in a high energy state in which the dark radiation component dominating the universe, $\chi\sim a^{-4}\gg a^{-2\alpha}\sim\rho^2$ where $\alpha<2$. Note that the energy exchange term T given by Eq. (9) is related with the spatial discontinuity of the

[1] The case of low-energy regime, i.e., $\rho<\sigma$, was studied in Ref. [21].

bulk metric $a(t,\eta)$ (here η represents the bulk coordinate) near the brane and must be positive in order the embedding of the 3-brane could be possible (see Refs. [18,20]). Therefore, a positive T gives $\alpha > 3(1+\omega)$, and together with the dominant dark radiation period, $\alpha < 2$, from which we get that $\omega < -1/3$, we could get an inflationary period. If, on the other hand, $\alpha < 3(1+\omega)$ and $\alpha > 2$ (which yields to $\omega > -1/3$) the universe is consistent only with a negative T, this energy transfer from the brane to the bulk cannot lead to periods of accelerated expansion on the brane [22]. Here $T \sim a^{-(2+\alpha)}$, so that

$$\chi \gg \sqrt{\gamma}\, T \gg \frac{\rho^2}{2\sigma} \gg \rho. \tag{14}$$

At this stage, the last term in Eq. (10) dominates, and thus, the Friedmann equation can be written as followed

$$H^2 \simeq 2\gamma\chi, \tag{15}$$

and the evolution of the dark radiation density is governed by the equation

$$\dot{\chi} + 4H\chi \simeq \frac{\rho}{\sigma} T(\rho). \tag{16}$$

An attractor solution for this configuration is that of a power-law scale factor $a(t) \sim t^n$, with $n < 1$. In this regime $\rho \simeq t^{-1}$ and $\chi \simeq t^{-2}$. The interaction term behaves as $T(t) \simeq t^{-2}$ which gives $T(\rho) \sim \rho^2$. This period is prolonged until a specific time, say t_e, that corresponds to when it is satisfied $\chi(t_e) \approx \sqrt{\gamma}\, T(t_e) \approx \frac{\rho^2(t_e)}{2\sigma} \sim a_e^{-2\alpha}$, where $a_e = a(t_e)$. Thus we have a dark radiation dominated period, which starts at some initial time, t_i, and finishes at the time t_e, i.e. we have $t_i \leqslant t \leqslant t_e$ for the dark radiation period.

We will restrict ourselves to a phenomenological expressions for the bulk-energy momentum tensor, T, previously considered in the literature from now on. One common choice for this is $T = 3\xi H\rho$, with ξ a positive-definite dimensionless constant, which is taken to be $\xi \ll 1$. This choice has been studied in detail in Ref. [23]. Then, the field equations become the dark radiation dominated era, i.e. for $\chi \gg \frac{\rho^2}{2\sigma}$

$$H^2 \simeq 2\gamma\chi, \tag{17}$$

$$\dot{\chi} + 4H\chi \simeq 0, \tag{18}$$

and

$$\dot{\rho} + 3H\rho(1+\omega) = -3\xi H\rho. \tag{19}$$

The apparent asymmetry between (18) and (19) needs a clarification. Assuming the dark radiation domination occurs in a high energy regime implies that $\chi \gg \rho(1+\rho/2\sigma)$, as can be see comparing Eq. (17) with Eq. (3). Using the interaction as $T = 3\xi H\rho$, the same factor appears at the right hand of Eq. (18), being negligible small compared to the homogeneous term $4H\chi$ in the same equation. So, the system (17)–(19) is a consistently high energy limit of the original system of equations, with an interaction term different from zero; i.e. $T(\rho) \neq 0$.

The corresponding solutions of these equations are given by

$$\rho(a) = \tilde{\rho}\left(\frac{\tilde{a}}{a}\right)^{3(\xi+\omega+1)}, \tag{20}$$

where tilde specifies some time well inside of the dark radiation period, i.e. $\tilde{t} \ll t_e$, and $\chi(a) \approx \frac{D}{a^4}$, with $a(t) \sim t^{1/2}$. Having described the period once was dominated by radiation, we will proceed to consider a posterior period of inflation. This will allow fixing some parameters that define our model in the radiation-dominated period, using astronomic data. We should stress here that the aim of the present Letter is to put some constraint on the dark radiation term ($\chi \sim D/a^4$) previous to the inflationary period (ρ^2) independently if the bulk-energy momentum tensor T could be vanished or not. We assume here that T is negligible in the two epochs, dark radiation and inflationary periods.

4. Inflationary phase

Following an approach similar to that described in Ref. [24], the proper transition from a dark radiation dominated regime to an accelerated one is through the solution[2]

$$a(t) = a_{in}\left[\sinh\left(C_2 \ln(1 + t/t_{in})\right)\right]^{1/n}, \tag{21}$$

which *interpolates* between the stage $t/t_{in} \ll 1$, where the scale factor follows the law

$$a(t) \simeq a_{in}(C_2)^{1/n}(t/t_{in})^{1/n}, \tag{22}$$

and the power-law inflationary phase, in which $t/t_{in} \gg 1$, and the scale factor becomes

$$a(t) \simeq a_{in}(1 + t/t_{in})^{C_2/n} \simeq a_{in}(t/t_{in})^{C_2/n} = a_{in}(t/t_{in})^p. \tag{23}$$

Here t_{in}, indicates the cosmological times at the beginning of inflation. It is clear that, in order to have a dark-radiation dominated stage, we need to take $n = 2$. On the other hand, note that we have chosen to work with a power-law inflation. We could relax this assumption, but the main consequences will not change. Then, imposing $\ddot{a} > 0$ leads to the constraint $p = C_2/2 > 1$.

We assume that, either in the dark-radiation era as in the inflationary era, the bulk-brane exchange energy term, T, becomes low enough, so that, it does not affect the evolution of the universe. For instance, in the dark radiation period, we have that $H \sim t^{-1} \sim a^{-2}$, and thus, the interaction term falls as $T \sim a^{-(2+\alpha)}$ (see Eq. (9)). Comparing this term with that corresponding to energy density, $a^{-3(1+\omega)}$, which starts to dominate when inflation comes into play, and since we have taken $\alpha > 3(1+\omega)$, then, we take the interaction term, T, to be negligible in this period. Note that this situation agrees with that in which the last term in Eq. (8) is associated to the solution of the inhomogeneous part of the energy density field equation (6). Thus, with these assumptions in mind, the onset of high energy inflationary scenarios will be discussed in detail.

During the dark radiation dominated phase, $H^2 \simeq 2\gamma\chi$, with χ dominated by the last term of Eq. (10), and using Eq. (22), we get

$$D \simeq \frac{a_{in}^4 p^2}{2\gamma t_{in}^2}. \tag{24}$$

Inflation starts at the high energy in which $H^2 \simeq \beta\rho^2$, where we have introduced the constant $\beta \equiv \frac{\gamma}{\sigma}$. From this expression we get that

$$\dot{H} \simeq \beta^{1/2}\dot{\rho}. \tag{25}$$

On the other hand, with the energy density given by the homogeneous term of Eq. (8), $\rho \sim \frac{B}{a^{3(1+\omega)}}$, we have

$$a_{in} \simeq \left[3(1+\omega)\sqrt{\beta} B t_{in}\right]^{1/3(1+\omega)}. \tag{26}$$

In the following, an homogenous single scalar field ϕ confined to the brane will be considered, whose energy density can lead to the accelerated expansion of the universe (inflation). Neglecting spatial gradients, the energy density and the pressure of the inflaton field are given by

[2] Here, at difference of Ref. [24], the solution (20) is not a solution of the complete gravity-dark radiation-inflation system, rather it is a simple interpolation between the dark radiation and inflationary periods.

$$\rho = \frac{\dot\phi^2}{2} + V(\phi), \qquad P = \frac{\dot\phi^2}{2} - V(\phi), \tag{27}$$

where $V(\phi)$ is the potential energy of the inflaton. The evolution equation of the inflaton field, ϕ, is obtained inserting Eqs. (27) into Eq. (1), neglecting the interaction term.

From Eqs. (1), (25) and (27) we obtain

$$\dot\phi^2 = -\frac{\dot H}{3H\beta^{1/2}} = \frac{1}{3\beta^{1/2}t}. \tag{28}$$

Using this latter expression together with Eq. (27), we obtain that the exact potential energy of the scalar field ϕ for power-law inflation becomes

$$V(\phi) = \frac{1}{\beta^{1/2}}\left[H + \frac{\dot H}{6H}\right] = \frac{2(6p-1)}{9\beta}\frac{1}{(\phi-\phi_{in})^2}, \tag{29}$$

where ϕ_{in}, indicates the value of the scalar field at the beginning of inflation. Of course, the form of this potential has to be considered just as an approximation of a more complex potential for the interval $\phi_{in} \leqslant \phi \leqslant \phi_f$.

In the following, the subscripts $*$ and f are used to denote to the epoch when the cosmological scales exit the horizon (as previously specified) and the end of inflation, respectively. The scalar perturbations for our model will be studied. We introduce comoving curvature perturbations, $\mathcal{R} = \psi + H\delta\phi/\dot\phi$, where $\delta\phi$ is the perturbation of the inflaton field, ϕ. The power spectrum associated to curvature perturbations is given by [25]

$$\mathcal{P}_R \simeq \frac{H^2}{\dot\phi^2}\left(\frac{H}{2\pi}\right)^2\bigg|_{k=k_*} \simeq \frac{3\beta^{1/2}p^4}{4\pi^2 t_*^3}, \tag{30}$$

where we have used Eq. (28). Here, k_* is referred to $k = Ha$, the value when the universe scale crosses the Hubble horizon during inflation. The recent WMAP five-year results [26] give the value for the scalar curvature spectrum $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$ with $k_* = 0.002\ \text{Mpc}^{-1}$.

On the other hand, the generation of tensor perturbations during inflation would produce gravitational waves and these perturbations in cosmology are more involved, since gravitons propagate in the bulk. The amplitude for tensor perturbations was evaluated in Ref. [27]

$$\mathcal{P}_g = 48\gamma\left(\frac{H}{2\pi}\right)^2 F^2(x) \simeq \frac{12\gamma}{\pi^2}\frac{p^2}{t^2}F^2(x), \tag{31}$$

where $x = Hm_p\sqrt{3/(4\pi\lambda)}$ and

$$F(x) = \left[\sqrt{1+x^2} - x^2\sinh^{-1}(1/x)\right]^{-1/2}.$$

Here, the function $F(x)$ appears from the normalization of a zero-mode. The spectral index n_g is given by $n_g = \frac{d\mathcal{P}_g}{d\ln k} = -\frac{2x_{,\phi}}{N_{,\phi}x}\frac{F^2}{\sqrt{1+x^2}}$.

From expressions (30) and (31) we may write the tensor-scalar ratio as

$$R = \left(\frac{\mathcal{P}_g}{P_{\mathcal{R}}}\right)\bigg|_{k=k_*} \simeq \frac{16\gamma t_*}{\beta^{1/2}p^2}F^2(t_*). \tag{32}$$

Combining WMAP five-year data [28] with the Sloan Digital Sky Survey (SDSS) large scale structure surveys [29], an upper bound for R is found given by $R(k_* \simeq 0.002\ \text{Mpc}^{-1}) < 0.28$ $(95\%CL)$, where $k_* \simeq 0.002\ \text{Mpc}^{-1}$ corresponds to $l = \tau_0 k \simeq 30$, with the distance to the decoupling surface $\tau_0 = 14{,}400$ Mpc. The SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes k in the range $0.016h\ \text{Mpc}^{-1} < k < 0.011h\ \text{Mpc}^{-1}$. The recent WMAP five-year results give the values for the scalar curvature spectrum $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$ and the scalar-tensor ratio $R(k_*) = 0.055$. These values we will be used to set constrains on the parameters of our model.

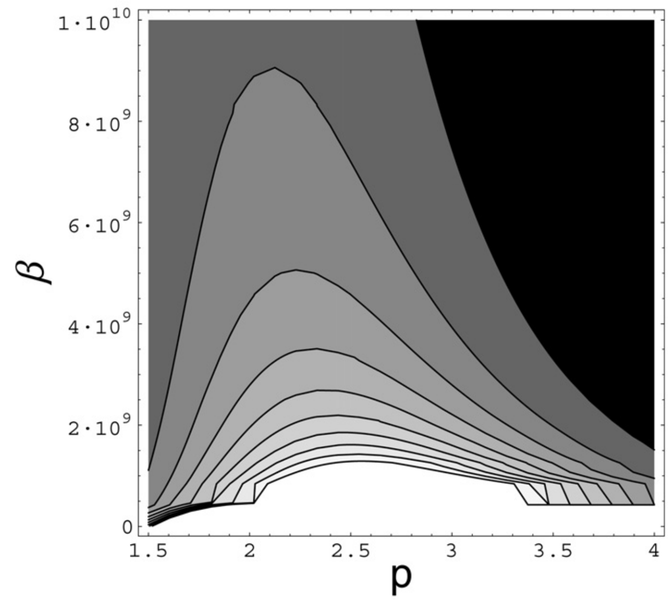

Fig. 1. Contour plot for the dimensionless values of the D/k_*^4 as a function of the parameters β and p, fitted from the cosmological times $t_{in} = 10^5/m_p$.

During inflation, from Eq. (8) we know that $B \simeq \rho_* a_*^{3(1+\omega)}$ at the crossing time and from Eq. (23) we find that

$$B \simeq \frac{k_*^{3(1+\omega)}}{\beta^{1/2}}\left(\frac{t_*}{p}\right)^{2+3\omega}. \tag{33}$$

So, having a value for t_*, we get an estimation for B, and through Eqs. (24) and (26) we obtain a value for the parameter D. The only constant that remains undetermined is a_{in}, however using that $k_* = a_* H_*$, we can write

$$a_{in} \simeq \frac{k_* t_{in}^p t_*^{1-p}}{p}. \tag{34}$$

From Eq. (30), and in the case of power-law inflation we get

$$t_* \simeq \left(\frac{3\sqrt{\beta}p^4}{4\pi^2\mathcal{P}_R}\right)^{\frac{1}{3}}. \tag{35}$$

The WMAP five-year data favors the tensor-scalar ratio $R \simeq 0.055$ and from Eqs. (32) and (35) we obtain a relation between the parameters β and p. In particular, for $p = 2$ we get the value for the brane tension $\beta \simeq 0.52 \times 10^{10} m_p^{-6}$. For the case in which $p = 10$, we obtain $\beta \simeq 0.15 \times 10^3 m_p^{-6}$. Here, we have used the WMAP five year data where $P_{\mathcal{R}}(k_*) \simeq 2.4 \times 10^{-9}$ and $k_* \simeq 0.002\ \text{Mpc}^{-1}$.

Finally, from Eqs. (24), (34) and (35), we obtain an estimation for the parameter D associated to the dark radiation energy density given by

$$D = \frac{k_*^4 p^{\frac{2(5-8p)}{3}}}{2\gamma}\left[\frac{3\sqrt{\beta}}{4\pi^2\mathcal{P}_R}\right]^{\frac{4(1-p)}{3}} t_{in}^{4p-2}. \tag{36}$$

In Fig. 1 contours curves corresponding to the same dimensionless number D/k_*^4 are plotted, as well as different combinations of the β and p parameters according to Eq. (36). Here, we have taken $t_{in} = 10^5/m_p$. From this plot, we see that, we can obtain the value of D/k_*^4, for a given values of β and p parameters. In particular, for $p = 2$ and $\beta = 0.52 \times 10^{10} m_p^{-6}$ we get $D/k_*^4 = 4.46 \times 10^{10}$. For $p = 10$ and $\beta = 0.15 \times 10^3 m_p^{-6}$ we have $D/k_*^4 = 1.03 \times 10^{36}$.

5. Summary

A possible transition from a dark radiation period to an inflationary phase has been here studied, based on a Randall–Sundrum II type brane-world. During the dark radiation period the energy density becomes $\chi \sim a^{-4}$, while in the inflationary phase the energy density associated to the inflaton field take the form $\rho \sim a^{-3(1+\omega)}$. During these periods the scale factor a evolves as $t^{1/2}$ and t^p (with $p > 1$) in the dark radiation period and the inflationary phase, respectively. The explicitly expression that connects these two patches is $a(t) = a_{in}[\sinh(C_2 \ln(1 + t/t_{in}))]^{1/n}$, where t_{in} represent the time when inflation begins. It is considered that the bulk-brane energy exchange, $T(\rho)$, plays no role in these two periods.

During the inflationary era the scale factor goes like $a(t) = a_{in}(t/t_{in})^p$ with the parameter p satisfying $p > 1$ in order to have $\ddot{a} > 1$. This period is quite interesting since we could fix some parameters appearing in the model by using astronomical data (coming, for instant, from WMAP 5 year). Actually, we succeeded in finding a relationship among the parameters related to the dark radiation energy (D), the brane tension parameter (β) and the power law parameter (p) for a given value of t_{in}. The relation of this parameters becomes given by the Eq. (36), i.e. $D = \frac{k_*^4 p^{\frac{2(5-8p)}{3}}}{2\gamma}[\frac{3\sqrt{\beta}}{4\pi^2 \mathcal{P}_R}]^{\frac{4(1-p)}{3}} t_{in}^{4p-2}$. This relation is summarized in the plot showed in Fig. 1. The interesting point here is that if we know the brane tension parameters, and p from Eqs. (32) and (35), we could fix an appropriated value for the parameter D associated to the dark radiation energy density. This might seem to be a very particular result, since the assumption of power law inflation is by itself a particular perform, but we think that relaxing this assumption our main results will not change very much. We hope to approach this problem in the near future.

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References

[1] J.B. Hartle, S.W. Hawking, Phys. Rev. D 28 (1983) 2960;
A. Vilenkin, Phys. Rev. D 30 (1984) 509;
A.D. Linde, Sov. Phys. JETP 60 (1984) 211.
[2] G.F.R. Ellis, R. Maartens, Class. Quantum Grav. 21 (2004) 223;
G.F.R. Ellis, J. Murugan, C.G. Tsagas, Class. Quantum Grav. 21 (2004) 233.
[3] A. Banerjee, T. Bandyopadhyay, S. Chakraborty, Grav. Cosmol. 13 (2007) 290.
[4] S. del Campo, R. Herrera, P. Labraña, JCAP 0711 (2007) 030.
[5] G. Veneziano, Phys. Lett. B 265 (1991) 287;
M. Gasperini, G. Veneziano, Astropart. Phys. 1 (1993) 317.
[6] N. Kaloper, A. Linde, R. Bousso, Phys. Rev. D 59 (1999) 043508.
[7] R.C. Tolman, Relativity, Thermodynamics and Cosmology, Clarendon Press, Oxford, 1934.
[8] J. Khoury, B.A. Ovrut, P.J. Steinhardt, N. Turok, Phys. Rev. D 64 (2001) 123522;
J. Khoury, B.A. Ovrut, N. Seiberg, P.J. Steinhardt, N. Turok, Phys. Rev. D 65 (2002) 086007;
P.J. Steinhardt, N. Turok, Science 296 (2002) 1436;
P.J. Steinhardt, N. Turok, Phys. Rev. D 65 (2002) 126003.
[9] N. Kanekar, V. Sahni, Y. Shtanov, Phys. Rev. D 63 (2001) 083520.
[10] J.E. Lidsey, D.J. Mulryne, N.J. Nunes, R. Tavakol, Phys. Rev. D 70 (2004) 063521.
[11] J.M. Cline, hep-th/0704.2198.
[12] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690.
[13] D. Langlois, Prog. Theor. Phys. Suppl. 148 (2003) 181;
P. Brax, C. van de Bruck, Class. Quantum Grav. 20 (2003) R201;
R. Maartens, Living Rev. Rel. 7 (2004) 1;
P. Brax, C. van de Bruck, A.C. Davies, Rep. Prog. Phys. 67 (2004) 2183.
[14] S. Mukohyama, Phys. Lett. B 473 (2000) 241.
[15] K. Ichiki, M. Yahiro, T. Kajino, M. Orito, G.J. Mathews, Phys. Rev. D 66 (2002) 043521;
J.D. Bratt, A.C. Gault, R.J. Scherrer, T.P. Walker, Phys. Lett. B 546 (2002) 19.
[16] E. Kiritsis, G. Kofinas, N. Tetradis, T.N. Tomaras, V. Zarikas, JHEP 0302 (2003) 035.
[17] A. Kehanias, E. Kiritsis, JHEP 9911 (1999) 022.
[18] P.S. Apostolopoulos, N. Tetradis, Phys. Lett. B 633 (2006) 409.
[19] P.S. Apostolopoulos, N. Tetradis, Phys. Rev. D 71 (2005) 043506.
[20] A. Hebecker, J. March-Russell, Nucl. Phys. B 608 (2001) 375;
D. Langlois, L. Sorbo, M. Rodriguez-Martinez, Phys. Rev. Lett. 89 (2002) 171301;
E. Leeper, R. Maartens, C.F. Sopuerta, Class. Quantum Grav. 21 (2004) 1125.
[21] N. Tetradis, Class. Quantum Grav. 21 (2004) 5221.
[22] E. Kiritsis, G. Kofinas, N. Tetradis, T.N. Tomaras, V. Zarikas, JHEP 0302 (2003) 035;
N. Tetradis, Phys. Lett. B 569 (2003) 1;
Y.S. Myung, J.Y. Kim, Class. Quantum Grav. 20 (2003) L169;
T.N. Tomaras, hep-th/0404142.
[23] S. del Campo, R. Herrera, G. Olivares, D. Pavón, Phys. Rev. D 74 (2006) 023501.
[24] A. Vilenkin, L.H. Ford, Phys. Rev. D 26 (1982) 1231.
[25] J.M. Bardeen, Phys. Rev. D 22 (1980) 1882;
B.A. Bassett, S. Tsujikawa, D. Wands, Rev. Mod. Phys. 78 (2006) 537.
[26] E. Komatsu, et al., WMAP5 Collaboration, astro-ph/0803.0547.
[27] D. Langlois, R. Maartens, D. Wands, Phys. Lett. B 489 (2000) 259.
[28] J. Dunkley, et al., WMAP Collaboration, arXiv: 0803.0586 [astro-ph];
G. Hinshaw, et al., arXiv: 0803.0732 [astro-ph].
[29] M. Tegmark, et al., Phys. Rev. D 69 (2004) 103501.