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Temperature in the throat

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Abstract

We study the temperature of extended objects in string theory. Rotating probe D-branes admit horizons and temperatures a la Unruh effect. We find that the induced metrics on slow rotating probe D1-branes in holographic string solutions including warped Calabi–Yau throats have distinct thermal horizons with characteristic Hawking temperatures even if there is no black hole in the bulk Calabi–Yau. Taking the UV/IR limits of the solution, we show that the world volume black hole nucleation depends on the deformation and the warping of the throat. We find that world volume horizons and temperatures of expected features form not in the regular confining IR region but in the singular nonconfining UV solution. In the conformal limit of the UV, we find horizons and temperatures similar to those on rotating probes in the AdS throat found in the literature. In this case, we also find that activating a background gauge field form the $U(1)$ R-symmetry modifies the induced metric with its temperature describing two different classes of black hole solutions.

1. Introduction

Gauge/gravity duality [1] has been at the frontier of research in string theory over the past decade. According to gauge/gravity duality, the nearby geometry of a stack of N D3-branes on the smooth ten-dimensional Minkowski space, the $AdS_5 \otimes S^5$ geometry, is dual to $\mathcal{N} = 4$ super Yang–Mills theory (SYM). This duality implies that $\mathcal{N} = 4$ SYM is described at high tempera-

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tures by black holes in AdS_5 [2]. The duality also led to the construction of more general and realistic holographic string solutions where some supersymmetry and/or conformal invariance are broken [3–5] (see also [6]). The prototypical example of such holographic string solutions is the warped throat geometry known as the Klebanov–Strassler (KS) throat, [3], produced by placing N regular D3-branes and M fractional D3-branes (wrapped D5-branes) not at a smooth point but at a generic Calabi–Yau singularity, the conifold point. In this setup, the presence of regular and fractional background branes, respectively, reduce the supersymmetry to $\mathcal{N} = 1$ and break conformal invariance, and the singularity is removed by conifold deformation. Finally, these branes dissolve into R–R and NS–NS fluxes producing a fully regular supergravity solution with a large hierarchy, consisting of a warped throat region that smoothly closes off in the IR, where warping becomes constant. The dual gauge theory is $SU(N + M) \times SU(N)$ and includes confinement and chiral symmetry breaking. Although the exact KS solution is noncompact, it is well approximated by a compact solution, [7], where the UV end of the throat is attached to the compact internal Calabi–Yau space. This sets the UV/IR hierarchy of the throat. Far from the tip of the throat, the KS solution is well-approximated by the Klebanov–Tseytlin (KT) solution, [4], where warping is continuous and varies logarithmically. The dual $SU(N + M) \times SU(N)$ gauge theory is chiral, but nonconformal and nonconfining including RG cascade. In the limit where the M fractional branes are removed, the KT solution reduces to the Klebanov–Witten (KW) throat solution, [5], dual to $\mathcal{N} = 1$ $SU(N)$ superconformal gauge field theory. The metrics of these Calabi–Yau throats asymptote AdS space in the UV and are equivalent in the mid throat region. In the IR region, however, the KT and KW solutions are singular at the tip while the KS caps off smoothly. Nonextremal generalizations of these holographic backgrounds have been considered in [8] where it was shown that at sufficiently high temperature the system develops a horizon with a corresponding Hawking temperature (see also [6]).

The extension of holographic backgrounds has also been considered by adding and using probes, [9], to model flavor physics [10,11], and quantum critical phenomena, [12], complementary to other works on charged AdS black holes, [13]. The remarkable property of the probe setup is that the dynamics of probe branes describe the strongly coupled dynamics of a known field theory. Furthermore, it has the advantage that the original supergravity background remains unchanged and the full dynamics is given by the world volume dynamics of the brane which solves easier than the Einstein equations. The other interesting phenomenon that has been studied in such setups is the appearance of horizons and temperatures on the world volume of *accelerating probe* D-branes in the background spacetime [14–21]. The appearance of horizons and temperatures on accelerated D-branes has been analyzed in [14] in general terms. The observation has been that the appearance of horizons and temperatures on accelerating probe branes is in fact the usual *Unruh effect*, [22], not for point particles but for extended objects. The presence of horizons on the world-sheet and on time-dependent D-branes in flat spacetime has been analyzed in [15–18] and [19], respectively. Horizons and temperatures on D-branes from accelerated observers in pure AdS spacetime have been studied in [20]. Moreover, in [21] the appearance of horizons and temperatures has been studied from the world volume dynamics of probe D-branes embedded in holographic backgrounds of the form $AdS_m \times S^n$, dual to defect or flavor conformal field theories. The main result of [21] has been that the induced world volume metrics on rotating probe D-branes in the $AdS_5 \otimes S^5$ solution have thermal horizons with characteristic Hawking temperatures even if there is no black hole in the bulk AdS. In the simplest example of a rotating probe D1-brane, the thermal horizon and Hawking temperature on the world volume of the rotating probe have been found to take a very simple form, proportional to the angular velocity ω, given by $r_H = \omega$ and $T_H = \frac{\omega}{2\pi}$. It has been argued there that these classical solutions describe thermal objects in the dual CFT, including a monopole or a quark at finite temperature T_H. It has also been shown that the application of gauge/gravity duality to such systems reveals thermal properties such as Brownian motion and AC conductivity in the dual field theories.

The aim of this work is to extend such previous analyses and study in detail the world volume horizons and temperatures of rotating probe branes in more general holographic string solutions called warped Calabi–Yau throats, [3–5], where some of the supersymmetry and/or conformal invariance are broken. The motivation is the fact that these holographic conifold backgrounds have distinct metrics and additional fluxes which modify the IR behavior of the AdS spacetime (singular KW, KT, regular KS). The induced world volume metric on the probe in such backgrounds is thereby expected to admit new examples of world volume black hole geometries characterized by world volume horizons and temperatures of distinct features. By gauge/gravity duality, these are expected to describe the temperature in the flavor sector of the corresponding $\mathcal{N}=1$ gauge theory. In addition, we note that in the UV and in the absence of fractional background branes the solution, [5], admits a local $U(1)$ R-symmetry with the associated gauge fields appearing as fluctuations of the metric and other background fields [23–25] (see also [6]). Thus, even in the UV where these string solutions asymptote AdS, the additional background gauge fields, when activated, modify the behavior of the background spacetime. Therefore the induced world volume metric on the probe is again expected to admit new examples of world volume black hole geometries described by world volume horizons and temperatures of distinct features. On the other hand, when the background gauge fields are turned off, one expects to find world volume horizons and temperatures similar to those of rotating probe branes in the $AdS_5 \otimes S^5$ solution, found in [21].

The model we consider consists of a slow moving probe D-brane which is rotating freely around spheres inside warped conifold throats which are at zero temperature. To obtain simple analytic rotating brane solutions from the action, we consider the simplest probe brane, a D1-brane, and take the small radii limit as well as the large radii limit of the full supergravity solution, respectively. The former limit corresponds to the very deep IR region of the KS throat whereas the latter to the UV solutions consisting of the KT and KW throats. The probe D1-brane, also called D-string or F-string, represents a monopole or a quark in the dual $\mathcal{N}=1$ gauge theories. We first solve the world volume dynamics of the slow rotating probe D1-brane in these throat backgrounds and derive the induced world volume metrics. We then derive from the induced world volume metrics the world volume horizons and temperatures.

We find an assertive affirmative result of these investigations. As with [21] considering the $AdS_5 \otimes S^5$ throat, we find that the induced metrics on the rotating probe in conifold throats also admit thermal horizons with characteristic Hawking temperatures even if there is no black hole in the bulk. However, more interestingly we obtain distinct world volume horizons and temperatures and find that world volume black hole nucleation with horizons and temperatures of expected features in conifold throats depends on the warping and the deformation of the throat.

In the very deep IR region of the KS throat where the warping is constant and the solution is regular, whereby the theory is confining and breaking chiral symmetry, we find that the induced metric on the probe is not given by the black hole geometry. We find the obstruction to the induced metric for admitting horizons and temperatures of expected features is due to the confinement or constant warping of the very deep IR. We derive the radius of the 'would be' world volume horizon and obtain a bound on the angular velocities scaling as the glueball masses. We find the induced radius shrinking continuously, instead of growing, with increasing the angular velocities. We also vary the parameters and find this behavior robust against such variations.

In the UV solution, including the KT throat where the warping varies logarithmically and the solution is singular, whereby the theory is nonconfining and chiral, we find, however, distinct world volume horizons and temperatures of expected features. We find the world volume horizons and temperatures increasing/decreasing continuously with increasing/decreasing the angular velocities. This behavior is qualitatively no surprise, though we see it here appearing in the KT and not at the bottom of the KS, as mentioned. However, the special feature we find is that the world volume horizon is described by the 'Lambert transcendental equation', solving to the 'Lambert function'. In our special case of slow rotations, we find that this has a logarithmic series representation, from which we determine the explicit closed form solution giving the world volume horizon. We find that the world volume horizon forms about the KT singularity in the IR and appears away in the UV with increasing the angular velocities. The other, surprising feature we find is that within certain limits in KT the world volume temperatures remain more or less constant despite varying the world volume horizons. Taking the limits of the world volume temperature, as the world volume horizon varies inward the UV/IR regions, we find that due to logarithmic warping the world volume temperature in KT is more or less constant and given in terms of the flux and deformation parameter. Another interesting feature we find within the UV/IR limits of KT is a large separation between world volume temperatures. We find this is due to logarithmic warping by which the flux in the UV can be chosen either large, or very small, unlike in the IR where the flux has to be large, in order to have a valid SUGRA solution. We also inspect the parameter dependence of the solution and find the scale and behavior of the world volume horizons and temperatures in KT subject to certain hierarchies of scales. Taking into account the backreaction of the rotating solution to the KT SUGRA background, we naturally expect the rotating D1-brane to yield a mini black hole in the bulk KT. This indicates that the rotating D1-brane describes a thermal object in the dual field theory whereby our configuration is dual to $\mathcal{N}=1$ gauge theory coupled to a monopole at finite temperature. Since the $\mathcal{N}=1$ gauge theory itself is at zero temperature while the monopole is at finite temperature we find that our configuration describes non-equilibrium steady states.

In the UV solution, including the KW throat, $AdS_5 \otimes T^{1,1}$, where logarithmic warping is removed, whereby the theory is conformal, we find that the induced metric on the probe coincides with that of the BTZ black hole with an angular coordinate suppressed. We find temperatures varying continuously with horizons, similar to those of rotating probes in $AdS_5 \otimes S^5$, [21], as expected. Taking into account the backreaction of the rotating solution to the KW SUGRA background, we naturally expect the rotating D1-brane to yield a mini black hole in the bulk KW. This shows us that the rotating D1-brane describes a thermal object in the dual conformal field theory whereby our configuration is dual to $\mathcal{N}=1$ conformal gauge theory coupled to a monopole at finite temperature. Since the $\mathcal{N}=1$ conformal gauge theory itself is at zero temperature while the monopole is at finite temperature we find that our configuration describes non-equilibrium steady states. In KW we also consider the case where a massless background gauge field is activated in AdS_5 due to $U(1)$ R-symmetry, including fluctuations in the ten-dimensional background metric. We choose a nontrivial gauge and solve the supergravity equation of motion describing the gauge field and obtain a solution of expected form. We find that our solution consists of a rank one massless field in AdS, corresponding to a dimension four operator or current in the gauge theory. This is just what would be expected from an R-current, to which gauge fields correspond. The new interesting feature we find is that in the presence of such a background gauge field the included world volume metric on the probe has thermal horizons and Hawking temperatures of the form and behavior similar to those of AdS–Reissner–Nordström and AdS–Schwarzschild black holes in five dimensions. The special feature we find here is that the Hawking temperature on the probe admits two distinct branches, describing two classes of black hole solutions. We find that there is one branch which, for large horizon size, the temperature goes with the quartic of the horizon. The other branch goes at small horizon size as the inverse cube of the cube of the horizon. These 'small' black holes have the familiar behavior of five-dimensional black holes in asymptotically flat spacetime, as their temperature decreases with increasing horizon size.

Our paper is organized as follows. In Sec. 2 we briefly introduce our general basic setup consisting of the general Type IIB supergravity background and the action of the probe Dp-brane in such backgrounds. In Secs. 3–5 we consider explicit examples of type IIB supergravity backgrounds including the KS, KT and KW backgrounds. We first construct rotating probe D1-brane solutions from the action and derive the induced world volume metrics on the rotating probe D1-brane in these backgrounds. We then derive and analyze the related world volume horizons and temperatures from the induced world volume metrics in these supergravity backgrounds. In Sec. 6 we discuss our results, summarize our findings and conclude with future outlook.

2. General basic setup: IIB theory and Dp-brane action

For our warped supergravity background, we consider the Calabi–Yau flux compactification of type IIB theory containing a warped throat region that smoothly closes off in the IR and is attached to the compact Calabi–Yau space at the UV end [7]. Type IIB string theory contains NS–NS fields, $\{g_{MN}, B_2, \Phi\}$, and R–R forms, $\{C_0, C_2, C_4\}$. The Type IIB action in the Einstein form takes the form

$$S_{\rm IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|g|}\left(\mathcal{R}_{10} - \frac{\partial_M\tau\cdot\partial^M\bar\tau}{2(\mathrm{Im}\tau)^2} - \frac{|G_3|^2}{12\,\mathrm{Im}\tau} - \frac{|\tilde F_5|^2}{4\cdot 5!}\right) + \frac{1}{8i\kappa_{10}^2}\int\frac{1}{\mathrm{Im}(\tau)}C_4\wedge G_3\wedge G_3^* + S_{\rm loc}. \tag{2.1}$$

Here $S_{\rm loc}$ stands for localized contributions from D-brane and orientifold planes; $G_3 = F_3 - \tau H_3$ is the combination of R–R and NS–NS field strengths, given respectively as $F_3 = dC_2$ and $H_3 = dB_2$; $\tau = C_0 + ie^{-i\Phi}$ is the axion–dilaton; $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7\alpha'^4 g_s^2$ is the ten-dimensional gravitational coupling with g_s the string coupling and $\alpha' = l_s^2$ the string scale; $\tilde F_5 = \star_{10}\tilde F_5 = dC_4 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$ is the self-dual five form with $\star_{10}$ being the ten-dimensional Hodge-star operator; $\mathcal{R}_{10}$ is the ten-dimensional Ricci-scalar. The ten-dimensional warped metric and the self-dual five-form read [7]

$$ds_{10}^2 = g_{MN}dX^M dX^N = h^{-1/2}(y)g_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(y)g_{mn}dy^m dy^n, \tag{2.2}$$

$$\tilde F_5 = (1+\star_{10})\Big[d\alpha(y)\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3\Big]. \tag{2.3}$$

Here $\alpha(y)$ is a scalar quantity and $h(y)$ is the warp factor with y^m being the internal coordinates. The Laplace equation and Bianchi identity imply that the localized sources to saturate BPS like identity and, $\alpha = h^{-1}$ and $\star_6 G_3 = iG_3$, which implies constant dilaton $\Phi = 0$. Such a string solution is called imaginary self-dual (ISD).

The action of a Dp-brane is the sum of the DBI and CS actions, which in the above background takes the form

$$S_{Dp} = -g_s T_p\int d^{p+1}\xi\, e^{-\Phi}\sqrt{-\det(\gamma_{ab}+\mathcal{F}_{ab})} + g_s T_p\int\sum_p C_{p+1}\wedge e^{\mathcal{F}_{ab}}. \tag{2.4}$$

Here C_{p+1} are the IIB R–R background fields coupling to the Dp-brane world volume; $\mathcal{F}_{ab} = \mathcal{B}_{ab} + 2\pi\alpha' F_{ab}$ is the gauge invariant field strength with F_2 the world volume gauge field and $\mathcal{B}_2$ the pullback of the NS–NS two-form background field, B_2, onto the world volume, $\mathcal{B}_{ab} = B_{MN}\partial_a X^M \partial_b X^N$, $\gamma_{ab} = g_{MN}\partial_a X^M \partial_b X^N$ the pullback of the ten-dimensional metric g_{MN} in string frame, and $\Phi = 0$. Lastly, ξ^a denote the world volume coordinates and $T_p = [(2\pi)^p g_s(\alpha')^{(p+1)/2}]^{-1}$ is the Dp-brane tension.

In what follows we first consider explicit examples of Calabi–Yau throats with known metric (2.2) and background fields and determine the explicit form of the probe Dp-brane action, (2.4), considering a rotating embedding ansatz for the world volume and assuming that there are no gauge fields on the world volume of the probe brane, $F_{ab} = 0$. We then solve the world volume dynamics from the action, considering slow rotations, and compute the induced metrics on the probe brane, from which we analyze and derive the world volume horizons and temperatures.

3. Temperature in the Klebanov–Strassler throat

3.1. The Klebanov–Strassler solution

As our first example, we take the Klebanov–Strassler (KS) throat geometry [3] (see also [6]), also known as the warped deformed conifold. The deformed conifold is a nonsingular and non-compact Calabi–Yau threefold defined by a hypersurface in $\mathbb{C}^4$

$$\sum_{a=1}^{4} z_a^2 = \epsilon^2, \quad z_a \in \mathbb{C}^4, \tag{3.1}$$

and by a radial coordinate

$$\sum_{a=1}^{4} |z_a|^2 = \epsilon^2 \cosh\eta = r^3. \tag{3.2}$$

Here η is the 'radial' coordinate on the conifold and ϵ is the deformation parameter of the conifold, which can be made real by phase rotation. In the limit $\epsilon \to 0$, Eq. (3.1) gives the singular conifold and describes a cone over a five-dimensional Einstein, base manifold $T^{1,1}$ of topology $S^2 \times S^3$ with both S^2 and S^3 spheres shrinking to zero size at the tip of the cone, $r = 0$. The topology of the base $T^{1,1}$ is parametrized in a standard way by a set of five Euler angles $\{\theta_i, \phi_i, \psi\}$, where $0 \le \theta_i \le \pi$, $0 \le \phi_i \le 2\pi$, $0 \le \psi \le 4\pi$ $(i = 1, 2)$. The $S^2 \times S^3$ topology can be then identified as:

$$S^2: \quad \psi = 0, \quad \theta_1 = \theta_2, \quad \phi_1 = -\phi_2; \qquad \text{and} \qquad S^3: \quad \theta_2 = \phi_2 = 0. \tag{3.3}$$

In the nonsingular limit, at the tip, $\eta \simeq 0$, the S^2 shrinks to zero size while the S^3 remains of finite size with radius $\epsilon^{2/3}$ amounting to the deformation by ϵ, which removes the singularity of the tip. The deformed conifold contains two independent three-cycles: the S^3 at the tip, known as the A-cycle, and the Poincaré dual three-cycle, known as the B-cycle.

The Kähler potential on the deformed conifold derived from (3.2) reads [26]

$$k(\eta) = \frac{\epsilon^{4/3}}{2^{1/3}} \int_0^{\eta} d\eta' [\sinh(2\eta') - 2\eta']^{1/3} \tag{3.4}$$

The metric on the deformed conifold, which is derived from this Kähler potential, takes the form [26,27]

$$ds_6^2 = \frac{1}{2}\epsilon^{4/3} K(\eta)\left[\frac{1}{3K(\eta)^3}\{d\eta^2 + (g^5)^2\} + \cosh^2\frac{\eta}{2}\{(g^3)^2 + (g^4)^2\} + \sinh^2\frac{\eta}{2}\{(g^1)^2 + (g^2)^2\}\right], \tag{3.5}$$

where

$$g^{1,3} = \frac{e^1 \mp e^3}{\sqrt{2}}, \quad g^{2,4} = \frac{e^2 \mp e^4}{\sqrt{2}}, \quad g^5 = e^5 \tag{3.6}$$

with

$$\begin{aligned} &e^1 = -\sin\theta_1 d\phi_1, \quad e^2 = d\theta_1, \quad e^3 = \cos\psi \sin\theta_2 d\phi_2 - \sin\psi d\theta_2, \\ &e^4 = \sin\psi \sin\theta_2 d\phi_2 + \cos\psi d\theta_2, \quad e^5 = d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2, \end{aligned} \tag{3.7}$$

and

$$K(\eta) = \frac{(\sinh(2\eta) - 2\eta)^{1/3}}{2^{1/3}\sinh\eta}. \tag{3.8}$$

In terms of this, the proper radial coordinate which measures the actual distance up the throat in the six-dimensional, internal metric is given by

$$r(\eta) = \frac{\epsilon^{2/3}}{\sqrt{6}}\int_0^\eta \frac{dx}{K(x)}. \tag{3.9}$$

The warping in this background is produced by placing N D3-branes, sourcing the self-dual R–R five-form field strength $\tilde{F}_5$, and M D5-branes wrapping a vanishing two-cycle given by the collapsing S^2 sphere, sourcing R–R three-form field strength F_3, on the conifold geometry. There is also a nontrivial NS–NS three-form field, H_3, and the R–R zero-form, C_0, and the dilaton field, Φ, vanish on this background. The background three-form fluxes of the KS solution are quantized

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_3 = M, \quad \frac{1}{(2\pi)^2\alpha'}\int_B H_3 = -K, \tag{3.10}$$

where $M \gg 1$ and $K \gg 1$ denote integers and the total D3-brane charge is $N = MK$. Due to the presence of these IIB fluxes, a large hierarchy of scales can be stabilized, $\epsilon^{1/3} \sim \exp(-2\pi K/3g_s M)$, [7], and there is a backreaction on the geometry, producing the ten-dimensional warped line element [3] (see also [6])

$$ds_{10}^2 = h^{-1/2}(\eta) ds_4^2 + h^{1/2}(\eta) ds_6^2. \tag{3.11}$$

Here $h(\eta)$ is the warp factor, ds_4^2 is the usual, four-dimensional Minkowski spacetime metric, and the internal metric ds_6^2, Eq. (3.5), is a strongly warped and deformed throat, which interpolates between a regular $\mathbb{R}^3 \times S^3$ tip in the IR, to an $\mathbb{R} \times T^{1,1}$ cone in the UV. At small r one has:

$$r \sim \frac{\epsilon^{2/3}}{3^{1/6} \cdot 2^{5/6}}, \quad K \simeq \left(\frac{2}{3}\right)^{1/3}, \tag{3.12}$$

where the internal metric (3.5) along the proper distance (3.9) smoothly rounds off with a finite S^3 of radius $\epsilon^{2/3}/(12)^{1/6}$. Inspection of the metric (3.5) in terms of the proper radial coordinate (3.9) shows that at large r, or for $\eta \simeq 10-15$, the throat explicitly takes the form of a cone $\mathbb{R}\times T^{1,1}$. Hence $\epsilon^{2/3}$ gives the radius of the nonsingular S^3 at the bottom of the throat, and the scale at which the throat asymptotes the $T^{1,1}$ cone: It sets the IR scale of the geometry. Like in the deformed conifold, the IR geometry is smooth and the A-cycle is finite in size, with radius $r_A=\sqrt{g_sM\alpha'}$, so the supergravity approximation remains valid near the tip provided that $g_sM \gg 1$. The other background fields are [3] (see also [6])

$$B_2=\frac{g_sM\alpha'}{2}\big[f(\eta)g^1\wedge g^2+k(\eta)g^3\wedge g^4\big], \tag{3.13}$$

$$H_3=\frac{g_sM\alpha'}{2}\big[d\eta\wedge(f'g^1\wedge g^2+k'g^3\wedge g^4)+\frac{1}{2}(k-f)g^5\wedge(g^1\wedge g^3+g^2\wedge g^4)\big], \tag{3.14}$$

$$C_2=\frac{M\alpha'}{2}\Big[\frac{\psi}{2}(g^1\wedge g^2+g^3\wedge g^4)+(1/2-F)(g^1\wedge g^3+g^2\wedge g^4) - \cos\theta_1\cos\theta_2\,d\phi_1\wedge d\phi_2\Big], \tag{3.15}$$

$$F_3=\frac{M\alpha'}{2}\big[g^5\wedge g^3\wedge g^4(1-F)+g^5\wedge g^1\wedge g^2F+F'd\eta\wedge(g^1\wedge g^3+g^2\wedge g^4)\big], \tag{3.16}$$

$$C_4=g_s^{-1}h^{-1}\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3, \tag{3.17}$$

$$\tilde{F}_5=\mathcal{F}_5+\star\mathcal{F}_5=B_2\wedge F_3+dC_4. \tag{3.18}$$

Here we note that $F_3=dC_2$ and $H_3=dB_2$, and the explicit form of the functions appearing above is [3] (see also [6]):

$$F(\eta)=\frac{\sinh\eta-\eta}{2\sinh\eta}, \tag{3.19}$$

$$f(\eta)=\frac{\eta\cosh\eta-1}{2\sinh\eta}(\cosh\eta-1), \tag{3.20}$$

$$k(\eta)=\frac{\eta\cosh\eta-1}{2\sinh\eta}(\cosh\eta+1), \tag{3.21}$$

$$h(\eta)=2^{2/3}(g_sM\alpha')^2\epsilon^{-8/3}I(\eta), \tag{3.22}$$

$$I(\eta)=\int_\eta^\infty dx\,\frac{x\cosh x-1}{\sinh^2 x}(\sinh x\cosh x-x)^{1/3}. \tag{3.23}$$

The integral (3.23) cannot be computed analytically, but one can readily find the two important limits of the solution for $\eta\to 0$ and $\eta\to\infty$ [3] ([6]). As discussed above, near the bottom of the throat (IR), the internal metric takes the form of an S^3 of finite radius whereas far from the bottom of the throat (UV), the metric takes the form of the KT solution, asymptoting AdS spacetime (see sections 4 & 5). The related limits of (3.23) are [3] (see also [6]):

$$I(\eta\to 0)\to a_0+\mathcal{O}(\eta^2)\ \text{ with }\ a_0\approx 0.71805, \tag{3.24}$$

$$I(\eta\to\infty)\to 3\cdot 2^{-1/3}(\eta-1/4)e^{-4\eta/3}. \tag{3.25}$$

In the next sections, we will consider the limits (3.24)–(3.25), in order to obtain simple analytic rotating brane solutions, which compute the induced world volume metrics, and world volume horizons and temperatures, in turn.

3.2. Induced metric and Hawking temperature in the KS throat

We now solve the world volume dynamics of a rotating probe D1-brane in the KS throat and derive the induced world volume metric from which we obtain the world volume horizon and temperature in turn. In order to obtain simple analytic rotating D1-brane solutions consistent with the IR limit of the supergravity solution, in this section we will consider the very small radii limit near the bottom of the throat, corresponding to the very deep IR limit of the KS solution. In this limit, (3.23) is given by (3.24), and accordingly, the warp factor, (3.22), is constant, and given by $h_0 = a_0(g_s M\alpha')^2\, 2^{2/3}\epsilon^{-8/3}$.[1] Hence the ten-dimensional background metric (3.5) takes the form [3] (see also [6])

$$ds_{10}^2 \to \frac{\epsilon^{4/3}}{(2)^{1/3} a_0^{1/2}(g_s M\alpha')}(dx^2 - dt^2) + a_0^{1/2}\, 6^{-1/3}(g_s M\alpha')\left\{\frac{1}{2}d\eta^2 + \frac{1}{2}(g^5)^2 + (g^3)^2 + (g^4) + \frac{\eta^2}{4}[(g^1)^2 + (g^2)^2]\right\}. \tag{3.26}$$

We also note that near the bottom of the throat the S^2 sphere shrinks to zero size while the S^3 sphere remains finite, as discussed. To obtain the explicit form of the metric near the bottom of the throat, we may therefore consider an S^3 round in (3.3). Since we are interested in rotating solutions in the background (3.26), we may also fix $\theta_1 = \theta = \pi/2$ (which together with (3.3) imply $g^{2,4} = 0$). The full background metric then reads

$$ds_{10}^2 \to \frac{\epsilon^{4/3}}{(2)^{1/3} a_0^{1/2}(g_s M\alpha')}(dx^2 - dt^2) + (2^{-1}\, a_0^{1/2}\, 6^{-1/3})(g_s M\alpha')\left\{d\eta^2 + d\psi^2 + B(\eta)d\phi^2\right\}, \tag{3.27}$$

where $B(\eta) = 1 + \eta^2/4$. We also note that in this background, (3.27), one has, according to (3.19), $F \simeq 1/2$, and so from (3.15) and (3.13) one infers that in the background (3.27) (where $g^{2,4} = 0$) C_2 and B_2 are locally vanishing.

To evaluate the action of the probe D1-brane in the background (3.27), we need to specify our embedding ansatz. We consider the probe D1-brane extending in both t- and η-directions, while localized at $x = 0$. Furthermore, take it spinning in the ψ- and/or ϕ-directions. Thus the D1-brane world volume is specified by $\phi(\eta, t)$ and/or $\psi(\eta, t)$. To evaluate (2.4) in the background (3.27) for this embedding ansatz, it suffices to consider the simplest single field case, setting either $\phi(\eta, t)$ or $\psi(\eta, t)$ constant. Setting $\psi = const.$, gives the action of the rotating D1-brane in the form:

$$S_{\mathrm{D1}} = -\frac{g_s T_{D1}\,\epsilon^{4/3}}{2(12)^{1/3}}\int dt\, d\eta\sqrt{1 + B(\eta)(\phi')^2 - \frac{a_0(g_s M\alpha')^2 B(\eta)}{(24\epsilon^4)^{1/3}}\dot{\phi}^2}. \tag{3.28}$$

[1] In the next sections, we will consider the very large radii limit where the explicit form of the warp factor (3.22) is determined by (3.25).

Here we note that in the limit of small velocities the higher-order non-canonical kinetic terms in (3.28) can be dropped (after Taylor expansion). The Lagrangian and equation of motion for the slow rotating D1-brane then take the form:

$$L \equiv 1 + \frac{1}{2} B(\eta)(\phi')^2 - \frac{a_0 (g_s M\alpha')^2 \, B(\eta)}{2(24\epsilon^4)^{1/3}} \dot{\phi}^2, \tag{3.29}$$

$$\frac{\partial}{\partial \eta}\left[B(\eta)\phi'(\eta,t)\right] = \frac{a_0 (g_s M\alpha')^2}{(24\epsilon^4)^{1/3}} \frac{\partial}{\partial t}\left[B(\eta)\dot{\phi}(\eta,t)\right]. \tag{3.30}$$

Now, consider the rotating solution of the form

$$\phi(\eta,t) = \omega t - 2\tan^{-1}\left(\frac{\eta}{2}\right). \tag{3.31}$$

Put these into the background and obtain the induced metric on the D1-brane as

$$\begin{aligned} ds^2_{ind} = &- \frac{a_0^{1/2}(g_s M\alpha')\,\Omega(\eta)}{2\cdot 6^{1/3}} dt^2 + \frac{a_0^{1/2}(g_s M\alpha')}{2\cdot 6^{1/3} B(\eta)} (1 + B(\eta))\, d\eta^2 \\ &+ \frac{a_0^{1/2}(g_s M\alpha')\omega}{2\cdot 6^{1/3}} dt d\eta, \quad \text{with} \quad \Omega(\eta) = \frac{(24\,\epsilon^4)^{1/3}}{a_0 (g_s M\alpha')^2} - B(\eta)\omega^2. \end{aligned} \tag{3.32}$$

To eliminate the cross term in this metric, introduce a new coordinate

$$\tau = t - \frac{a_0^{1/2}(g_s M\alpha')\omega}{2\cdot 6^{1/3}} \int \frac{d\eta}{\Omega(\eta)}. \tag{3.33}$$

The metric (3.32) then becomes

$$\begin{aligned} ds^2_{ind} = &- \frac{a_0^{1/2}(g_s M\alpha')\Omega(\eta)}{2\cdot 6^{1/3}} d\tau^2 \\ &+ \frac{a_0^{1/2}(g_s M\alpha')}{2\cdot 6^{1/3} B(\eta)\Omega(\eta)} \left\{ (1 + B(\eta)) \left[\frac{a_0^{1/2}(g_s M\alpha')\Omega(\eta)}{2\cdot 6^{1/3}} \right] + B(\eta)\omega^2 \right\} d\eta^2. \end{aligned} \tag{3.34}$$

One can go on and find the location of the horizon in the induced metric by the usual methods, setting $g^{\eta\eta} = 0$. However, as we will show in the following, some unexpected features appear in this case. First note that this equation has no solutions for η if both angular momenta are zero. Therefore with no rotations there will be no world volume horizon as expected. Next consider the case with $\omega \neq 0$. One can see that in the limit of small ω, the horizon moves away from the bottom of the throat. As ω increases, the horizon moves towards the bottom and in the limit of large ω it will eventually hit the very bottom of the throat.

This tells us that the world volume black hole is likely to nucleate away from the bottom of the throat with an horizon that grows by increasing the angular momentum. The horizon grows in two directions, one away from the bottom of throat and one towards it. The former part is outside the regime of validity of the metric we have considered in this section and the latter is described by the induced metric above. The feature just described predicts a specific value for angular momentum for which the horizon reaches the bottom of the throat. When this happens, the world volume of the brane that is rotating in the IR throat region is completely inside the world volume black hole.

All of this can be seen by studying the following equation

$$g^{\eta\eta}(\eta_0) = 0 \to \eta_0 = \left|\frac{2}{\omega}\right| \sqrt{\frac{(24\,\epsilon^4)^{1/3}}{a_0(g_s M\alpha')^2} - \omega^2} \tag{3.35}$$

This relation already shows the upper bound on the angular velocities

$$\omega^2 < \frac{(24\,\epsilon^4)^{1/3}}{a_0(g_s M\alpha')^2}. \tag{3.36}$$

One should have in mind that, as explained above, the bound on ω is due to the maximum radius of horizon in the IR region before hitting the bottom of throat. It is also interesting to note that according to our bound (3.36) the angular velocity scales less than or equal to the mass of glueball and Kaluza–Klein states, given by $m^2_{\text{glueball}} \sim m^2_{\text{KK}} \sim \epsilon^{2/3}/(g_s M\alpha')$ [6]. In the special case where the angular velocities scale as glueball masses and the inequality is saturated, one has $\eta_0 = 0$. This occurs precisely at the IR location where the radial coordinate (3.2), corresponding to $\eta_0 = 0$, is given by the radius of the finite S^3, $r_0 = \epsilon^{2/3}$. In this special case one may be inclined to conclude that the world volume temperature is identically zero, and equals the background temperature. However, in general and as discussed above, Eq. (3.35) does not describe the world volume horizon at the bottom of the throat since by Eq. (3.35) η_0 has no consistent and continuous limits with varying the angular velocity. Note that if $\omega \lesssim m_{\text{glueball}}$ then $\eta_0 > 0$ and $r_0 \lesssim \epsilon^{2/3}$, which is less than the minimum radius of the nonvanishing S^3 at the bottom of the throat. If $\omega \ll m_{\text{glueball}}$ then $\eta_0 \to$ large, giving a very large r_0, outside the validity rage of the IR metric. It is also straightforward to consider the alternative embedding ansatz for the rotating D1-brane, where the worldvolume dynamics is described by ψ instead of by ϕ, and show that the induced metric on the rotating D1-brane will never describe a world volume black hole. This is due to the fact that in the IR region of the throat, the ψ angle will have a constant (η independent) warp factor. However, once the linear velocity in the ψ direction reaches that of speed of light the metric degenerates as expected. These features of the world volume theory are likely due to the fact that in the metric (3.26) for small radii the warp factor approaches constant, unlike the case for large radii where the KS is well approximated by KT.

It is also instructive to inspect the parameter dependence of η_0 and T_0 by looking at their behavior as a function of ω for different values of ϵ and $g_s M$. Inspection of (3.35) and T_0 (see footnote 2) shows that varying ϵ, while keeping $g_s M$ fixed, only shifts the value of ω at which η_0 and T_0 hit its minimum value at the bottom of the throat (see Fig. 1 [upper panel]). The same behavior appears when doing the opposite; when varying $g_s M$, while keeping ϵ fixed (see Fig. 1 [lower panel]). From both cases it is clear that the behavior of η_0 and T_0 are quite robust against varying the parameters of the theory, as changing these only changes the rate of decrease in η_0 and T_0 with ω, but not their overall, decreasing behavior with increasing ω.

The upshot of this section is that all nontrivial features of the world volume metric that arise because of accelerating the brane, especially for slow rotations, seem to occur away from the bottom of the throat which can be explained, for instance, by studying the KT throat. This will be the subject of next section.[2]

[2] The nucleation of black hole away from the bottom of the throat and appearance of an horizon that approaches the bottom of throat for larger rotations can also be seen by a naive calculation of the horizon temperature. If one *wrongly* interprets η_0 as appearing in (3.35) as the radius of horizon, a standard calculation of the horizon temperature gives

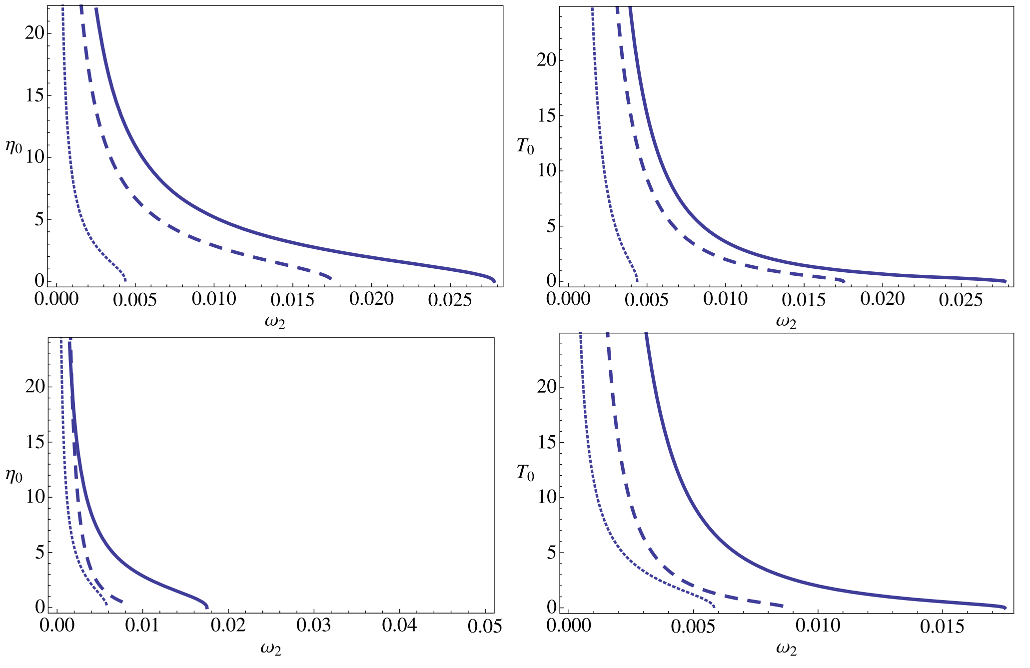

Fig. 1. [Upper panel] The behavior of η_0 and T_0 for $\epsilon = 0.1$ (dotted), $\epsilon = 0.8$ (dashed), $\epsilon = 1.6$ (solid). [Lower panel] The behavior of η_0 and T_0 for $\omega_1 = 10^{-5}$, $\epsilon = 0.8$, $L^2 = 3 \times 10^2$ (dotted), $L^2 = 2 \times 10^2$ (dashed), $L^2 = 10^2$ (solid).

4. Temperature in the Klebanov–Tseytlin throat

4.1. The Klebanov–Tseytlin solution

Far from the tip of the cone, where η is large, the deformation of the conifold can be neglected and Eq. (3.1) reduces the constraint equation of the singular conifold

$$\sum_{a=1}^{4} z_a^2 = 0. \tag{4.1}$$

In this limit, one may introduce another radial coordinate r through

$$r^2 = \frac{3}{2^{5/3}} \epsilon^{4/3} \exp(2\eta/3). \tag{4.2}$$

$$T_0 = \frac{(g^{\eta\eta})'}{4\pi}\bigg|_{\eta=\eta_0} = \frac{6^{1/3}}{2\pi a_0^{1/2} (g_s M\alpha')\,\omega} \sqrt{\frac{(24\epsilon^4)^{1/3}}{a_0 (g_s M\alpha')^2} - \omega^2}.$$

This predicts that for larger rotations we will have smaller temperatures which may seem inconsistent, when compared with the temperature of typical rotating black hole geometries. However, in other interesting study in the literature, ref. [28], using similar D-brane systems, it has been shown that when an electric field is turned on (in place of rotation, or R-charge, considered here), in certain codimensions, the induced world volume temperature of the probe is given by a decreasing function of the electric field (see Eq. (24) in ref. [28]).

The Kähler potential is then given by [26]

$$k = \frac{3}{2}\left(\sum_{a=1}^{4}|z_a|^2\right)^{2/3} = \frac{3}{2}r^2 = \hat{r}^2. \tag{4.3}$$

The metric on the singular conifold, which is derived from this Kähler potential, takes the form [26]

$$ds_6^2 = d\hat{r}^2 + \hat{r}^2 ds_{T^{1,1}} \quad \text{with} \quad ds^2_{T^{1,1}} = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2, \tag{4.4}$$

where g^i's are given by Eqs. (3.6)–(3.7). Far from the bottom of the throat (tip of the cone), where the deformation parameter can be neglected, by which Eq. (3.1) reduces to Eq. (4.1), the KS throat solution joins the Klebanov–Tseytlin (KT) throat solution [4] (see also [6]). In this limit, the ten-dimensional warped line element is that of the KT throat and takes the form [4] (see also [6])

$$ds_{10}^2 = h^{-1/2}ds_4^2 + h^{1/2}(d\hat{r}^2 + \hat{r}^2 ds^2_{T^{1,1}}), \tag{4.5}$$

where $ds^2_{T^{1,1}}$ is given by (4.4). The warp factor, h, and other background fields take the form [4] (see also [6])

$$\begin{aligned}
B_2 &= \frac{3g_s M\alpha'}{4}\left[\ln\frac{\hat{r}}{\hat{r}_{\rm UV}}\right](g^1\wedge g^2 + g^3\wedge g^4), && (4.6)\\
H_3 &= dB_2 = \frac{3g_s M\alpha'}{4\hat{r}}d\hat{r}\wedge(g^1\wedge g^2 + g^3\wedge g^4), && (4.7)\\
C_2 &\to \frac{M\alpha'\psi}{2}(g^1\wedge g^2 + g^3\wedge g^4), && (4.8)\\
F_3 &= dC_2 = \frac{M\alpha'}{4}g^5\wedge(g^1\wedge g^2 + g^3\wedge g^4), && (4.9)\\
\tilde{F}_5 &= \mathcal{F}_5 + \star\mathcal{F}_5, && (4.10)\\
\star\mathcal{F}_5 &= dC_4 = g_s^{-1}d(h^{-1}(\hat{r}))\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3, && (4.11)\\
\mathcal{F}_5 &= B_2\wedge F_3 = 27\pi(\alpha')^2 N_e \mathrm{Vol}(T^{1,1}), && (4.12)\\
N_e &= N + \frac{3(g_s M)^2}{2\pi}\ln\frac{\hat{r}}{\hat{r}_{\rm UV}}, && (4.13)\\
h(\hat{r}) &= \frac{27\pi\alpha'^2}{4\hat{r}^4}\left[g_s N + \frac{3(g_s M)^2}{2\pi}\left(\ln\frac{\hat{r}}{\hat{r}_{\rm UV}} + \frac{1}{4}\right)\right]. && (4.14)
\end{aligned}$$

4.2. Induced metric and Hawking temperature in the KT throat

We now solve the world volume dynamics of a slow rotating probe D1-brane well above the IR limit the KS throat and derive the induced world volume metric from which we obtain the world volume horizon and temperature in turn. We obtain simple analytic rotating D1-brane solutions by considering the very large radii limit, (3.25), away the bottom of the throat, yet well inside the throat, $\epsilon^{2/3} \ll \hat{r} \ll \hat{r}_{\rm UV}$, where the KS throat is well approximated by the UV solution, consisting of the KT solution. Here we also note that according to (4.2) for $\eta_{\rm UV} \simeq 10$–15 [3] one has $\hat{r}_{\rm UV} \simeq 10^2\,\epsilon^{2/3}$, which sets the maximum radial distance for the throat, the UV scale of the geometry, where the throat is attached to the compact Calabi–Yau space. In the large-η radii limit, (3.23) is given by (3.25), and, by (4.2), we may write the warp factor, (4.14), in the form [4] (see also [6]):

$$h(\hat{r}) = \frac{L^4}{\hat{r}^4}\ln(\hat{r}/\epsilon^{2/3}), \qquad L^4 = \frac{81(g_s M\alpha')^2}{8}, \tag{4.15}$$

giving the ten-dimensional background metric (4.5) of the form [4] (see also [6])

$$ds^2 = \frac{\hat{r}^2}{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}(dx^2 - dt^2) + \frac{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}{\hat{r}^2}d\hat{r}^2 + \frac{L^2}{\hat{r}^2}\sqrt{\ln(\hat{r}/\epsilon^{2/3})}ds^2_{T^{1,1}}. \tag{4.16}$$

Note that due to logarithmic dependence this metric becomes singular at $\hat{r} = \epsilon^{2/3}$. Also, note that because $T^{1,1}$ expands slowly toward large $\hat{r}$, the curvatures decrease there so that corrections to the supergravity solution become negligible. Thus even when $g_s M$ is very small, the supergravity solution considered here is reliable for sufficiently large radii where $g_s N_e \gg 1$ (see Eq. (4.13)) [4] (see also [6]).

Considering the same S^3 cycle as in the previous section, we obtain this full background metric of the form

$$ds^2_{10} = \frac{\hat{r}^2}{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}(dx^2 - dt^2) + \frac{L^2\sqrt{\ln(\hat{r}/\epsilon^{2/3})}}{\hat{r}^2}\left(d\hat{r}^2 + \frac{\hat{r}^2}{6}d\phi^2 + \frac{\hat{r}^2}{9}d\psi^2\right). \tag{4.17}$$

We also note that in the background (4.17), from (4.8) and (4.6) one infers that in the background (4.17) (where $g^{2,4} = 0$) C_2 and B_2 are locally vanishing.

To evaluate the action of the probe D1-brane (4.17), we consider the same embedding ansatz as before, but now working with the radial coordinate $\hat{r}$. Thus the D1-brane world volume is again specified by $\phi(\hat{r}, t)$ and/or $\psi(\hat{r}, t)$. To evaluate (2.4) in the background (4.17) for the embedding ansatz, it suffices to set either ϕ or ψ constant. Setting $\phi = const.$, gives the action of the rotating D1-brane in the form:

$$S_{\rm D1} = -g_s T_{D1}\int dt\, d\hat{r}\sqrt{1 + \frac{\hat{r}^2(\psi')^2}{9} - \frac{L^4}{9\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\psi}^2}. \tag{4.18}$$

As before, we note that in the limit of small velocities the higher-order non-canonical kinetic terms in (4.18) can be dropped (after Taylor expansion). The Lagrangian and equation of motion for the slow rotating D1-brane then take the form:

$$L = 1 + \frac{\hat{r}^2(\psi')^2}{18} - \frac{L^4}{18\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\psi}^2, \tag{4.19}$$

$$\frac{\partial}{\partial\hat{r}}\left[\frac{\hat{r}^2\psi'(\hat{r}, t)}{9}\right] = \frac{\partial}{\partial t}\left[\frac{L^4}{9\hat{r}^2}\ln\left(\hat{r}/\epsilon^{2/3}\right)\dot{\psi}(\hat{r}, t)\right]. \tag{4.20}$$

[3] See also Section 3, the discussion below (3.12).

Consider the simple rotating solution as

$$\psi(\hat{r},t)=\omega t-\frac{\omega}{\hat{r}}+\psi_0. \tag{4.21}$$

Putting this into the background, gives the induced metric on the brane as

$$\begin{aligned} ds^2_{ind}=&-\frac{\left[\hat{r}^2-L^4\ln(\hat{r}/\epsilon^{2/3})\overline{\omega}^2\right]}{\sqrt{L^4\ln\left(\hat{r}/\epsilon^{2/3}\right)}}dt^2+\sqrt{L^4\ln\left(\hat{r}/\epsilon^{2/3}\right)}\left(\frac{1}{\hat{r}^2}+\frac{\overline{\omega}^2}{\hat{r}^4}\right)d\hat{r}^2\\ &+\frac{2\overline{\omega}^2}{\hat{r}^2}\sqrt{L^4\ln\left(\hat{r}/\epsilon^{2/3}\right)}dt\,d\hat{r},\quad \text{with}\quad \overline{\omega}^2=\frac{\omega^2}{9}. \end{aligned} \tag{4.22}$$

To eliminate the cross term in this metric, we may consider a coordinate transformation of the form

$$\tau=t-\overline{\omega}^2\int\frac{d\hat{r}}{\hat{r}^2(\hat{r}^2-L^4\ln(\hat{r}/\epsilon^{2/3})\overline{\omega}^2)}. \tag{4.23}$$

The induced metric (4.22) then takes the form

$$\begin{aligned} ds^2_{ind}=&-\frac{\left[\hat{r}^2-L^4\ln(\hat{r}/\epsilon^{2/3})\overline{\omega}^2\right]}{\sqrt{L^4\ln\left(\hat{r}/\epsilon^{2/3}\right)}}d\tau^2\\ &+\sqrt{L^4\ln\left(\hat{r}/\epsilon^{2/3}\right)}\left[\frac{\overline{\omega}^2+\hat{r}^2-L^4\ln(\hat{r}/\epsilon^{2/3})\,\overline{\omega}^2}{\hat{r}^2(\hat{r}^2-L^4\ln(\hat{r}/\epsilon^{2/3})\,\overline{\omega}^2)}\right]d\hat{r}^2. \end{aligned} \tag{4.24}$$

Here we note that, up to ln-dependence, the metric (4.24) has the form given by the BTZ black hole with the angular coordinate suppressed (see next sec. Eq. (5.14)).

To obtain the horizon, we set from this metric $g^{\hat{r}\hat{r}}=0$, which gives:

$$g=\hat{r}_H^2-L^4\overline{\omega}^2\ln(\hat{r}_H/\epsilon^{2/3})=0. \tag{4.25}$$

This equation can have at most two (real positive) zeros. The value and number of these zeros depends on the value of the conserved charge. Clearly, at $\overline{\omega}=0$ no zero of (4.25) appears and there will be no horizon. Now let $\overline{\omega}_c$ be the minimum, critical value of conserved charge for which at least one zero of (4.25) appears. Inspection of (4.25) shows that for $\overline{\omega}_c\simeq\epsilon^{2/3}/L^2$, the equation (4.25) will have one solution in the IR region, close to the tip of the throat and near the singularity of KT, $\hat{r}_c\simeq\epsilon^{2/3}$. We can also see this more directly by obtaining an explicit solution of Eq. (4.25). By expanding around $\overline{\omega}=0$ and consider the leading terms, we obtain the solution[4]:

$$\hat{r}_{\mathrm{H}}=L^2\overline{\omega}\sqrt{\left|\ln(\sqrt{2}\,\epsilon^{2/3}/L^2\overline{\omega})\right|}. \tag{4.26}$$

Here again it is clear that for $\overline{\omega}_c\simeq\epsilon^{2/3}/L^2$ we get $\hat{r}_c\simeq\epsilon^{2/3}$. Now, increase $\overline{\omega}$. For any value of conserved charge $\overline{\omega}>\overline{\omega}_c$, we will have two solutions for (4.25), one smaller than $\hat{r}_c$ and

[4] Here we note that Eq. (4.25) can be rearranged into a Lambert's transcendental equation of the form $\ln\hat{X}_H=(\epsilon^{4/3}/L^4\overline{\omega}^2)\hat{X}_H^2$ whose solution is given in terms of Lambert W-function as $\hat{X}_H=\exp[-W(-2\epsilon^{4/3}/L^4\overline{\omega}^2)/2]$. The expansion of the Lambert W-function $W(X)$ about $X=\infty$, or $\overline{\omega}=0$, is $W(X)=-\ln[1/X]-\ln[-\ln[1/X]]-\ln[-\ln[1/X]]/\ln[1/X]^2-\cdots$. Considering the first two leading terms in this expansion and replacing X by the argument of the exponent of $\hat{X}_H$ one obtains (4.26), after rearranging back.

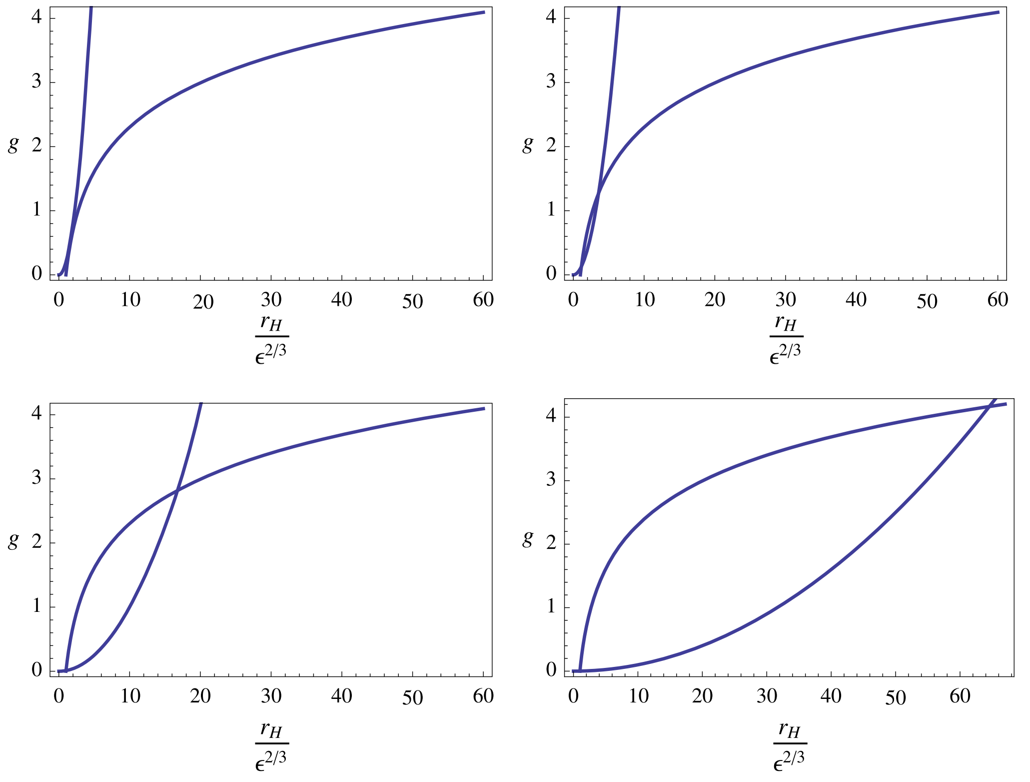

Fig. 2. [Upper panel] The plots of g for $\overline{\omega}^2 = \epsilon^{4/3}/L^4$ (left), $\overline{\omega}^2 = 10\epsilon^{4/3}/L^4$ (right). [Lower panel] The plots of g for $\overline{\omega}^2 = 50\epsilon^{4/3}/L^4$ (left), $\overline{\omega}^2 = 100\epsilon^{4/3}/L^4$ (right). The zeros of g, given by the intersecting points, represent the location of the world volume horizon.

one larger, denoted by $\hat{r}_<$ and $\hat{r}_>$ respectively. Increasing the value of $\overline{\omega}$ continuously, further decreases (increases) the value of $\hat{r}_<$ ($\hat{r}_>$). One may first be tempted to interpret this result as the appearance of a double horizon. However, one should keep in mind that the zero at $\hat{r}_<$ is located about the KT singularity away from the UV. The conclusion of this analysis is that by rotating the brane inside the throat, a world volume black hole nucleates around the KT singularity and by increasing the angular momentum the horizon grows in size. For rotations $\overline{\omega} \simeq (10^2\text{–}10^3)^{1/2}\,\epsilon^{2/3}/L^2$, the horizon approaches the UV region, far from the tip and the KT singularity, and will be of the size of the UV scale of the geometry, $\hat{r}_H \to 10^2\epsilon^{2/3}$ (see Fig. 2).

To obtain the Hawking temperature, we Wick-rotate τ into a Euclidean time, and after a straightforward calculation we get:

$$T_{\rm H} = \frac{(g^{\hat{r}\hat{r}})'}{4\pi}\bigg|_{\hat{r}=\hat{r}_{\rm H}} = \frac{\hat{r}_{\rm H}(2\hat{r}_H^2 - L^4\overline{\omega}^2)}{4\pi(\overline{\omega}L)^2\sqrt{\ln\left(\hat{r}_{\rm H}/\epsilon^{2/3}\right)}}, \tag{4.27}$$

where $\hat{r}_H$ is the horizon discussed above. Inspection of (4.27) shows that the temperature of the black hole solution on the probe, T_H, is always real, positive definite and finite. This is because within intermediate scales $\hat{r}_H$ neither hits the KT singularity in the IR nor extends beyond the

UV cutoff, with the temperature (4.27) scaling roughly as $T_H \gtrsim L^2\,\epsilon^{2/3}$ in the UV limit where $\hat{r}_H \to 10^2\epsilon^{2/3}$, and as $T_H \lesssim L^2\,\epsilon^{2/3}$ in the IR limit where $\hat{r}_H \to \epsilon^{2/3}$. This means that away from the mid throat region, where $\hat{r}_H$ approaches the UV/IR ends of the throat, the temperature of the probe, T_H, is uniformly continuous and more or less constant. One can also see that T_H increases/decreases continuously with expanding/shrinking $\hat{r}_H$, as expected.

If we consider the backreaction of the above solution to the KT supergravity background, it is natural to expect such D1-branes to form mini black holes in the bulk KT. This indicates that the rotating D1-brane describes thermal object with temperature T_H in the dual field theory. The configuration is dual to $\mathcal{N}=1$ gauge theory coupled to a monopole at finite temperature. The $\mathcal{N}=1$ gauge theory is itself at zero temperature while the monopole is at finite temperature T_H. Thus such configurations are in non-equilibrium steady states.

The acceptable values of T_H follow from the validity range for $\hat{r}_H$. In addition, since the KT supergravity solution is reliable for sufficiently large radii even when $g_sM \ll 1$, the temperature scales smaller, or larger, depending on whether $g_sM \gg 1$ (making $L^2 \gg 1$), or $g_sM \ll 1$ (making $L^2 \ll 1$) is considered. In particular, since the KT solution is singular and consistent with very small deformations ($\epsilon \ll 1$), if $g_sM \ll 1$ then $T_H \gtrsim L^2\epsilon^{2/3}$ is vanishingly small. Conversely, if $g_sM \gg 1$ then T_H is finitely (nonvanishingly) small, when g_sM is large enough. It is conclusive then that the size of T_H in KT at sufficiently large radii, in the far UV limit, depends on the choice of flux. On the other hand, in the IR limit, we may only consider $g_sM \gg 1$, in which case $T_H \lesssim L^2\,\epsilon^{2/3}$ is always finitely small for large enough g_sM, similar to T_H in the far UV limit, when g_sM is large enough. Hence it is also conclusive that in KT, when $g_sM \gg 1$, the size of T_H away from the mid throat region, in the UV/IR limits of $\hat{r}_H$, is essentially same, always finitely small.

It is also instructive to inspect the parameter dependence of the world volume black hole solution more closely and quantitatively. Inspection of (4.27) and (4.26) shows that for the reasonable choice of parameters $L^2 = 10^{-2}$ and $\epsilon = 10^{-5}$ the horizon and temperature of the world volume black hole increase continuously and monotonically with increasing the angular velocity (see Fig. 3 [upper panel]), as they should. However, inspection of (4.26) shows that by increasing the value of ϵ, while keeping L^2 fixed small, as before, the world volume horizon develops three branches. It first increases, then decreases, and finally it increases continuously (see again Fig. 3 [upper panel]). Due to the behavior of the world volume horizon in the middle branch (where it decreases despite increasing the angular velocity), it seems that only for certain values of ϵ the world volume horizon behaves consistently, i.e., increases continuously with increasing angular velocities. Nonetheless, inspection of (4.27) and (4.26) shows that when the value of L^2 is also increased, the world volume horizon and temperature increase continuously with increasing the angular velocities despite increasing the value of ϵ (see Fig. 3 [lower panel]). It is also interesting to see that in this case increasing the value of ϵ decreases the world volume horizon and temperature. We therefore conclude that the scale and behavior of the world volume horizon and temperature in KT depends on the choice of parameters. We conclude, in particular, that world volume horizons and temperatures of expected features form in the KT throat subject to certain hierarchies of scales, $\epsilon/g_sM \simeq 10^3$–$10^4$.

We also note that for $g_sM \ll 1$ our Hawking temperature (4.27) takes the general form $T_{\mathrm{H}} \simeq \hat{r}_{\mathrm{H}}^3/L^2\sqrt{\ln\left(\hat{r}_{\mathrm{H}}/\epsilon^{2/3}\right)}$. The denominator of this equals the denominator of the Hawking temperature discussed in [8] (*cf.* Eq. (89) of [6]), $T_{\mathrm{H}} \simeq \hat{r}_{\mathrm{H}}/L^2\sqrt{\ln\left(\hat{r}_{\mathrm{H}}/\epsilon^{2/3}\right)}$, but its numerator is very different. Here it grows with the cube of $\hat{r}_{\mathrm{H}}$ whereas there (*cf.* Eq. (89) of [6]) the

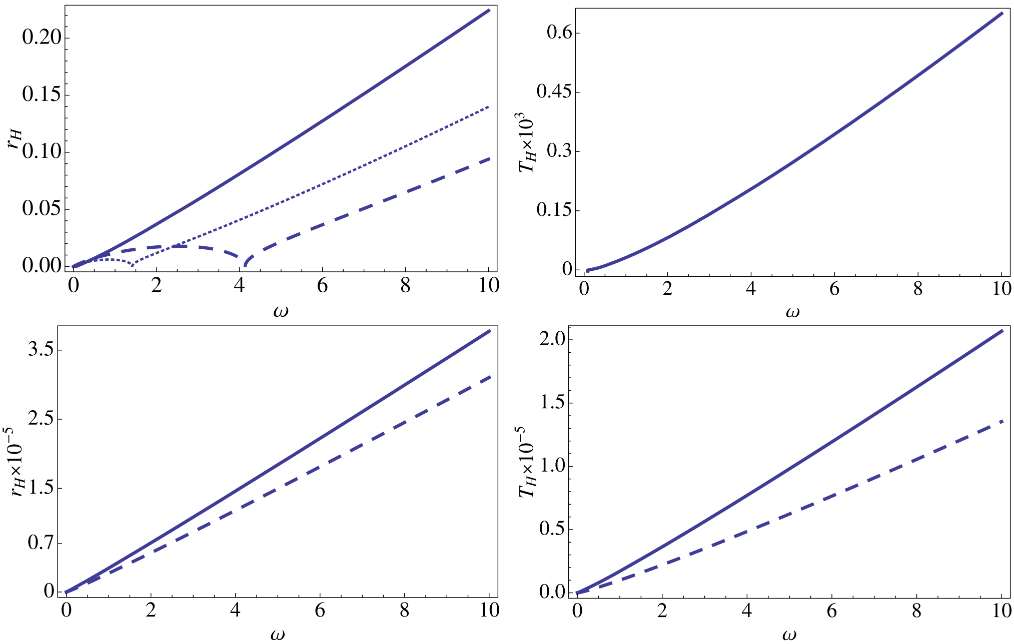

Fig. 3. [Upper panel] The behavior of the horizon and temperature for $L^2 = 10^{-2}$, $\epsilon = 10^{-5}$ (solid), $\epsilon = 10^{-3}$ (dotted), $\epsilon = 5 \times 10^{-3}$ (dashed). [Lower panel] The behavior of the horizon and temperature for $L^2 = 10^2$, $\epsilon = 10^{-5}$ (solid), $\epsilon = 10^{-2}$ (dashed).

numerator of T_H grows linearly with $\hat{r}_H$. Thus these temperatures, though similar, are not the same. Remember that the black hole here lives on a rotating probe brane whereas in [8] there are no probes and the black hole lives in the supergravity background itself. Note also that here we considered the pure, original KT supergravity solution, neither modifying it nor considering backreaction effects, as in the probe limit. But, in [8] the KT solution is modified and generalized where it was shown that at sufficiently high temperature the system develops a horizon with a corresponding Hawking temperature. It is interesting to note that here we have found that the induced metric on the rotating probe in KT has thermal horizon and Hawking temperature even when there is no such black hole in the bulk.

In our analysis above, the backreaction of the D1-brane to the supergravity background solution has been neglected since we considered the probe limit. It is also instructive to see to what extend this can be justified. To see this, we note that the total energy of our rotating D1-brane in the KT supergravity solution is given by the following relation[5]:

$$E = T_{D1} \int_{r_{IR}}^{r_{UV}} dr\, \sqrt{-g} T_t^t = T_{D1} \int_{r_{IR}}^{r_{UV}} d\hat{r} \left(1 + \frac{\overline{\omega}^2}{2\hat{r}^2}\right). \tag{4.28}$$

[5] Here we are using the energy–stress tensor defined by $T_N^M = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ML}} g_{LN}$, where g_{MN} denotes the bulk metric with M and N running over all ten coordinates of the ten-dimensional spacetime. This satisfies the equation of motion $\nabla_M T_N^M = 0$. For static spacetime, this reduces to $\partial_M (T_t^M \sqrt{-g}) = 0$, which leads to the energy $E = \int dr \sqrt{-g} T_t^t$.

Since in KT ϵ is very small (such that $\epsilon^2 = 0$), from Eq. (4.28) one can see that the energy density becomes very large, when taking the IR limit $\hat{r} \to \epsilon^{2/3}$. In addition, the total angular momentum of our rotating D1-brane in KT is given by:

$$Q = \frac{\delta S}{\delta \dot{\psi}} = \frac{2L^4 T_{D1}\overline{\omega}}{3} \int_{r_{IR}}^{r_{UV}} \frac{d\hat{r}}{\hat{r}^2} \ln\left(\frac{\hat{r}}{\epsilon^{2/3}}\right). \tag{4.29}$$

Performing the integral in (4.29) for $r_{IR} \simeq \epsilon^{2/3}$ and $r_{UV} \simeq 10^3 \epsilon^{2/3}$ shows that the total angular momentum is given by $Q \simeq T_{D1} L^4 \overline{\omega}/\epsilon^{2/3}$, which is large like its energy. Thus in the IR limit the backreaction of the probe D1-brane to the supergravity background is non-negligible even when $\overline{\omega}$ is not large. It is then reasonable to conclude that the large backreaction will result in the formation of a black hole in the bulk of KT, centered in the IR.

5. Temperature in the Klebanov–Witten throat

5.1. The Klebanov–Witten solution

In the absence of M wrapped D5-branes the KT throat solution joins the Klebanov–Witten throat solution [5]. The KW throat is the simplest conifold throat background. The ten-dimensional metric and the self-dual five-form on the KW throat takes the form [5]

$$ds_{10}^2 = h(\hat{r})^{-1/2} dx_n dx^n + h(\hat{r})^{1/2} \hat{r}^2 \left[\frac{d\hat{r}^2}{\hat{r}^2} + \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2\right], \tag{5.1}$$

$$\tilde{F}_5 = dC_4 = (1 + \star_{10})\left[d(h^{-1}(\hat{r})) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\right], \tag{5.2}$$

where the warp factor reads

$$h(\hat{r}) = \frac{L^4}{\hat{r}^4}, \quad \text{and} \quad L^4 \equiv \frac{27\pi}{4} g_s N (\alpha')^2. \tag{5.3}$$

Note that the asymptotic UV metric, (5.1), has a $U(1)$ symmetry associated with the rotations of the angular coordinate $\beta = \psi/2$, normalized such that β has period 2π. This is the R-symmetry of the dual gauge theory. In the absence of fractional branes there are no background three-form fluxes, so the $U(1)$ R-symmetry is a true symmetry of the field theory. Because the R-symmetry is realized geometrically by invariance under a rigid shift of the angle β, it becomes a local symmetry in the full gravity theory, and the associated gauge fields $A = A_\mu dx^\mu$ appear as fluctuations of the ten-dimensional metric and RR four-form potential [23–25]. The metric and the self-dual five form take modified the form [6]:

$$ds_{10}^2 = h(\hat{r})^{-1/2} dx_n dx^n + h(\hat{r})^{1/2} \hat{r}^2 \left[\frac{d\hat{r}^2}{\hat{r}^2} + \frac{1}{9}(g^5 - 2A)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2\right] \tag{5.4}$$

$$\begin{aligned}\tilde{F}_5 = dC_4 = \frac{1}{g_s} d^4x \wedge dh^{-1} + \frac{\pi\alpha' N}{4}\Big[&(g^5 - 2A) \wedge g^1 \wedge g^2 \wedge g^3 \wedge g^4 \\ &- dA \wedge g^5 \wedge dg^5 + \frac{3}{L} \star_5 dA \wedge dg^5\Big].\end{aligned} \tag{5.5}$$

Here note that this metric is of the familiar Kaluza–Klein form, and $\star_5$ is the five-dimensional Hodge star operator defined with respect to the AdS_5 metric $ds_5^2 = h^{-1/2} dx_n\, dx^n + h^{1/2} dr^2$. The supergravity field equation $d\tilde{F}_5 = 0$ implies [6]:

$$d \star_5 dA = 0. \tag{5.6}$$

Hence the A field satisfies the equation of motion for a massless vector in AdS_5 space.

5.2. Induced metric and Hawking temperature in the KW throat

In this section we derive the world volume horizon and temperature of the rotating probe in the background (5.1)–(5.2), where the background gauge field A is not activated. Considering the same S^3 cycle as in previous sections, we obtain this full background metric on the KT throat in the form

$$ds_{10}^2 = \frac{\hat{r}^2}{L^2}\left(dx^2 - dt^2\right) + \frac{L^2}{\hat{r}^2}\left(d\hat{r}^2 + \frac{\hat{r}^2}{6} d\phi^2 + \frac{\hat{r}^2}{9} d\psi^2\right). \tag{5.7}$$

To evaluate the action of the brane, we consider the same embedding ansatz as in the previous sections with $\phi(\hat{r}, t)$ and/or $\psi(\hat{r}, t)$ specifying the world volume of the brane. To Evaluate (2.4) in the background (5.7) (where $M = 0$ and $g^{2,4} = 0$, giving $C_2 = B_2 = 0$) for the embedding ansatz, it suffices to set either ϕ or ψ constant. Setting $\phi = const.$, gives the action of the rotating D1-brane in the form:

$$S_{\mathrm{D1}} = -g_s\, T_{D1} \int dt\, d\hat{r} \sqrt{1 + \frac{\hat{r}^2 (\psi')^2}{9} - \frac{L^4\, \dot{\psi}^2}{9\, \hat{r}^2}}. \tag{5.8}$$

As in the KT and KS throats, we note that in the limit of small velocities the higher-order non-canonical kinetic terms in (5.8) may be dropped. The Lagrangian and equation of motion for the slow rotating D1-brane take the simple form:

$$L = 1 + \frac{\hat{r}^2 (\psi')^2}{18} - \frac{L^4\, \dot{\psi}^2}{18\, \hat{r}^2}, \tag{5.9}$$

$$\frac{\partial}{\partial \hat{r}}\left[\frac{\hat{r}^2 \psi'(\hat{r}, t)}{9}\right] = \frac{\partial}{\partial t}\left[\frac{L^4\, \dot{\psi}(\hat{r}, t)}{9\hat{r}^2}\right]. \tag{5.10}$$

As before, consider solutions of the form

$$\psi(\hat{r}, t) = \omega t - \frac{\omega}{\hat{r}} + \psi_0. \tag{5.11}$$

Putting these into the background, gives the induced metric on the brane as

$$ds_{ind}^2 = -\frac{[\hat{r}^2 - L^4 \overline{\omega}^2]}{L^2} dt^2 + L^2\left(\frac{1}{\hat{r}^2} + \frac{\overline{\omega}^2}{\hat{r}^4}\right) d\hat{r}^2 + \frac{2\overline{\omega}^2}{\hat{r}^2} L^2 dt\, d\hat{r}, \quad \overline{\omega}^2 = \frac{\omega^2}{9}. \tag{5.12}$$

To eliminate the cross term in this metric, we may consider a coordinate transformation of the form

$$\tau = t - L^4 \overline{\omega}^2 \int \frac{d\hat{r}}{\hat{r}^2 (\hat{r}^2 - L^4 \overline{\omega}^2)}. \tag{5.13}$$

The induced metric (5.12) then takes the form

$$ds_{ind}^2 = -\frac{\left[\hat{r}^2 - L^4\overline{\omega}^2\right]}{L^2}d\tau^2 + L^2\left[\frac{\overline{\omega}^2 + \hat{r}^2 - L^4\overline{\omega}^2}{\hat{r}^2(\hat{r}^2 - L^4\overline{\omega}^2)}\right]d\hat{r}^2. \tag{5.14}$$

It is interesting to note that (for $L = 1$) the induced metric, (5.14), has the form of the BTZ black hole with the angular coordinate suppressed. Thus it has a world volume horizon and Hawking temperature given by:

$$\hat{r}_H^2 - L^8\overline{\omega}^2 = 0, \qquad T_{\rm H} = \frac{\hat{r}_H}{2\pi} = \frac{L^4\overline{\omega}}{2\pi}. \tag{5.15}$$

Clearly, $\hat{r}_H$ in (5.15) has one (real positive) zero, forming a single horizon, $\hat{r}_H = L^2\overline{\omega}$. Since the validity range of the solution is the same as in KT, $\hat{r}_H$ and T_H, (5.15), in KW are constrained by UV/IR scales of the throat. Comparing (5.15) to (4.26) shows that $\hat{r}_H$ in KW has a form similar to $\hat{r}_H$ in KT for very small rotations, but shrinking/expanding linearly with $\overline{\omega}$, since there is no logarithmic warping, and therefore changing more rapidly. This implies that T_H in KW increases/decreases faster than T_H in KT. Note also from the previous section that in the IR, $\hat{r}_H \to \epsilon^{2/3}$, and UV, $\hat{r}_H \to 10^2\epsilon^{2/3}$, limits the world volume temperature in KT is more or less constant and given by $T_H \sim L^2\,\epsilon^{2/3}$. (Also recall from the previous section that in the IR limit of the KT solution the flux has to be $L^2 \gg 1$, in order to have a valid SUGRA solution, whereas in the UV limit, at sufficiently large radii, the solution remains valid even if $L^2 \ll 1$.) On contrary, (5.15) shows T_H in KW linearly increases/decreases with $\hat{r}_H$ in these limits. In addition to these qualitative differences, there are also quantitative differences in the two examples in their UV/IR limits. For $\hat{r}_H \to \epsilon^{2/3}$, T_H in KW is about L^2 times less than T_H in KT, noting that in the IR limit of KT $L^2 \gg 1$. For $\hat{r}_H \to (10)^2\epsilon^{2/3}$, T_H in KW is less than T_H in KT if $L^2 \gg 1$ is large enough whereas if $L^2 \ll 1$ T_H in KW is always greater than T_H in KT, as in this case T_H in KT is vanishingly small. It is conclusive then that in the IR limit there is always a large separation between T_H in KW and T_H in KT, by the addition of flux in KT, whereas in the UV limit the separation may persist, or go away, depending on the choice of flux in KT. It is also clear from Eq. (5.15) that for rotating probe D1-branes in KW, $\hat{r}_H$ and T_H have a form very similar to $\hat{r}_H$ and T_H for rotating D1 probes in $AdS_5 \otimes S^5$, [21], in particular, increasing/decreasing linearly with $\overline{\omega}$.

If one considers the backreaction of the above solution to the KW supergravity background, it is natural to expect such D1-branes to form mini black holes in the bulk KW. This indicates that the rotating D1-brane describes thermal object with temperature T_H in the dual field theory. The configuration is dual to $\mathcal{N} = 1$ conformal gauge theory coupled to a monopole at finite temperature. The $\mathcal{N} = 1$ gauge theory is itself at zero temperature while the monopole is at finite temperature T_H. Thus such configurations are in non-equilibrium steady states.

In our analysis above, the backreaction of the D1-brane to the supergravity background has been neglected since we considered the probe limit. It is also instructive to see to what extend this can be justified. We note that the total energy of our rotating D1-brane in KW is given by:

$$E = T_{D1}\int dr\,\sqrt{-g}T_t^t = T_{D1}\int d\hat{r}\left(1 + \frac{\overline{\omega}^2}{2\hat{r}^2}\right). \tag{5.16}$$

From Eq. (5.16) one can see that in the IR the energy density becomes very large, when taking the IR limit. Furthermore, the total angular momentum of our rotating D1-brane in KW reads:

$$Q = \frac{\delta S}{\delta \dot{\psi}} = \frac{2L^4 T_{D1}\overline{\omega}}{3} \int \frac{d\hat{r}}{\hat{r}^2}. \tag{5.17}$$

From Eq. (5.17) one can show that the total angular momentum is large like its energy. Thus in the IR limit the backreaction of the probe D1-brane to the supergravity background is non-negligible even when $\overline{\omega}$ is not large. It is then reasonable to conclude that the large back-reaction will result in the formation of a black hole in the bulk of KW, centered at the IR location.

Now we would like to derive the world volume horizon and temperature of the rotating probe in the background (5.4)–(5.5), where the background gauge field A is activated and described by (5.18). To derive A from Eq. (5.18), we note that the four noncompact spatial coordinates combine with the radial coordinate of the conifold to span AdS_5 with its metric given by $ds_5^2 = h^{-1/2}dx_n\,dx^n + h^{1/2}dr^2$. Using this metric, and choosing the gauge $\mathcal{A}^{\mathbf{x}} = \mathcal{A}^r = 0$ and $\mathcal{A}^t = \Phi(r)$ one has componentwise as $A_i = g_{ij}\mathcal{A}^j = (h^{-1/2}\mathcal{A}^t, h^{-1/2}\mathcal{A}^{\mathbf{x}}, h^{1/2}\mathcal{A}^r) = (\widetilde{\Phi}(r), 0, 0, 0)$, where $h = L^4/r^4$. A straightforward but tedious computation shows that for the above gauge configuration the supergravity equation of motion, (5.18), takes the form:

$$d\star_5 dA = \frac{3!}{L^3}\partial_r\big[r^3\,\partial_r\widetilde{\Phi}(r)\big]dx\wedge dy\wedge dz\wedge dr = 0. \tag{5.18}$$

It is easy to see that the solution of (5.18) takes the simple form:

$$\widetilde{\Phi}(r) \simeq \frac{1}{r^2}. \tag{5.19}$$

We would like to remark that our solution (5.19) is of expected form. We note that an electric field will be supported by a potential of the from $\widetilde{\Phi}(r) = A_t \simeq r^{-2}$. As this is a rank one massless field in AdS, it must correspond to a dimension four operator or current in the gauge theory. This is just what one would expect from an R-current, to which gauge fields correspond.

In the presence of the above gauge field the background metric (5.4) for our usual S^3 round then takes the form:

$$ds_{10}^2 = -\frac{\hat{r}^2}{L^2}\left(1 - \frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2}\right)dt^2 + \frac{L^2}{\hat{r}^2}\left(d\hat{r}^2 + \frac{\hat{r}^2}{6}d\phi^2 + \frac{\hat{r}^2}{9}d\psi^2 + \frac{4\hat{r}^2}{9}\widetilde{\Phi}(\hat{r})dt\,d\psi\right). \tag{5.20}$$

As in the pure KW, we may set $\phi = const.$ and obtain the action of the rotating D1-brane in the background (5.20) as:

$$S_{\mathrm{D1}} = -g_s T_{D1}\int dt\,d\hat{r}\sqrt{1 - \frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2} + \frac{\hat{r}^2}{9}(\psi'(\hat{r},t))^2\left(1 - \frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2}\right) - \frac{L^4\dot{\psi}^2}{9\hat{r}^2}}. \tag{5.21}$$

As before, we note that in the limit of small velocities the higher-order non-canonical kinetic terms in (5.21) can be dropped (after Taylor expansion). The Lagrangian and equation of motion for the slow rotating D1-brane then take the form:

$$L=\left(1-\frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2}\right)^{1/2}\left[1+\frac{\hat{r}^2\,(\psi'(\hat{r},t))^2}{18}-\frac{L^4\dot{\psi}^2}{18\hat{r}^2}\left(1-\frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2}\right)^{-1}\right], \tag{5.22}$$

$$\frac{\partial}{\partial\hat{r}}\left[\left(1-\frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2}\right)^{\frac{1}{2}}\frac{\hat{r}^2\psi'(\hat{r},t)}{9}\right]=\frac{\partial}{\partial t}\left[\left(1-\frac{4L^4\widetilde{\Phi}^2(\hat{r})}{9\hat{r}^2}\right)^{-\frac{1}{2}}\frac{L^4\dot{\psi}(\hat{r},t)}{9\hat{r}^2}\right]. \tag{5.23}$$

Consider the rotating solution of the form:

$$\psi(\hat{r},t)=\omega t+u(\hat{r}),\quad u'(\hat{r})=\frac{\omega}{\hat{r}^2(1-4L^4\widetilde{\Phi}^2(\hat{r})/9\hat{r}^2)^{1/2}}. \tag{5.24}$$

Inspection of Eqs. (5.24) and (5.23) shows that in the absence of background gauge field the brane equation of motion and rotating brane solution reduce to Eqs. (5.11) and (5.10), respectively, as they should.

Putting the solution (5.24) into the background metric (5.20), gives the induced metric in the form:

$$\begin{aligned} ds^2_{ind.}=&-\frac{(\hat{r}^2-L^4\overline{\omega}^2-4L^4\widetilde{\Phi}(\hat{r})(\widetilde{\Phi}(\hat{r})+\omega)/9)}{L^2}dt^2\\ &+L^2\left[\frac{\overline{\omega}^2}{\hat{r}^4(1-4L^4\widetilde{\Phi}(\hat{r})^2/9\hat{r}^2)}+\frac{1}{\hat{r}^2}\right]d\hat{r}^2+\frac{2L^2\omega(\omega+2\widetilde{\Phi}(\hat{r}))}{9\hat{r}^2(1-4L^4\widetilde{\Phi}(\hat{r})^2/9\hat{r}^2)}d\hat{r}dt. \end{aligned} \tag{5.25}$$

To eliminate the cross-term, consider a coordinate transformation of the form:

$$\tau=t-\frac{L^4\omega}{9}\int\frac{d\hat{r}\,(\omega+2\tilde{\Phi}(\hat{r}))}{\hat{r}^2(1-4L^4\widetilde{\Phi}^2(\hat{r})/9\hat{r}^2)^{1/2}(\hat{r}^2-L^4\overline{\omega}^2-4L^4\widetilde{\Phi}(\hat{r})(\widetilde{\Phi}(\hat{r})+\omega)/9)}. \tag{5.26}$$

The induced metric then takes the form:

$$\begin{aligned} ds^2_{ind.}=&-\frac{\mathcal{G}(\hat{r})}{L^2}d\tau^2\\ &+L^2\left[\frac{[9\hat{r}^2-4L^4\widetilde{\Phi}^2(\hat{r})+\omega^2]\mathcal{G}(\hat{r})+L^4\overline{\omega}^2[\omega+2\widetilde{\Phi}(\hat{r})]^2}{\hat{r}^2[9\hat{r}^2-4L^4\widetilde{\Phi}^2(\hat{r})]\mathcal{G}(\hat{r})}\right]d\hat{r}^2. \end{aligned} \tag{5.27}$$

Here $\mathcal{G}(\hat{r})=\hat{r}^2-L^4\overline{\omega}^2-4L^4\widetilde{\Phi}(\hat{r})(\widetilde{\Phi}(\hat{r})+\omega)/9$. It is clear that the induced metric depends on the gauge field component $\tilde{\Phi}(\hat{r})$, given by (5.19). Inspection of Eqs. (5.25)–(5.27) shows that in the absence of the background gauge field the induced world volume metrics reduce to (5.12)–(5.14), with the induced world volume horizons and temperatures given by (5.15). According to our solution (5.19), this case corresponds to the very large radii limit, $\hat{r}\to\infty$.

Now we would like to take the small radii limit, $\hat{r}\to 0$, where the conifold singularity is approached, where massless degrees of freedom become important (*cf.* [30]). In this limit, our solution (5.19) shows that the additional massless gauge field, $\tilde{\Phi}(\hat{r})$, contributes nontrivially to induced world volume metric, given by the black hole geometry, (5.27). The world volume horizon from (5.27), $\mathcal{G}(\hat{r}_H)=-g_{\tau\tau}=g^{\hat{r}\hat{r}}=0$, is described by:

$$\begin{aligned} &\hat{r}_H^2-L^4\overline{\omega}^2-4L^4\widetilde{\Phi}(\hat{r}_H)(\widetilde{\Phi}(\hat{r}_H)+\omega)/9=0,\\ &\hat{r}_H^6-L^4\overline{\omega}^2\hat{r}_H^4-(4L^4\omega/9)\hat{r}_H^2-4L^4/9=0. \end{aligned} \tag{5.28}$$

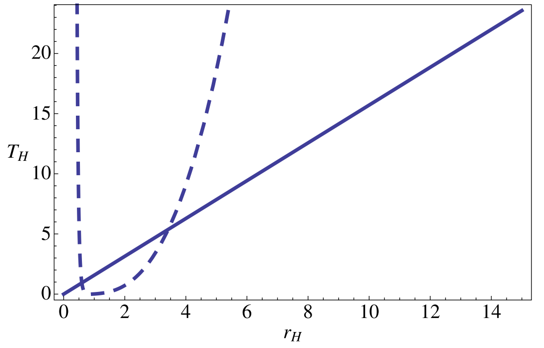

Fig. 4. The behavior of the world volume Hawking temperature for $L = 1$ with (dashed) and without (solid) background gauge field turned on.

Here in the second line we inserted the gauge field component given by our solution (5.19). It is clear from (5.28) that near the conifold point the massless gauge field contributes nontrivially to the world volume horizon. It is interesting to note that (5.28) has the form of the horizon equation of the AdS–Reissner–Nordström black hole (*cf.* [29]). Note though, as before, that here the horizon (5.28) is on the world volume obtained from the induced metric, (5.27), on the rotating a probe in the KW, without having a black hole in the KW background. Equation (5.28) is a 'sextic equation' in $\hat{r}_H$. To solve this equation, we reduced it to a 'depressed cubic' and use trigonometric method to find the three distinct roots. However, we note that the horizon of the solution is located at the largest root of $g^{\hat{r}\hat{r}} = 0$, which is:

$$\hat{r}_H^+ = \frac{L^4\varpi^2}{3} + \frac{2L^2}{3}\sqrt{4\omega/3 + L^4\varpi^4} \times \cos\left\{(1/3)\arccos\left[3\left(\frac{6 + 2L^4\varpi^2\omega + L^8\varpi^6}{4\omega + 3L^4\varpi^4}\right)\sqrt{\frac{3}{4\omega + 3L^4\varpi^4}}\right]\right\}. \tag{5.29}$$

The Hawking temperature at this horizon takes the form:

$$\begin{aligned} T_H &= \left.\frac{r^2[9\hat{r}^2 - 4L^4\widetilde{\Phi}^2(r)][2\hat{r} + 4L^4(2\widetilde{\Phi}(\hat{r}) + \omega)\tilde{\Phi}'(\hat{r})/9]}{2\pi L^6\varpi^2[\omega + 2\widetilde{\Phi}(r)]^2}\right|_{\hat{r}=\hat{r}_H^+} \\ &= \frac{r^2[9(\hat{r}_H^+)^2 - 4L^4/(\hat{r}_H^+)^4][2\hat{r}_H^+ - 4L^4(2/(\hat{r}_H^+)^2 + \omega)/9(\hat{r}_H^+)^3]}{2\pi L^6\varpi^2[\omega + 2/(\hat{r}_H^+)^2]^2}. \end{aligned} \tag{5.30}$$

Here in the second line we inserted the gauge field component given by our solution (5.19), and $\hat{r}_H^+$ is radius of the horizon given by (5.29). By comparing (5.30) and (5.29) with (5.15), it is clear that the presence of the background gauge field has modified the world volume horizon and temperature significantly. It is also straightforward to see that by setting the background gauge field zero, $\widetilde{\Phi}(r) = 0$, the above temperature (5.30) reduces to the temperature (5.15), as it should. Our expression (5.30) is quite interesting and very distinct from our previous world volume temperatures (see Fig. 4). This is because for a given temperature T_H, there are in fact *two* values of $\hat{r}_H^+$, which solve the relation (5.30). We also see that there are two classes of world volume black hole solutions. There is one branch which, for large $\hat{r}_H^+$, the temperature T_H goes with $(\hat{r}_H^+)^3$. The other branch goes at small $\hat{r}_H^+$ as the inverse of $(\hat{r}_H^+)^4$. These 'small' black holes have the familiar behavior of five-dimensional black holes in asymptotically flat spacetime, as their temperature decreases with increasing horizon size. (The term 'small' is appropriate, as they are smaller than the characteristic size set by the AdS scale L, and so they have the characteristics of the asymptotically Minkowskian black holes.) In a similar way, the 'large' black holes may be obtained when L is small compared to the horizon size. We also note that increasing the flux further merely changes the scale of the temperature but leaves its overall behavior unchanged (as in Fig. 4). We therefore conclude that when in the KW solution the background gauge field from the $U(1)$ R-symmetry is turned on, the induced metric on the rotating probe in the KW background admits two classes of world volume black hole solutions. These are characterized by two qualitatively distinct world volume temperatures in the large and small world volume horizon radii limits, respectively. This is similar to the behavior of the temperature in AdS–Schwarzschild black hole (*cf.* [29]), though note that the black hole solution we obtained here, as in previous examples, is on the world volume of the rotating probe in spite of the absence of real black holes in the bulk.

In our analysis above, the backreaction of the D1-brane to the supergravity background has been neglected since we considered the probe limit. It is also instructive to see to what extend this can be justified. We note that the total energy of our rotating D1-brane in KW with background gauge field turned on is given by:

$$\begin{aligned}
E &= T_{D1}\int dr\,\sqrt{-g}T^t_t \\
&= 2T_{D1}\int d\hat{r}\left(1-\frac{4L^4\widetilde{\Phi}^2(r)}{9\hat{r}^2}\right)^{1/2}\left[1+\frac{\varpi^2}{2\hat{r}^2(1-4L^4\widetilde{\Phi}^2(r)/9\hat{r}^2)}\right] \\
&= 2T_{D1}\int d\hat{r}(9\hat{r}^6-4L^4)^{1/2}\left[1+\frac{\hat{r}^4\omega^2}{2(9\hat{r}^6-4L^4)}\right].
\end{aligned} \tag{5.31}$$

It is clear that when the background gauge field is turned off, $\widetilde{\Phi}(r)=0$, Eq. (5.31) reduces to Eq. (5.16), as it should. From Eq. (5.31) one can see that the energy density becomes very large, when taking the IR limit. Furthermore, the total angular momentum of our rotating D1-brane in KW reads:

$$Q=\frac{\delta S}{\delta\dot{\psi}}=T_{D1}\frac{L^4\omega}{3}\int\frac{d\hat{r}}{\hat{r}^2[1-4L^4\widetilde{\Phi}^2(r)/9\hat{r}^2]^{1/2}}=T_{D1}\frac{L^4\omega}{3}\int\frac{\hat{r}\,d\hat{r}}{(9\hat{r}^6-4L^4)^{1/2}}. \tag{5.32}$$

It is clear that when the background gauge field is turned off, $\widetilde{\Phi}(r)=0$, Eq. (5.32) reduces to Eq. (5.17), as it should. From Eq. (5.32) one can see that in the IR the angular momentum is large like its energy. Thus in the IR limit the backreaction of the probe D1-brane to the supergravity background is non-negligible even when ω is not large. It is then reasonable to conclude that the large backreaction will result in the formation of a black hole in the bulk of KW, centered in the IR region.

6. Discussion

In this paper, we studied the induced world volume metrics on rotating probes in warped Calabi–Yau throats and provided the first examples of world volume black hole solutions in such throat backgrounds. Such examples have been found in the literature in $AdS_5\otimes S^5$ background dual to $\mathcal{N}=4$ SYM theory. The aim of our work was to extend such examples to more general supergravity solutions including Calabi–Yau throat backgrounds dual to $\mathcal{N}=1$ gauge theories.

The motivation of our study was to find examples of world volume black hole solutions in backgrounds where some supersymmetry and/or conformal invariance are broken whereby the IR behavior of AdS spacetime is modified. By gauge/gravity duality, the temperature of such black hole solutions correspond to the temperatures of flavors in the dual gauge field theory. We considered different examples of conifold throat backgrounds and computed the induced metrics on the world volume of rotating probe branes in these backgrounds. We performed a UV/IR consistent analysis and found interesting novel black hole solutions that can form on the world volume of the rotating probe brane in these backgrounds in spite of the absence of black holes in the bulk Calabi–Yau.

We began by considering the KS solution as the first example, taking its very small radii limit corresponding to the very deep IR limit of the solution. We then modified the background by taking the very large radii limit corresponding to UV solutions, considering respectively the KT and KW solutions as next examples. Expectedly, the world volume horizons and temperatures in these two UV solutions present some similarities, due to the fact that warping has similar features in these solutions. Nonetheless, comparing these, in particular, with the analysis in the deep IR limit of the KS, our study revealed interesting differences. These differences are related both to the varying forms of the warp factors in the three cases, and to the different manifolds in which the angular coordinates are compactified. We also found interesting differences between world volume horizons and temperatures in KW, when the background gauge field due to $U(1)$ R-symmetry is activated.

In the very deep IR limit of the KS solution, where the warp factor is constant and the solution is regular, whereby the dual field theory is confining and breaking chiral symmetry, we derived the induced world volume metric on the rotating probe and found no world volume black hole nucleation with horizons and temperatures of expected features. On contrary, in the UV solution, including the KT throat, where the warp factor varies logarithmically and the solution is singular, whereby the dual field theory is non-confining and chiral, we found from induced world volume metric on the rotating probe world volume black hole solutions with horizons and temperatures of expected features. In the UV solution, including the KW throat, where the warp factor is that of AdS and the dual field theory is conformal, we also found that the induced world volume metric on the probe is given by the black hole geometry with horizons and temperatures of expected features.

In both KT and KW examples, we found that the induced world volume metrics on the rotating probes have single world volume horizons and finite world volume temperatures. In the KT solution, we found that world volume horizon on the rotating probe is given by the Lambert's transcendental equation solving to the Lambert function. We found that the world volume horizon forms about the singularity with the horizon size growing with increasing the angular momenta. We found that certain angular momenta guarantee the world volume horizons and temperatures to form in the UV away from the KT singularity in the IR. Taking the limits of the world volume temperature, as the world volume horizon approaches the UV/IR regions of the throat, we found that due to logarithmic warping the world volume temperature of the rotating probe in KT is more or less constant and determined by the flux: For large enough flux and world volume horizons approaching the UV/IR, we found that the world volume temperatures of the slow rotating probe are finitely small. On contrary, for small flux and world volume horizons approaching the UV we found that the world volume temperature of the slow rotating probe is vanishingly small. Hence we found that the size of the world volume temperature in KT in the far UV limit depends on the choice of flux and accordingly compares to the size of the world volume temperature in the IR limit. In KT we also examined the parameter dependence of the solution and found that the scale and behavior of the world volume horizons and temperatures are subject to certain hierarchies between flux and the deformation parameter. In KW, where logarithmic warping is removed, we found that the induced world volume metric on the rotating probe has the form of the BTZ black hole. We found the world volume horizons and temperatures varying linearly with angular velocities. These were different from world volume horizons and temperatures of rotating probes in KT, but similar to those of rotating probes in $AdS_5 \otimes S^5$ found in the literature. Taking the limits of world volume temperatures, as the world volume horizon approaches the UV/IR, we found that in the IR limit the world volume temperatures in KW and in KT are largely separated by the additional flux in KT whereas in the UV limit we found this difference depending on the choice of flux in KT. Furthermore, in KW we saw that turning on a background gauge field due to $U(1)$ R-symmetry modifies the induced world volume metric on the rotating probe. We found that the world volume horizons and temperature have features and behaviors similar to those of AdS–Reissner–Nordström and AdS–Schwarzschild black holes. We showed that in this particular case the related Hawking temperature on the probe has two distinct branches which describe two different classes of black hole solutions. These included ‘small’ black holes characterized by temperatures which decrease with increasing horizon size. In both KT and KW, we also discussed the backreaction of our rotating D1-brane to the SUGRA background. We noted that by taking into account the backreaction of the rotating brane solution to the SUGRA background the rotating solution is expected to produce mini black holes in the bulk. We noted that these describe thermal objects in the dual field theory, including a monopole at finite temperature coupled to the corresponding $\mathcal{N} = 1$ gauge theory which itself is at zero temperature.

We conclude that the induced world volume metrics on slow rotating probe D1-branes in warped Calabi–Yau throats have thermal horizons with characteristic Hawking temperatures despite the absence of black holes in the bulk Calabi–Yau. We conclude that this world volume black hole nucleation depends on the warping and the deformation of the throat with world volume horizons and temperatures of expected features forming not in the regular confining IR region where warping is constant, but in the singular UV solutions where warping varies. Furthermore, we conclude that in the singular UV solutions world volume black hole solutions with distinct horizons and temperatures form due to distinct choices of warping and background gauge fields. By gauge/gravity duality, we may also conclude that the $\mathcal{N} = 1$ gauge field theories at zero temperature couple to thermal objects at finite temperature, hence producing systems in non-equilibrium steady states, if the supergravity dual containing rotating probes is away from the confining IR limit.

There are limitations and approximations considered in our work. First, in order to obtain simple analytic rotating brane solutions, we considered the simplest probe brane, a probe D1-brane, and took the very small and the very large radii limits of the geometry corresponding to the IR and UV solutions, respectively, where the explicit analytic form of the warp factors is known. Though these are the two important limits of the full SUGRA solution, they rather solve the world volume dynamics in specific conical regions. A more generic analysis for the world volume dynamics would require the consideration of the full KS warp factor, with the brane equations of motion solved numerically. Second, in order to keep our analysis of world volume horizon and temperature as simple as possible, we deliberately turned off gauge fields on the probe D1-brane and considered slow rotations about cycles inside the throat for which the two-form background fields have a vanishing contribution to the D1-brane action. This rendered the world volume dynamics of the probe D1-brane easy to solve. We expect, in particular, that taking other cycles parametrized differently, such that the contribution of the two-forms to the world volume

D1-brane action is locally nontrivial, to significantly complicate our analysis, and possibly lead to some alterations of our results. However, we expect our main conclusions remain unchanged. Third, we did not discuss in this work the details of thermalization of dual gauge field theory of our results. For instance, we could consider gauge theory on the probe in the KW theory, compute the related temperature and find agreement with the supergravity result for the temperature on the probe discussed here. The gauge theory approach can shed light on the temperature for extended objects which we may consider in a separate work. Fourth, the other aspect that we did not study is the connection between the Hawking and the Unruh temperatures of the probe brane. One could derive the Hawking temperature of the rotating probe in supergravity and compare the local temperature with Unruh temperature.

Our analysis in this paper can be extended in several ways. One extension would be to study world volume horizons and temperatures of higher dimensional rotating branes and make comparison with the results obtained in this paper. The other possible extension would be to study horizons and temperatures on rotating probe branes in the throat subject to moduli stabilization, which induce corrections to the ISD supergravity solution and hence to the action of the probe brane. Finally, we note that the complete understanding of the temperature of probes would require the consideration of the full square root structure of the probe action, including higher order non-canonical kinetic terms as well as the contribution of world volume gauge fields. We leave the investigation of these and the above issues for future study.

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