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Mass generation for Abelian spin-1 particles via a symmetric tensor

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ABSTRACT

In the topologically massive BF model (TMBF) the photon becomes massive via coupling to an antisymmetric tensor, without breaking the $U(1)$ gauge symmetry. There is no need of a Higgs field. The TMBF model is dual to a first-order (in derivatives) formulation of the Maxwell–Proca theory where the antisymmetric field plays the role of an auxiliary field. Since the Maxwell–Proca theory also admits a first-order version which makes use of an auxiliary symmetric tensor, we investigate here a possible generalization of the TMBF model where the photon acquires mass via coupling to a symmetric tensor. We show that it is indeed possible to build up dual models to the Maxwell–Proca theory where the $U(1)$ gauge symmetry is manifest without Higgs field, but after a local field redefinition the vector field eats up the trace of the symmetric tensor and becomes massive. So the explicit $U(1)$ symmetry can be removed unlike the TMBF model.

1. Introduction

In the usual description of massive spin-1 particles via a Maxwell–Proca (MP) action the gauge symmetry is explicitly broken. It is of interest to search for alternatives to the Higgs mechanism to preserve the gauge symmetry while generating a mass for a spin-1 particle specially for the non-Abelian case. Here we address this question in the simpler case of the Abelian $U(1)$ gauge symmetry. Dualization methods can help in investigating this problem. It is convenient for those methods to rewrite the Maxwell action in a first-order form by using auxiliary fields. In $D = 1 + 1$ we can achieve that with help of a scalar field which interacts with the vector field via a topological term $\phi\epsilon^{\mu\nu}\partial_\mu A_\nu$. By using the master action approach of [1] as a dualization procedure it can be shown [2] that the first-order MP theory in $D = 1 + 1$ is dual to a local action with manifest $U(1)$ gauge symmetry. It corresponds to the bosonized form of the Schwinger model whose effective action, after elimination of the auxiliary scalar field, is written down in our formula (15). Although non-local, the effective action is manifest $U(1)$ invariant.

In $D = 2+1$ we replace the scalar field by a vector field B_μ and the topological coupling term becomes $\epsilon^{\mu\nu\alpha}B_\mu\partial_\nu A_\alpha$. After some trivial field redefinition we end up, see master action in [3] with equal masses, with a dual theory to MP which consists of a couple of non-interacting Maxwell–Chern–Simons actions with the same mass but with opposite helicities. This theory is manifest $U(1)$ symmetric and represents one massive spin-1 particle with helicities ± 1 just like the MP theory in $D = 2 + 1$.

In $D = 3 + 1$ we can use an antisymmetric tensor with the so-called topological BF coupling $\epsilon^{\mu\nu\alpha\beta}B_{\mu\nu}\partial_\alpha A_\beta$. The theory dual to MP is the topologically massive BF model (TMBF), also named Cremmer–Scherk model [4]. It can be obtained from the first-order MP theory via both master action [5] and Noether gauge embedment [6]. The TMBF model is unitary [7] and explicitly $U(1)$ invariant. Unfortunately, as shown in [8], a non-Abelian generalization of the TMBF model without extra fields will necessarily lead to power-counting non-renormalizable couplings as in [9], see however [10,11], where the extra field is non-propagating and [12] for a recent suggestion which makes use of tensor gauge fields. In [13] the geometrical origin of tensor gauge connections is investigated. Thus, it is welcome to try alternatives to the TMBF model. Here we follow this route for the Abelian $U(1)$ case as a laboratory for a possible non-Abelian generalization.

In fact, in [14] there appears a new first-order form of the Maxwell action which makes use of a symmetric auxiliary field $W_{\mu\nu} = W_{\nu\mu}$. By adding the Proca mass term we build up a first-order version of the MP theory, see [15]. Now we have the coupling term $W_{\mu\nu}\partial^\mu A^\nu$, though non-topological, this term by itself has no particle content. In Section 2 we use this first-order formulation of the MP theory as a starting point to obtain via master action and Noether gauge embedment alternative dual theories to the MP theory. In Section 3 we start with an Ansatz quadratic in the fields A_μ and $W_{\mu\nu}$ and second-order in derivatives. We analyze its particle content and the presence of $U(1)$ gauge symmetry. In Section 4 we draw our conclusions.

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2. Master action and Noether gauge embedment

In analogy with the derivation of the TMBF model via master action as given in [5] we define here a master action depending on four different fields[1]:

$$S_M[A,\tilde{A},W,\tilde{W}] = \int d^D x\bigg[W^{\mu\nu}W_{\mu\nu} - \frac{W^2}{D-1} + 2W^{\mu\nu}\partial_{(\mu}A_{\nu)} - \frac{m^2}{2}A^\mu A_\mu - 2\big(W^{\mu\nu} - \tilde{W}^{\mu\nu}\big)\big(\partial_{(\mu}A_{\nu)} - \partial_{(\mu}\tilde{A}_{\nu)}\big)\bigg]. \quad (1)$$

The first four terms of (1) correspond to a first-order form of the Maxwell–Proca theory, see [14,15], while the last term mixes the (A,W) fields with the duals $(\tilde{A},\tilde{W})$. After the shift $\tilde{A}_\mu \to \tilde{A}_\mu + A_\mu$ and $\tilde{W}_{\mu\nu} \to \tilde{W}_{\mu\nu} + W_{\mu\nu}$ the last term of (1) decouples and becomes $\mathcal{L}_{\tilde{A},\tilde{W}} = -2\tilde{W}^{\mu\nu}\partial_{(\mu}\tilde{A}_{\nu)}$. Thus, the particle content of the master action (1) corresponds to one massive spin-1 particle plus the content of $\mathcal{L}_{\tilde{A},\tilde{W}}$. Minimizing the action[2] $S_{A,W} = \int d^D x\,\mathcal{L}_{A,W}$ we have the equations of motion:

$$\partial_\mu A_\nu + \partial_\nu A_\nu = 0, \quad (2)$$

$$\partial^\mu W_{\mu\nu} = 0. \quad (3)$$

It is easy to convince oneself, assuming vanishing fields at infinity, that the solution of (2) is trivial $A_\mu = 0$ while (3) is solved [16] by $W_{\mu\nu} = \partial^\alpha\partial^\beta R_{\mu\alpha\beta\nu}$ where $R_{\mu\alpha\beta\nu}$ is a tensor with the index symmetries of the Riemann curvature tensor but otherwise arbitrary. However, since the action $S_{A,W}$ is itself invariant under $\delta_\Lambda W_{\mu\nu} = \partial^\alpha\partial^\beta\Lambda_{\mu\alpha\beta\nu}$ where $\Lambda_{\mu\alpha\beta\nu}$ has the same properties of $R_{\mu\alpha\beta\nu}$ we can say that the general solution of (3) is pure gauge. Therefore, the last term of (1) has no particle content and the whole master action (1) contains only one massive spin-1 particle in the spectrum. Following the master action approach, if we Gaussian integrate over the fields (A,W) we have the dual action to the first-order Maxwell–Proca model:

$$S^*_{MP(I)} = \int d^D x\bigg[-\frac{1}{4}F^2_{\mu\nu}(\tilde{A}) + \frac{2}{m^2}\big(\partial^\alpha\tilde{W}^\nu_\alpha\big)^2 - 2\tilde{W}^{\mu\nu}\partial_{(\mu}\tilde{A}_{\nu)}\bigg]. \quad (4)$$

The action $S^*_{MP(I)}$ is invariant under the gauge transformations $\delta_\Lambda\tilde{W}_{\mu\nu} = \partial^\alpha\partial^\beta\Lambda_{\mu\alpha\beta\nu}$ with $\delta_\Lambda\tilde{A}_\mu = 0$. The equations of motion of (4) are:

$$\partial_\mu\tilde{q}_\nu + \partial_\nu\tilde{q}_\mu = 0, \quad (5)$$

$$\Box\theta_{\mu\nu}\tilde{A}^\nu - m^2\tilde{A}_\mu + m^2\tilde{q}_\mu = 0 \quad (6)$$

where $\tilde{q}_\mu = \tilde{A}_\mu + 2\partial^\alpha\tilde{W}_{\alpha\mu}/m^2$ is gauge invariant. As in (2), due to the boundary conditions at infinity, we have the solution $\tilde{q}_\mu = 0$ of (5) which allows us to eliminate $\tilde{W}_{\mu\nu}$ in terms of $\tilde{A}_\mu$ up to gauge transformations, i.e., $\partial^\alpha\tilde{W}_{\alpha\beta} = -m^2\tilde{A}_\beta/2$. Back in (6) we recover the Maxwell–Proca equation confirming that $S^*_{MP(I)}$ contains only one massive particle of spin-1 in the spectrum but contrary to the TMBF model it has no $U(1)$ gauge symmetry. The key point is that the mixing term (BF term) in the master action of [5] carries $U(1)$ gauge symmetry differently from the mixing term of the master action (1). So, the master action approach is only partially successful.

Another way of obtaining the TMBF model from the Maxwell–Proca theory is by means of a Lagrangian Noether gauge embedding (NGE) procedure as in [6,17], see also [18] for a Hamiltonian embedding. Let us apply the same Lagrangian procedure here. The first four terms of (1) define the first-order Maxwell–Proca theory:

$$S^{(0)} = \int d^D x\bigg[W^{\mu\nu}W_{\mu\nu} - \frac{W^2}{D-1} + 2W^{\mu\nu}\partial_{(\mu}A_{\nu)} - \frac{m^2}{2}A^\mu A_\mu\bigg]. \quad (7)$$

The first three terms of (7) are invariant under the $U(1)$ gauge transformations:

$$\delta_\phi A_\mu = \partial_\mu\phi; \qquad \delta_\phi W_{\mu\nu} = \Box\theta_{\mu\nu}\phi. \quad (8)$$

Where we define the projection operators:

$$\theta_{\alpha\beta} = (\eta_{\alpha\beta} - \omega_{\alpha\beta}), \qquad \omega_{\alpha\beta} = \frac{\partial_\alpha\partial_\beta}{\Box}. \quad (9)$$

In order to preserve the $U(1)$ symmetry broken by the mass term in (7) we modify the action $S^{(0)}$ through an iterative procedure with help of auxiliary fields B_μ and $C_{\mu\nu}$ which transform as $\delta_\phi B_\mu = -\delta_\phi A_\mu$ and $\delta_\phi C_{\mu\nu} = -\delta_\phi W_{\mu\nu}$. We end up with the gauge invariant action:

$$S^{(2)} = S^{(0)} + \int d^D x\bigg(K_\mu B^\mu - m^2\frac{B^\mu B_\mu}{2} + C^{\mu\nu}M_{\mu\nu}\bigg) \quad (10)$$

where the Euler tensors are given by

$$K_\mu = \frac{\delta S^{(0)}}{\delta A^\mu} = -m^2 A_\mu - 2\partial^\nu W_{\mu\nu}, \quad (11)$$

$$M_{\mu\nu} = \frac{\delta S^{(0)}}{\delta W^{\mu\nu}} = 2\bigg[\partial_{(\mu}A_{\nu)} + W_{\mu\nu} - \eta_{\mu\nu}\frac{W}{D-1}\bigg]. \quad (12)$$

After a functional integral over the auxiliary fields we end up with the Podolsky [19] action

$$S_P = \frac{1}{4}\int d^D x\,F_{\mu\nu}\bigg(1 - \frac{\Box}{m^2}\bigg)F^{\mu\nu}. \quad (13)$$

Besides the massive spin-1 physical particle, as expected from the embedding procedure, the Podolsky theory contains also one massless ghost which violates unitarity. This is also not totally surprisingly from the point of view of the NGE procedure as explained in [20].

In summary, the master action and the NGE procedures, which give a systematic derivation of the TMBF model from the Maxwell–Proca theory, have led us to different results in the case of the symmetric tensor $W_{\mu\nu}$ and none of them is satisfactory. There is either lack of explicit $U(1)$ symmetry or lack of unitarity. In Section 3 we use another approach for a broader investigation of this question.

3. A general Ansatz

Another way of figuring out the particle content of the TMBF model is to integrate in the two form field $B_{\mu\nu}$ in the path integral and obtain an effective action for the vector field A_μ. One ends up, see [21], with a four-dimensional version of the well-known Schwinger model which appears in $D = 1+1$ dimensions due to the non-conservation of the axial current, namely:

$$\exp^{iS_{eff}[A]} = \int \mathcal{D}B_{\mu\nu}\exp^{iS_{TMBF}[A,B]} \quad (14)$$

where

[1] In this work we use mostly plus D-dimensional signature $\eta_{\mu\nu} = (-,+,\dots,+)$

[2] We quit the tildes for while.

$$S_{eff}[A] = S_{Schw} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}\frac{(\Box - m^2)}{\Box}F^{\mu\nu}. \tag{15}$$

The Schwinger model is of course $U(1)$ gauge invariant and a careful analysis of the analytic properties of the propagator reveals that we have only one massive (spin-1) particle in the spectrum as in the initial TMBF model. In what follows we start with a more general (second-order in derivatives) Ansatz for a local quadratic action containing the fields $(A_\mu, W_{\mu\nu})$ and integrate over $W_{\mu\nu}$ in order to deduce a D-dimensional effective action for the vector field.

Let us start with the Ansatz:

$$\begin{aligned} S[A,W] = \int d^Dx \big[& a(\partial\cdot A)^2 + b(\partial_{(\mu}A_{\nu)})^2 + c_1\big(\partial^\nu W_{\mu\nu}\big)^2 \\ &+ c_2\partial^\nu W\partial^\mu W_{\mu\nu} + c_3\partial^\mu W\partial_\mu W \\ &+ c_4\partial^\alpha W_{\mu\nu}\partial_\alpha W^{\mu\nu} + dW_{\mu\nu}W^{\mu\nu} + eW^2 \\ &+ fW_{\mu\nu}\partial^\mu A^\nu + gW\partial\cdot A\big] \end{aligned} \tag{16}$$

where (a, b, c_i, e, f, g) are so far unknown real constants. We can rewrite the Ansatz as:

$$\begin{aligned} S[A,W] = \int d^Dx \big[& a(\partial\cdot A)^2 + b(\partial_{(\mu}A_{\nu)})^2 \\ &+ W_{\mu\nu}G^{\mu\nu}{}_{\alpha\beta}W^{\alpha\beta} + W_{\alpha\beta}T^{\alpha\beta}\big] \end{aligned} \tag{17}$$

where

$$T^{\alpha\beta} = f\partial^{(\alpha}A^{\beta)} + g\eta^{\alpha\beta}\partial\cdot A, \tag{18}$$

$$\begin{aligned} G^{\mu\nu}{}_{\alpha\beta} = \Big\{ & (d - c_4\Box)P^{(2)}_{SS} + \left(d - \frac{c_1\Box}{2} - c_4\Box\right)P^{(1)}_{SS} \\ &+ \big[d + e - (c_1 + c_2 + c_3 + c_4)\big]P^{(0)}_{WW} \\ &+ \big[d - c_4\Box + (e - c_3\Box)(D-1)\big]P^{(0)}_{SS} \\ &+ \sqrt{D-1}\left(e - c_3\Box - \frac{c_2\Box}{2}\right)\big(T^{(0)}_{SW} + T^{(0)}_{WS}\big)\Big\}^{\mu\nu}_{\alpha\beta} \end{aligned} \tag{19}$$

where the projection operators $P^{(s)}_{IJ}$ of spin-s and the transition operators $T^{(0)}_{SW}$, $T^{(0)}_{WS}$ are defined as:

$$\big(P^{(2)}_{SS}\big)^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}\big(\theta^\lambda{}_\alpha\theta^\mu{}_\beta + \theta^\mu{}_\alpha\theta^\lambda{}_\beta\big) - \frac{\theta^{\lambda\mu}\theta_{\alpha\beta}}{D-1}, \tag{20}$$

$$\big(P^{(1)}_{SS}\big)^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{2}\big(\theta^\lambda{}_\alpha\omega^\mu{}_\beta + \theta^\mu{}_\alpha\omega^\lambda{}_\beta + \theta^\lambda{}_\beta\omega^\mu{}_\alpha + \theta^\mu{}_\beta\omega^\lambda{}_\alpha\big), \tag{21}$$

$$\big(P^{(0)}_{SS}\big)^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{D-1}\theta^{\lambda\mu}\theta_{\alpha\beta}, \qquad \big(P^{(0)}_{WW}\big)^{\lambda\mu}{}_{\alpha\beta} = \omega^{\lambda\mu}\omega_{\alpha\beta}, \tag{22}$$

$$\big(T^{(0)}_{SW}\big)^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{D-1}}\theta^{\lambda\mu}\omega_{\alpha\beta},$$

$$\big(T^{(0)}_{WS}\big)^{\lambda\mu}{}_{\alpha\beta} = \frac{1}{\sqrt{D-1}}\omega^{\lambda\mu}\theta_{\alpha\beta}. \tag{23}$$

From (17), integrating over the fields $W_{\mu\nu}$ in the path integral we obtain the effective action

$$\begin{aligned} S_{eff}[A] = \int d^Dx \Big[& a(\partial\cdot A)^2 + b(\partial_{(\mu}A_{\nu)})^2 \\ &- \frac{1}{4}T_{\mu\alpha}(A)\big(G^{-1}\big)^{\mu\alpha}{}_{\gamma\beta}T^{\gamma\beta}(A)\Big], \end{aligned} \tag{24}$$

where, suppressing the indices for convenience, we have

$$\begin{aligned} G^{-1} = & \frac{P^{(2)}_{SS}}{(d - c_4\Box)} + \frac{P^{(1)}_{SS}}{d - \Box(c_4 + \frac{c_1}{2})} \\ &+ \frac{[d - c_4\Box + (e - c_3\Box)(D-1)]P^{(0)}_{WW}}{K} \\ &+ \frac{[d + e - (c_1 + c_2 + c_3 + c_4)\Box]P^{(0)}_{SS}}{K} \\ &+ \frac{\sqrt{D-1}}{K}\left(e - c_3\Box - \frac{c_2\Box}{2}\right)\big(T^{(0)}_{SW} + T^{(0)}_{WS}\big) \end{aligned} \tag{25}$$

with

$$\begin{aligned} K = & \big[d + e - (c_1 + c_2 + c_3 + c_4)\Box\big] \\ & \times \big[d - c_4\Box + (e - c_3\Box)(D-1)\big] \\ & - (D-1)\left(e - c_3\Box - \frac{c_2\Box}{2}\right)^2. \end{aligned} \tag{26}$$

Working out the expression (24) we have

$$\begin{aligned} S_{eff}[A] = \int d^Dx \Big[& a(\partial\cdot A)^2 + b(\partial_{(\mu}A_{\nu)})^2 + (\partial\cdot A)H(\Box)(\partial\cdot A) \\ &- \frac{1}{16}F_{\mu\nu}\frac{f^2}{d - \Box(c_4 + \frac{c_1}{2})}F^{\mu\nu}\Big] \end{aligned} \tag{27}$$

where

$$\begin{aligned} H(\Box) = \frac{-1}{4K}\big\{ & (D-1)g^2\big[d + e - (c_1 + c_2 + c_3 + c_4)\Box\big] \\ &+ (f+g)^2\big[d - c_4\Box + (e - c_3\Box)(D-1)\big] \\ &+ 2(D-1)g(f+g)\big[(c_3 + c_2/2)\Box - e\big]\big\}. \end{aligned} \tag{28}$$

In order to have $U(1)$ gauge invariance in (27) the constants in our Ansatz (16) must be such that

$$H(\Box) = -(a+b). \tag{29}$$

Consequently we end up with the gauge invariant theory

$$S_{eff}[A] = -\frac{1}{16}\int d^Dx\, F_{\mu\nu}\frac{4b[(c_4 + \frac{c_1}{2})\Box - d] + f^2}{d - (c_4 + \frac{c_1}{2})\Box}F^{\mu\nu}. \tag{30}$$

By adding a gauge fixing term we can obtain the propagator and calculate the saturated two point amplitude in momentum space $A(k)$ from which we can read off the particle content of the theory. Explicitly,

$$\begin{aligned} A(k) &= J^*_\mu(k)\big\langle A^\mu(-k)A^\nu(k)\big\rangle J_\nu(k) = -\frac{i}{2}J^*_\mu(k)\big[G^{-1}(k)\big]^{\mu\nu}J_\nu(k) \\ &= -\frac{i}{2}\frac{J^*(k)\cdot J(k)[(c_4 + \frac{c_1}{2})k^2 + d]}{k^2[4b(c_4 + \frac{c_1}{2})k^2 + 4bd - f^2]}. \end{aligned} \tag{31}$$

Note that the contribution of the gauge fixing term $\lambda(\partial\cdot A)^2$ drops out from $A(k)$ due to the transverse nature of the sources ($k\cdot J = 0$) as required by gauge invariance.

We may have one or two poles in $A(k)$. Since our aim is to obtain only one physical massive particle in the spectrum we impose henceforth:

$$d = 0; \qquad b\left(c_4 + \frac{c_1}{2}\right) \neq 0; \qquad f \neq 0. \tag{32}$$

In this case:

$$A(k) = -\frac{iJ^*\cdot J}{8b(k^2 + m^2)} \tag{33}$$

where

$$m^2 = -\frac{f^2}{4b(c_4 + c_1/2)}. \tag{34}$$

The imaginary part of the residue of $A(k)$ at the pole $k^2 = -m^2$ becomes $-J^*(k) \cdot J(k)/(8b)$ evaluated at $k^2 = -m^2$. In the rest frame $k_\mu = (m, 0, \dots, 0)$, due to $k \cdot J(k) = 0$, we must have $J_0(k) = 0$. So we can easily check that the frame independent quantity $J^*(k) \cdot J(k)$ is positive. Consequently, in order to have a physical particle as required by unitarity ($\mathrm{Im}\,\mathrm{Res}(A(k)) > 0$) and be free of tachyons, see (34), we must further assume that:

$$b < 0; \qquad c_4 + \frac{c_1}{2} > 0. \tag{35}$$

According to the above requirements the effective action (30) becomes exactly, fixing $b = -1$, the Schwinger model effective action (15). Clearly, we have to inspect the restrictions imposed by the gauge invariance condition (29). Namely,

$$(D-1)e = 0, \tag{36}$$

$$(a+b)\big[(D-1)(c_2 f - 2c_1 g) - 2c_4(f + Dg)\big] = 0, \tag{37}$$

$$(D-1) f^2 c_3 = g(D-1)(c_2 f - c_1 g) - c_4\big(f^2 + 2fg + Dg^2\big). \tag{38}$$

For future use we recall that in deducing (36), (37) and (38) we have assumed $d = 0$, $c_4 \neq 0$, $c_4 + c_1/2 \neq 0$ and that $K \neq 0$ which means, using $e = 0$ according to (36), that

$$K = \Box^2 \bigg\{ (c_1 + c_2 + c_3 + c_4)\big[(D-1)c_3 + c_4\big] - (D-1)\left(c_3 + \frac{c_2}{2}\right)^2 \bigg\} \neq 0. \tag{39}$$

Although there are several solutions of (37) and (38) some of them are related via trivial field redefinitions in our Ansatz (16). Here we stick to a subset of solutions which is the simplest one. Namely,

$$b = -a = -1; \qquad c_2 = 0 = c_4; \qquad c_3 = -\left(\frac{g}{f}\right)^2 c_1, \tag{40}$$

which leads to the family of actions:

$$S_I = \int d^D x \bigg\{ -\frac{1}{4} F_{\mu\nu}^2 + f W_{\mu\nu} \partial^\mu A^\nu + g W \partial \cdot A + c_1 \bigg[\left(\partial^\mu W_{\mu\nu}\right)^2 - \frac{g^2}{f^2} \partial^\mu W \partial_\mu W \bigg] \bigg\}. \tag{41}$$

We must have, see (32), (35) and (40), $c_1 > 0$ and $f \neq 0$ while g is an arbitrary real constant. Since $c_4 = 0$ and $d = 0$ too, see (32), there will be a zero in the denominator of (25). This indicates the appearance of a local symmetry not initially considered. Indeed, the reader can check that S_I is invariant under the local higher derivative transformations:

$$\delta_\Lambda A_\mu = 0, \qquad \delta_\Lambda W_{\mu\nu} = \big[\Box^2 P_{SS}^{(2)}\big]_{\mu\nu}{}^{\alpha\beta} \Lambda_{\alpha\beta} \quad \to \quad \delta_\Lambda W = 0 \tag{42}$$

where $\Lambda_{\alpha\beta} = \Lambda_{\beta\alpha}$ is an arbitrary symmetric tensor. Thus, in order to integrate over $W_{\mu\nu}$ we must add an appropriate gauge fixing term to break (42). We can add to (41) for instance,

$$\mathcal{L}_{GF}^{(2)} = \lambda_2 \big(\Box^2 \big[P_{SS}^{(2)}\big]_{\mu\nu}{}^{\alpha\beta} W_{\alpha\beta}\big)^2. \tag{43}$$

After integrating over $W_{\mu\nu}$ now we obtain an effective action independent of λ_2 which is of the Schwinger type (15). Thus, confirming that S_I describes a massive spin-1 particle in a gauge invariant way as originally desired. However, the $U(1)$ symmetry of S_I is not the usual one but rather a higher derivative form of it, i.e.,

$$\delta_\phi^{hd} A_\mu = \partial_\mu \Box \phi, \tag{44}$$

$$\delta_\phi^{hd} W_{\mu\nu} = \frac{f}{2c_1(D-1)g}\big[(f + gD)\partial_\mu \partial_\nu \phi - (f+g)\eta_{\mu\nu}\Box\phi\big]. \tag{45}$$

It is clear from (44) and (45) that $f = -Dg$ is the most interesting case. In this special case we can choose, recalling (34), without loss of generality $f = m^2 = -Dg$ and $c_1 = m^2/2$. Back in (41) we have our main result:

$$S_{II} = \int d^D x \bigg[-\frac{1}{4} F_{\mu\nu}^2 + m^2 \left(W_{\mu\nu} - \frac{W}{D}\eta_{\mu\nu}\right)\partial^\mu A^\nu + \frac{m^2}{2}\left(\partial^\mu W_{\mu\nu} \partial^\alpha W_\alpha{}^\nu - \frac{1}{D^2}\partial^\mu W \partial_\mu W\right)\bigg]. \tag{46}$$

The action S_{II} is explicitly invariant under usual (first-order) $U(1)$ gauge transformations:

$$\delta_\phi A_\mu = \partial_\mu \phi; \qquad \delta_\phi W_{\mu\nu} = \eta_{\mu\nu}\phi. \tag{47}$$

As we have already mentioned, integrating over $W_{\mu\nu}$ in the path integral (with appropriate gauge fixing of the higher spin symmetry (42)) we end up with the effective action of the Schwinger type, see (15). Thus, S_{II} is a new action which describes a massive spin-1 particle with manifest usual $U(1)$ symmetry.

Regarding the remaining case $f \neq -Dg$ with unusual $U(1)$ symmetry, it can be shown that the redefinition of the fields

$$A_\mu = \tilde{A}_\mu - \frac{2c_1 g}{f(f+gD)}\partial_\mu \tilde{W}, \tag{48}$$

$$W_{\mu\nu} = \tilde{W}_{\mu\nu} - \frac{g}{f+gD}\eta_{\mu\nu}\tilde{W} \tag{49}$$

is equivalent to set $g = 0$ in S_I. The new vector field $\tilde{A}_\mu$ is gauge invariant. So, after the field redefinition we loose the manifest unusual $U(1)$ gauge symmetry. The action $S_I(g=0)$ corresponds, with the normalization $c_1 = 2/m^2$ and $f = -2$, to the dual model $S_{MP(I)}^*$ obtained in Section 2 from the MP theory via master action, see (4).

One might ask whether it is also possible to redefine fields and hide the explicit usual $U(1)$ symmetry in the more interesting case $f = -Dg$. In fact, since $\delta_\phi W = D\phi$ we can always change variables to a gauge invariant vector field $A_\mu \to A_\mu - \partial_\mu W/D$ and loose the manifest $U(1)$ symmetry. The action S_{II} after this field redefinition becomes:

$$S_{MP(II)}^* = \int d^D x \bigg[-\frac{1}{4}F_{\mu\nu}^2 + m^2\left(W_{\mu\nu} - \frac{W}{D}\eta_{\mu\nu}\right)\partial^\mu A^\nu + \frac{m^2}{2}\left(\partial^\mu W_{\mu\nu} - \frac{\partial_\nu W}{D}\right)^2\bigg]. \tag{50}$$

One can say that the initial massless vector field A_μ has eaten up the trace W and became massive as in the usual Stuckelberg mechanism. Notice also that in $S_{MP(II)}^*$ only the traceless piece of $W_{\mu\nu}$ effectively appears contrary to (46). The action $S_{MP(II)}^*$ is invariant under the spin-2 local transformations (42) and under Weyl transformations $\delta_\phi W_{\mu\nu} = \eta_{\mu\nu}\phi$. After convenient gauge fixing of those symmetries by adding a gauge fixing term like (43) and another one for the Weyl symmetry, like for instance

$$\mathcal{L}_{GF}^{(0)} = \lambda_0\big[\Box^2\big(P_{WW}^{(0)}\big)_{\alpha\beta}{}^{\mu\nu}W_{\mu\nu}\big]^2, \quad (51)$$

we can integrate over $W_{\mu\nu}$ and obtain an effective action for the vector field, independent of λ_2 and λ_0, which becomes exactly the Maxwell–Proca theory:

$$\mathcal{L}_{eff}[A] = \mathcal{L}_{MP} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}A_\mu A^\mu. \quad (52)$$

Therefore in both cases $f \neq -Dg$ and $f = -Dg$ we can redefine the fields, get rid of the explicit $U(1)$ symmetry and deduce new dual models to the Maxwell–Proca theory which correspond respectively to $S^*_{MP(I)}$ and $S^*_{MP(II)}$. We have found interesting to check the equations of motion of (50) which can be written as

$$\Box\theta_{\mu\nu}A^\nu - m^2 v_\mu = 0; \qquad \partial_\mu q_\nu + \partial_\nu q_\mu = \frac{2}{D}\eta_{\mu\nu}\partial\cdot q \quad (53)$$

where

$$v_\mu = \partial^\alpha W_{\alpha\mu} - \frac{\partial_\mu W}{D}; \qquad q_\mu = A_\mu - v_\mu. \quad (54)$$

Note that the vectors A_μ, v_μ and consequently q_μ are $U(1)$ gauge invariant. General coordinate transformations in a flat space–time changes the metric tensor according to $\delta_\xi g_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$. Conformal transformations require that $\delta_\xi g_{\mu\nu} = \phi g_{\mu\nu}$ whose trace implies $\phi = 2(\partial\cdot\xi)/D$. Therefore, the general solution to the second equation of (53) corresponds exactly to conformal transformations

$$q_\mu = A_\mu - v_\mu = a_\mu + \Lambda_{\mu\nu}x^\nu + \lambda x_\mu + 2x_\mu(x\cdot c) - x^2 c_\mu, \quad (55)$$

where the antisymmetric matrix $\Lambda_{\mu\nu}$ and a_μ, c_μ, λ are constant parameters. Since the fields must vanish at infinity, all those constant parameters must vanish. So $q_\mu = v_\mu - A_\mu = 0$ allows us to replace v_μ by A_μ in the first equation of (53) which becomes, as expected from the effective action, the Maxwell–Proca equation $\Box\theta_{\mu\nu}A^\nu - m^2 A_\mu = 0$.

After eliminating v_μ in terms of A_μ we are still left with degrees of freedom in $W_{\mu\nu}$ which are not present in the combination v_μ however, those are exactly the pure gauge degrees of freedom related to the symmetries of (50). So the duality between (50) and the Maxwell–Proca theory is also established at classical level as expected.

In summary, there is an important difference between our new $U(1)$ invariant model S_{II} describing a massive spin-1 particle and the TMBF model. Namely, in the former case it is always possible to get rid of the explicit $U(1)$ symmetry by a local field redefinition unlike the TMBF model. Technically, this is possible by combining the vector fields A_μ and $\partial_\mu W$ and defining a gauge invariant vector field.

One might blame the choice of parameters (40) for the existence of a $U(1)$ gauge invariant vector field. Next we give a symmetry argument to show that even for the general Ansatz (16) it is always possible to define a gauge invariant vector field. Namely, the $U(1)$ gauge transformation which leaves the Ansatz (16) invariant must be of the general form

$$\delta_\phi A_\mu = \partial_\mu\phi; \qquad \delta_\phi W_{\mu\nu} = r\phi\eta_{\mu\nu} + s\Box\phi\eta_{\mu\nu} + t\partial_\mu\partial_\nu\phi \quad (56)$$

where (r, s, t) are real constants. The variation of the Ansatz includes the following independent terms:

$$\delta S = \int d^Dx\big[2r(De+d)W\phi + r(f+Dg)\partial^\mu A_\mu\phi + (f - 2rc_1 - Drc_2 + 2dt)\partial^\mu\partial^\nu W_{\mu\nu}\phi + \cdots\big]. \quad (57)$$

Therefore, among other constraints, we have the following ones

$$r(De+d) = 0, \quad (58)$$

$$r(f+Dg) = 0, \quad (59)$$

$$r(2c_1 + Dc_2) - 2dt = f. \quad (60)$$

For only one massive particle in the spectrum we must have $d = 0$ and $f \neq 0$, therefore $r \neq 0$ so we can rescale $r \to 1$. It also follows that $e = 0$ and $f = -Dg$ which is in agreement with our previous results S_I and S_{II} since we have demanded usual (first-order) $U(1)$ transformations for the vector field.

On the other hand, the field redefinition

$$W_{\mu\nu} = \tilde{W}_{\mu\nu} + s\eta_{\mu\nu}\partial\cdot A + t\partial_{(\mu}A_{\nu)} \quad (61)$$

will absorb the t and s factors such that $\delta_\phi\tilde{W}_{\mu\nu} = \eta_{\mu\nu}\phi$, i.e., we can set $s = 0 = t$ in (56). Therefore, we conclude that we are always able to make a field redefinition $A_\mu = \tilde{A}_\mu + \partial_\mu\tilde{W}/D$ to a gauge invariant vector field $\delta_\phi\tilde{A}_\mu = 0$ which jeopardizes the manifest $U(1)$ symmetry.

In practice we have checked for other choices different from (40) that is always possible to redefine the fields and end up without manifest $U(1)$ symmetry.

4. Conclusion

In the topologically massive BF model (TMBF), also named Cremmer–Scherk model, the photon acquires mass without need of a Higgs field while keeping the $U(1)$ gauge symmetry manifest in the action. It is not possible in this case to remove the $U(1)$ symmetry from the action by any local field redefinition. In this model the vector field is coupled to an antisymmetric tensor. Motivated by the TMBF model we have investigated here the possibility of generating mass for the photon, in a $U(1)$ invariant way, by coupling the vector field to a symmetric rank-2 tensor instead. Since the TMBF model can be obtained via dualization methods like master action and Noether gauge embedment (NGE) from the Maxwell–Proca theory, we have applied in Section 2 the same techniques to a first-order form of the Maxell–Proca theory, see [14], where a symmetric rank-two tensor replaces the totally antisymmetric tensor of the TMBF model. The NGE procedure has led us to a non-unitary theory while the master action approach has furnished the model (4) which is in fact dual to the Maxwell–Proca theory in arbitrary D dimensions without however, manifest $U(1)$ gauge symmetry.

In Section 3 we have applied a more general procedure which starts from a rather complete second-order (in derivatives) Ansatz, see (16), involving quadratic terms in the vector and tensor fields. We have integrated in the path integral over the tensor field and obtained an effective action for the vector field. Requiring that the effective vector theory be $U(1)$ invariant and contain only one massive spin-1 particle in the spectrum we have deduced a set of constraints on the couplings. In particular, the constraints solution given in (40) has led us to the family of $U(1)$ invariant actions S_I given in (41). However, it turns out that in the general case the $U(1)$ transformations are not the usual ones, see (44) and (45). A further restriction on the parameters space ($f = -Dg$) is required to recover the usual (first order) $U(1)$ transformations. In this case we obtain the gauge invariant description of a massive spin-1 particle given in (46) which is our main result.

It turns out both for $f \neq -Dg$ and $f = -Dg$ that after a local redefinition of the fields involving the trace $W = W^\mu{}_\mu$, the manifest $U(1)$ symmetry can be removed very much like in the usual Stückelberg formalism although our action is rather different from the usual Stückelberg form of the Maxwell–Proca theory. In our case the trace W is eaten up by the vector field which becomes massive. After those field redefinitions we obtain new dual theories to the Maxwell–Proca model given in (4) and (50). We have also tried other solutions of the constraint equations but it turns out that it is always possible to eat up the trace and end up without explicit $U(1)$ symmetry. We have given a symmetry argument explaining that point. Clearly, one might try to include higher derivative (above second-order) terms in the Ansatz but they are expected to jeopardize unitarity.

In the TMBF model it is not possible to remove the $U(1)$ symmetry by any local field redefinition. The key difference seems to be that the $U(1)$ gauge symmetry of the vector field does not need to be compensated by any transformation of the auxiliary two-form field unlike the case investigated here where the symmetric rank-2 tensor must transform nontrivially.

We are currently investigating a non-Abelian extension of our results. Moreover, in [22] the coupling of higher spin particles to the electromagnetic field has been studied leading to some apparently universal conclusions. In [22] the usual Stückelberg formalism has been employed. It is desirable to check the universality of their results via an alternative gauge invariant formulation for massive particles as given here. We are working on a generalization of our approach to higher spin charged particles.

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