Calculation of Aircraft Exhaust System Infrared Radiation Using Temperature Corrected Turbulence Model

Xi-xi Li\textsuperscript{a}, Eriqitai\textsuperscript{a}, Qiang Wang\textsuperscript{a}

\textsuperscript{a} School of Jet Propulsion, Beihang University, Beijing, 100191, China

Abstract

The standard two-equation turbulence model has no provision for large density gradient when used to compute the mean flow and the infrared radiation of hot jet. Stability consideration indicates that density gradient in a turbulent flow would add to instability due to local accelerations in the turbulent velocity field. Such instability would lead to faster mixing and spreading of jet flow. By choosing the total temperature gradient to represent the density gradient, a temperature corrected two-equation turbulence model would take into account the spatial instability. The coupled calculations for flow field, species concentration field and gas radiation transfer/energy equations based on Narrow Band k-distribution in non-gray absorbing-emitting were employed to simulate accurately the infrared signature of the aircraft exhaust system. The final infrared signature has considered the atmosphere effect, and homochromous atmospheric transmittance under various conditions was obtained by LOWTRAN 7. The standard two-equation model, Jones-Launder k-\textit{\textalpha}\textsuperscript{I} formulation, was also investigated for comparison with the temperature corrected turbulence model. All of the models were investigated for a reference nozzle producing heated jets at a low Mach number to avoid complications of large compressibility effects. The primary deficiency of the standard models was the delayed initial jet mixing rate relative to experimental data. The temperature corrected turbulence model provided improved mean flow and infrared radiation predictions relative to the standard models.

Keywords: exhaust system; infrared radiation; Narrow Band k-distribution; turbulence model; temperature corrected

1. Introduction

The aircraft exhaust system, composed by the high temperature parts after turbine, cavity and jet flow, is the one of the most significant infrared radiation sources [1]. With the deep development of infrared homing missile and infrared detector, the extensive numerical studies of infrared radiation (IR) concerning exhaust system have been done in recent decades. A simple descriptive model for infrared analysis was presented to predicting the detailed flow field from a two-dimensional convergent-divergent nozzle plume by Chu et al [2]. A comprehensive scheme for the prediction of radiation from an exhaust system, based on the combination of radiation from the cavity and the gaseous plume, was given by Heragu et al [3, 4]. A three-dimensional infrared radiation code for exhaust system was developed by the finite volume method coupled with Narrow Band model in non-gray absorbing-emitting media by HaiYang Hu [5].

While Large-Eddy Simulations (LES) offer promise for the future by directly calculating large scale turbulence, Reynolds-averaged Navier-Stokes (RANS) techniques will be required for the foreseeable future, especially for the
analysis of complex nozzle geometries. Within the class of RANS methods, two-equation turbulence models have been used most frequently for jet flow analyses because of their capability to provide mean flow fields necessary for subsequent infrared radiation analysis. However, the prediction by standard model was not so accurate. Capturing the initial jet growth region remains a difficulty for all of these RANS models with the calculated jet mixing rates generally being much slower than that exhibited by experimental data. Additionally, far downstream of the end of the jet potential core, it had been generally found that the computed farfield mixing rate became too high [6]. To improve the accuracy of the prediction by two-equation turbulent models, Pope [7] proposed a correction of vortex stretching to account for the effects of compressibility. For high Mach number jet flows, Sarkar [8] proposed another modification for the compressibility of gas. According to these two modifications, Tam and Thies [9] proposed a \( k-\epsilon \) model with different closure coefficients compared with standard \( k-\epsilon \) models. These modified closure coefficients were recalibrated using a series of jet flows. It is determined that the density difference between the ambient gas and that of a hot jet would promote strong flow instabilities, which, in turn, lead to faster mixing and spreading of the jet flow. Based on this observation, Tam and Ganesan [10] extended the work of Thies and Tam by incorporating a correction for heated jets. The modification on the Tam-Thies model significantly improves the accuracy of its prediction. However Tam-Ganesan model is strictly intended for free shear flows and therefore not applicable to the simulation for aircraft exhaust system.

With exhaust system it is particularly important that the model apply to both free shear and wall bounded flows. Experimental studies by Seiner et al [11] showed that the high total temperature gradient led to faster mixing and spreading of jet flow. Based on these observations, a minimally invasive correction only to turbulent eddy viscosity was proposed to rectify the mixing deficiency for high temperature jets by SJ Massey et al [12]. The temperature corrected turbulence model that we refer to as the PAB Temperature Correction (PAB TC) was built upon the Jones-Lauder [13] \( k-\epsilon \) model. Stability consideration indicated that density gradient in a turbulent flow would add to instability due to local accelerations in the turbulent velocity field. A layered fluid medium was statically unstable when the dot product of the density gradient vector and the acceleration vector is negative. This criterion was met perhaps half the time in a turbulent flow with a density gradient created by a temperature gradient [12]. To insure that the PAB TC turbulence model returns to the original model for cold jets and that it remains accurate for wall bounded flows, the total temperature gradient was chosen as the variable for the additional eddy viscosity dependence, instead of using directly the density gradient. Moreover, the modification would be insensitive to density and temperature gradients due to Mach wave expansion and the embedded shocks in supersonic jet. Consequently, the computational stability was greatly enhanced. The commonly accepted compressibility effect has also been taken into account for the PAB TC modification. The results by this model showed good agreement with experimental results. However, it was still unknown whether the modified model contributed to improve the prediction accuracy of infrared radiation for aircraft exhaust system.

Our objective in the present work is to investigate the relationship between the turbulence model and the simulation precision about exhaust system infrared signatures. The standard two-equation model, Jones-Lauder \( k-\epsilon \) formulation, was also investigated for comparison with the temperature corrected turbulence model. All of the models were investigated for a reference nozzle producing heated jets at a low Mach number to avoid complications of large compressibility effects. The primary deficiency of the standard models was the delayed initial jet mixing rate relative to experimental data. The temperature corrected turbulence model provided improved mean flow and infrared radiation predictions relative to the standard models. Accordingly, the simulation precision about exhaust system infrared signatures was improved.

In this paper, the temperature, the pressure and the concentration distribution of exhaust system were calculated by numerical simulation of the three-dimensional flow field by employing Finite Volume Method (FVM) and two-equation turbulence models, which coupled with conduction, convection and radiation [14]. FVM is also used to simulate the IR for coupling the radiation with the flow field which is evaluated by the same method. In the study, it is assumed that the engine operating in the non-afterburning state and the combustion of hydrocarbon fuel is complete. That is to say, the density of solid particles is rather low. Thus, the mixture of \( \text{CO}_2 \) and \( \text{H}_2\text{O} \) but no particles is taken into account when simulating the plume radiation. The gas radiation parameter was calculated by narrow band \( k \)-distributions. The attenuation of the exhaust system infrared signatures due to atmosphere has been considered, and the homochromous atmospheric transmittance under various conditions was obtained by LOWTRAN 7 [15].

2. Turbulence Modeling Details

The FICC-BUAA RANS solver was used for all of the turbulence models investigations described in this paper. In Ref. 5, FICC-BUAA was found to provide nearly identical results to those obtained from other similar production CFD
solvers for jet flow predictions when the same turbulence model was employed. Accordingly, it is expected that the results obtained here with FICC-BUAA are representative of those that would be obtained from other similar CFD solvers. As a standard turbulence model, Jones-Launder k-ε formulation was investigated for comparison with the temperature corrected turbulence model. The PAB Temperature Correction or “PAB TC” k-ε formulation was built upon the Jones-Launder k-ε model in FICC-BUAA. Details of the equation sets for all of the turbulence models and associated corrections are provided subsequently.

2.1. Standard Turbulence Model

The Jones-Launder k-ε model [13], which is considered the “standard k-ε model”, solves an equation set that for regions away from walls, and the compressibility effect has been taken into account. The k and ε equations are shown as follows:

\[
\frac{D(\rho k)}{Dt} = \tau_{ij} \frac{\partial u_i}{\partial x_j} - \rho \varepsilon + \frac{\partial}{\partial x_j} \left( \left( \frac{\mu + \mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right)
\]  

(1)

\[
\frac{D(\rho \varepsilon)}{Dt} = C_{\varepsilon 1} \frac{\varepsilon}{k} \tau_{ij} \frac{\partial u_i}{\partial x_j} - C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j} \left( \left( \frac{\mu + \mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right)
\]  

(2)

The eddy viscosity is calculated as:

\[
\mu_t = C_\mu \rho \frac{k^2}{\varepsilon}
\]  

(3)

The Sarkar compressibility correction [8] modifies the dissipation rate term in the k-equation (see Eq. (1)) via the expression (4). The ε_s is the solenoidal dissipation rate solved via Eq. (2).

\[
\varepsilon = \varepsilon_s \left( 1 + \alpha M_{a_s}^2 \right)
\]  

(4)

The turbulent Mach number, \(M_{a_s}\), is defined as:

\[
M_{a_s}^2 = 2k / a^2
\]  

(5)

The Jones-Launder model closure coefficients are \(C_\mu = 0.09\), \(\sigma_k = 1.0\), \(\sigma_\varepsilon = 1.3\), \(C_{\varepsilon 1} = 1.44\), \(C_{\varepsilon 2} = 1.92\), and \(\alpha = 1.0\). The turbulent Prandtl number, \(Pr_t = 0.7\). This setting for \(Pr_t\) was used for PAB TC turbulence models described in this section.

2.2. PAB TC Turbulence Model

The PAB Temperature Corrected turbulence model [12] that we refer to as the PAB TC was built upon the Jones-Launder k-ε model. The same equations for k and ε as shown in Eqs. (1) and (2) with corresponding closure coefficients are used here. The correction modifies the coefficient, \(C_\mu\), in Eq. (3) for jet flows with a stagnation temperature gradient. The normalized stagnation temperature gradient is defined as:

\[
T_g = \frac{\nabla T_i}{T_i} \left( \frac{k^{3/2}}{\varepsilon} \right)
\]  

(6)

The coefficient \(C_\mu\) then becomes a function of this stagnation temperature gradient:

\[
C_\mu = 0.09 \left( 1 + \frac{T_g^3}{0.041 + f(M_t)} \right)
\]  

(7)

Where

\[
f(M_t) = (M_t^2 - M_{\infty}^2)H(M_t - M_{\infty})
\]  

(8)
H(x) is the Heaviside step function; and \( f(M_t) = 0 \) for no compressibility correction. We have selected \( M_{\infty} = 0.1 \) for the present model. To avoid too large a value, \( C_p \) was capped to not exceed 5 times the standard value of 0.09 in Ref. 13 and this same restriction was used for the calculations in this paper.

### 3. Results and Discussions

#### 3.1. Potential Core Length for different turbulence models

In order to confirm that the PAB TC model could provide better agreement in high temperature flows compared to the standard turbulence model, a test point from the NASA Glenn Research Center’s Acoustic Reference Nozzle (ARN) database [16] were investigated for the turbulence models described in the previous section. Fig. 1 (a) shows the computational grid near the nozzle exit. Exactly the same grid packing strategy was used in ref. 6. The stagnation temperature and pressure were specified as boundary conditions at the nozzle entrance. The nozzle stagnation pressure was set to 132576Pa, 1.308 times the freestream static pressure. And the stagnation temperature was set equal to 522.99K, 1.915 times the freestream static temperature. The stagnation pressure and temperature set at the inflow of the freestream zone were set to the freestream static values to model the ambient conditions surrounding the jet. In the computations, the second order Roe’s scheme is employed to calculate fluxes at cell faces. The third boundary condition is employed to deal with the coupled problem between flow field/solid temperature fields. The temperature, the pressure and the concentration distribution were calculated by a computational fluid dynamics (CFD) code FICC-BUAA, which solving the three-dimensional Navier-Stokes (N-S) equations by employing FVM coupling with conduction, convection and radiation.

A comparison of centerline axial velocities obtained from the two turbulence modeling approaches is made with experimental data for the heated case in Fig. 1 (b). The Jones-Lauder standard turbulence model produces much longer potential core compared to experiment. In contrast, the PAB TC provides much better agreement with experimental data in the potential core length. The result indicates that the temperature correction for PAB TC produces the correct trend in faster mixing due to jet heating, and provides good agreement with the experimentally measured velocities.

#### 3.2. Exhaust System IR for different turbulence models

One convergent nozzle model with a central cone is calculated by the method presented above to validate this work. The emissivity of the nozzle cavity and the entrance of the nozzle which is assumed as a solid surface is a constant (\( \varepsilon_C = 0.8 \)), the total temperature, the total pressure and the Mach number of nozzle inlet are set to 810K, 11000.0Pa and 0.15, respectively, and the environmental pressure is 101325.0Pa. The total number of spatial control volumes in this simulation is 750,000, and the solid angle for radiation calculation is divided into \( N_\theta \times N_\phi = 12 \times 18 \). Fig. 2 (a) shows a sketch of the nozzle geometry configuration, and Fig. 2 (b) shows a view of the grid for flow field. Fig. 2 (c) is a sparse grid for radiation simulation specially, and Fig. 2 (d) shows a sketch of the infrared radiation detection points.

The comparison of calculating and experiment IR data is shown in Fig. 3. Where \( I_0 \) is experiment result, \( I_1 \) and \( I_2 \) represent the computational values employing Jones-Lauder standard model and PAB TC model, respectively. The value of \( I_1 \) or \( I_2 \) is an integral of the spectral radiation over the 3-5\( \mu \)m band. As shown in Fig. 3, the shape of cavity-plume infrared radiation is like a half ‘pear’, when the zenithal angle is increasingly close to 0°, the value increases quickly at first then decreases slowly. The IR of the cavity and the central cone is the most critical factor for this phenomenon, because the cavity and the central cone emit radiation continuously, and it contributes most part of the total radiation.

Table 1 presents the relative errors between experimental and calculating values, \( e_1 \) and \( e_2 \) are the relative error of \( I_1 \) and \( I_2 \), the expression of them is \( e_{1/2} = (I_{1/2}-I_0)/I_0 \), and \( e_{12} = e_1 - e_2 \). As Table 1 shown, with the increase of the detection angle \( \theta \), \( e_1 \) and \( e_2 \) become larger. The plume radiation share of total IR becomes larger and oppositely the nozzle cavity radiation share gets smaller with the increase of zenithal angle. Hence, the larger the zenithal angle is, the greater the proportion of plume radiation in the infrared radiation. If the jet potential core of the simulation results longer than the experimental data, jet radiation would be greater than the actual value. As mentioned previously, PAB TC model could provide much better agreement in high temperature flows compared to the standard turbulence model. When the detection angle is equal to 90 degree, the share of jet radiation increased to maximum at this point, the maximum error of \( I_2 \) is less than 15\%, but the maximum relative error of \( I_1 \) reaches up to 64.1\%. That implies that the role of turbulence model is crucial when simulating the IR signature of the aircraft exhaust system, especially when simulating the plume IR signature.
Table 1. Error comparison for different turbulence models

<table>
<thead>
<tr>
<th>θ</th>
<th>I₀/W/Sr</th>
<th>I₁/W/Sr</th>
<th>I₂/W/Sr</th>
<th>ε₁/°</th>
<th>ε₂/°</th>
<th>ε₁₂/°</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10.2</td>
<td>11.195</td>
<td>10.217</td>
<td>9.8</td>
<td>0.2</td>
<td>9.6</td>
</tr>
<tr>
<td>10</td>
<td>10.5</td>
<td>11.330</td>
<td>10.283</td>
<td>7.9</td>
<td>-2.1</td>
<td>10.0</td>
</tr>
<tr>
<td>15</td>
<td>9.5</td>
<td>11.296</td>
<td>10.227</td>
<td>18.9</td>
<td>7.6</td>
<td>11.3</td>
</tr>
<tr>
<td>30</td>
<td>8.7</td>
<td>10.188</td>
<td>9.329</td>
<td>17.1</td>
<td>7.2</td>
<td>9.9</td>
</tr>
<tr>
<td>45</td>
<td>6.7</td>
<td>7.978</td>
<td>7.337</td>
<td>19.1</td>
<td>9.5</td>
<td>9.6</td>
</tr>
<tr>
<td>60</td>
<td>4.9</td>
<td>5.965</td>
<td>5.377</td>
<td>21.7</td>
<td>9.7</td>
<td>12.0</td>
</tr>
<tr>
<td>75</td>
<td>2.9</td>
<td>3.985</td>
<td>3.325</td>
<td>37.4</td>
<td>14.7</td>
<td>22.7</td>
</tr>
<tr>
<td>90</td>
<td>1.4</td>
<td>2.298</td>
<td>1.601</td>
<td>64.1</td>
<td>14.3</td>
<td>49.8</td>
</tr>
</tbody>
</table>

4. Conclusions

A code for cavity-plume combination infrared radiation in three-dimensional enclosures based on FVM was developed. The temperature field, the pressure field and the concentration field of exhaust system were calculated by a computational fluid dynamics code FICC-BUAA, which coupled with conduction, convection and radiation. The infrared radiation of exhaust system in the band 3–5μm is obtained by employing Jones-Launder standard model and PAB TC model, respectively. By choosing the total temperature gradient to represent the density gradient, the PAB TC two-equation turbulence model would take into account the spatial instability, and accordingly provide much better agreement in high temperature flows compared to the standard turbulence model. Thus, the simulation precision about exhaust system infrared signatures was improved.

As the detection angle increases, the plume radiation share of total IR becomes larger. When the detection angle is equal to 90 degree, the maximum error of I₂ is less than 15%, but the maximum relative error of I₁ reaches up to 64.1%. That implies that the role of turbulence model is crucial when simulating the IR signature of the aircraft exhaust system, especially when simulating the plume IR signature.

5. Author Artwork

Fig.1. (a) Computational grid for ARN; (b) Centerline velocity decay for different turbulence models

Fig.2. (a) Nozzle geometry configuration; (b) Flow field computational grid; (c) IR computational grid; (d) IR detection points
Fig.3. Comparison of calculating and testing values

References