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Modeling of wire electrochemical micromachining

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Abstract

Wire electrochemical micromachining (WECMM) is a promising method for the fabrication of various metal parts. In recent years, WECMM has attracted increasing interest, especially for treatment of complex-shaped microworkpieces. By now, the regularities of electrochemical shaping for the complex-shaped workpieces have not been adequately investigated, because the majority of the works, which are devoted to WECMM, are experimental.

In this work WECMM is studied theoretically. The Laplace equation for the electric potential and the equation of workpiece surface evolution are used as the mathematical model of the process. A scheme of computer simulation involves the numerical solution of the Laplace equation by the boundary element method; the determination of a new position of workpiece surface with regard for possible topological changes; and the motion of wire tool-electrode along a prescribed trajectory. Various schemes of shaping for the tool-electrodes with various cross-section shapes and various types of motions are analyzed. As a result of simulation, the dependences of the front and side interelectrode gaps on the machining parameters are obtained. They can be used for determining the path of wire tool-electrode in order to obtain the prescribed shape and sizes of workpiece surface.

Keywords: Electrochemical micromachining; Wire tool electrode; Numerical simulation

1. Introduction

For cutting the complex-shaped parts made of difficult-to-work materials, various methods of machining (diamond wire cutting [1], wire electrical discharge machining [2], laser beam cutting [3], plasma arc cutting [4], fluid jet machining [5]) are widely used. Along with the methods of mechanical, chemical, and physical treatment, various schemes of electrochemical machining (ECM) are used to fabricate complex-shaped and microstructured surfaces. The ECM offers several advantages: the absence of mechanical and heat effect on the workpiece, absence of tool-electrode (TE) wear, a possibility of controlling the shape and dimensions of workpiece surface by varying the machining conditions and trajectory of TE movement [6]. The following schemes of electrochemical machining are widely used to fabricate the complex-shaped and functional surfaces:

1. With the use of a stationary non-profiled TE and a mask placed on the anode or cathode.
2. With a profiled TE moving towards the workpiece surface.
3. With non-profiled TE, which moves along the workpiece surface by the prescribed trajectory with the aid of numerically controlled system. The term “non-profiled” means that the shape and sizes of TE do not correspond to those of workpiece surface.

\textbf{Nomenclature}

\begin{itemize}
  \item $B_{TE}$ dimensionless length of TE cross-section
  \item $d_{TE}$ diameter of the circuncircle for the TE cross-section
  \item $E$ electrode potential
  \item $H$ dimensionless width of slit
  \item $H_{TE}$ dimensionless height of TE cross-section
  \item $i$ current density
  \item $L$ natural parameter of workpiece surface
\end{itemize}
The application of non-profiled TE enables one to improve the conditions of removal of machining products, because the instantaneous machining surface area is only a small fraction of the total machining surface area. The required shape and sizes of workpiece surface are provided by the motion of TE along a certain trajectory. By now, the technologies of fabrication of tool-microelectrodes with rather complicated cross-section shape have been developed [7]. The application of such wire TE enables one to enhance the efficiency of machining by combining the merits of profiled and non-profiled TE. This method significantly extends the capabilities of electrochemical machining for fabricating complex-shaped surfaces and the microelements of various shapes and sizes.

Though the schemes of wire electrochemical cutting have been known rather long [8], insufficient accuracy of the machining limited its application. However, in recent years, the technologies of fabrication of tool-microelectrodes with rather complicated cross-section shape have been developed [7]. The application of such wire TE enables one to enhance the efficiency of machining by combining the merits of profiled and non-profiled TE. This method significantly extends the capabilities of electrochemical machining for fabricating complex-shaped surfaces and the microelements of various shapes and sizes.

The model of the wire electrochemical machining (Fig. 1) involves the Laplace equation for the electric field potential, equation of workpiece surface evolution, and equations of motion of wire TE. Let us present the mathematical model in the dimensionless form. The diameter of the circumscribed circle for the TE cross-section (TE) was taken as a unit length; the characteristic size of TE or workpiece. However, in many cases, especially in the WECMM, the gap is comparable to the characteristic size of TE. In this case, the distribution of current density should be calculated by using the Laplace equation. Earlier [18], we developed a method of modeling ECM with a wire TE, which moves by an arbitrary trajectory along the workpiece surface, using the boundary element method. Here, the earlier-proposed method is further developed in order to predict the shape and sizes of complex-shaped surfaces and microstructures, which form on the workpiece surface machined with a moving wire TE with various shapes of cross-section; the tool-electrode can move forward, rotate, and execute transverse oscillations.

2. Mathematical Model

The model of the wire electrochemical machining (Fig. 1) involves the Laplace equation for the electric field potential, equation of workpiece surface evolution, and equations of motion of wire TE. Let us present the mathematical model in the dimensionless form. The diameter of the circumscribed circle for the TE cross-section (dTE) was taken as a unit length; the characteristic size of TE or workpiece. The characteristic applied voltage (U''), as a unit electric potential; and the characteristic TE feed rate (v''), as a unit rate. Then, the following system of dimensionless equations is obtained:

\[
\begin{align*}
\text{div}(\text{grad} \, \Phi) &= 0, \\
\frac{dX_{\text{TE}}}{d\tau} &= \eta S_\text{TE} \frac{d\Phi}{dX}, \\
\frac{dY_{\text{TE}}}{d\tau} &= \eta S_\text{TE} \frac{d\Phi}{dY}, \\
\frac{dX}{d\tau} &= V_{\text{in}}, \ (r), \\
\frac{dY}{d\tau} &= V_{\text{in}}, \ (r), \\
\frac{dX}{d\tau} &= \Omega_{\text{in}}, \ (r), \\
\frac{dV_{\text{in}}}{d\tau} &= X_{\text{in}} (r) + X_{\text{TE}} (L_{\text{in}}) \sin[\alpha(r)] \cos[\alpha(r)] - V_{\text{in}} (L_{\text{in}}) \sin[\alpha(r)], \\
\frac{dX_{\text{in}}}{d\tau} &= Y_{\text{in}} (r) + Y_{\text{TE}} (L_{\text{in}}) \cos[\alpha(r)] + V_{\text{in}} (L_{\text{in}}) \sin[\alpha(r)].
\end{align*}
\]

Equations (1) involve the dimensionless parameter Sf, which characterizes the machining conditions. This parameter is calculated by the following equation:

\[
S_f = \frac{U''}{d_{\text{in}} \nu''}.
\]
\( S_y \) corresponds to the steady-state value of frontal interelectrode gap in the electrochemical machining with a plane TE, which moves along a normal to the workpiece surface with a constant rate \( V^* \), in the case of 100% current efficiency of machining.

\[
X_{WP}(L, 0) = R_c \cos (L / R_c), \quad Y_{WP}(L, 0) = R_c \sin (L / R_c),
\]

\[
X_T(0) = 0, \quad Y_T(0) = 0, \quad \alpha(0) = \alpha_c,
\]

\[
\Phi_{ WP, \alpha} = \frac{Y_{WP}}{d_{WP}} \left( \frac{\partial \Phi}{\partial Y_{WP}} \right)_{WP},
\]

\[
\Phi_{ T, \alpha} = -\frac{Y_T}{d_{T}} \left( \frac{\partial \Phi}{\partial Y_T} \right)_{T},
\]

where \( W_{ WP, \alpha} = \frac{X}{d_{WP}} \left( \frac{\partial \Phi}{\partial X} \right)_{WP} \) and \( W_{ T, \alpha} = \frac{X}{d_{T}} \left( \frac{\partial \Phi}{\partial X} \right)_{T} \) are the Wagner numbers, which take into consideration the polarization of workpiece and TE, respectively; \( \sigma \) is the dimensionless voltage applied to the electrodes.

The boundary-value problem for equations (1) - (3) belongs to the class of problems with a moving boundary. In this case, a simultaneous solution of equations, which describe the transport processes and the motion of computational region boundary, is required. This involves considerable difficulties and, as a rule, requires the application of numerical methods. The numerical solution is conveniently performed by the boundary element method, because, in this case, the discretization of only computational region boundary is required (Fig. 2b). Small circles in Fig. 2b show the nodes of boundary elements.

3. Method of Numerical Simulation

The numerical solution is frequently simplified by using the quasi-steady-state approximation. Within this approximation, the entire time of machining is divided into a certain number of steps. For each time step:

1) the distribution of electric field (at the electrode geometry corresponding to the beginning of step) is calculated by the numerical solution of Laplace equation with the boundary conditions (3);

2) a new shape of workpiece surface (at the distribution of current density corresponding to the beginning of step) is determined;

3) a new position of TE center is determined;

4) a new value of angle of TE rotation is determined.

The Laplace equation is solved by the boundary element method. The method allows one to reduce the dimensionality of the problem and simplify significantly the remeshing, which is required for taking into consideration the motion of computational region boundaries. As a result of solution of Laplace equation, we obtain the values of normal derivative of potential that enables one to determine the anodic dissolution rate of workpiece surface in all computational points, which, in its turn, is used for determining a new position of workpiece surface.

The highest current density is observed on the workpiece surface areas, which are closer to the TE surface. The current density steeply decreases with increasing distance between the TE and workpiece and becomes virtually zero at the distances exceeding the characteristic size of TE by 3 - 5 times. This regularity of current density distribution was used to reduce the volume of computation: the size of computational region was limited by a circle with radius \( R \) and the center coinciding with the center of TE (Fig. 3). In this case, the outer boundary of computational region consisted of a portion of workpiece surface.
surface inside the circle and a segment of circle, where the condition of absence of current was prescribed, i.e. it was assumed that the circle was dielectric (Fig. 3b). This method allows one to reduce significantly the volume of computations, especially in the modeling of machining of long surfaces.

Fig. 3. Scheme of reducing the computational region in order to raise the efficiency of numerical solution: (a) the initial computational region and (b) the reduced computational region; R is the radius of reduced computational region

In the course of machining, the topological changes of the workpiece surface can occur, which manifest themselves as the self-intersections of workpiece surface (Fig. 4). Actually, these self-intersections are not observed; they are caused by the existence of several solutions of equations describing the workpiece surface evolution; however, only one solution is physically meaningful.

Fig. 4. A scheme of treatment of topological changes of workpiece surface: (a) a scheme of machining; (b) and (c) the workpiece surface at the topological change and its treatment; (d) the workpiece surface after the topological change

To eliminate the self-intersections of workpiece surface, i.e. to treat the topological changes, the following approach was used. For each time step, the existence of self-intersection points of workpiece surface was determined. As a rule, at the initial stage of machining, no self-intersection of workpiece surface is observed. In the course of machining, the self-intersection of workpiece surface can occur (Fig. 4b). To eliminate the self-intersection, the coordinates of intersection points are determined (Fig. 4c). Commonly, two intersection points arise (Fig. 4c, points 1 and 2). Taking into account that we go clockwise along the outer boundary of computational region, the self-intersection of workpiece surface should be eliminated by using the intersection point 1, where a difference between the numbers of intersecting boundary elements is the largest. For example, assume that the number of boundary element is determined by the number of its initial point, and the boundary elements with the numbers \( i < j \) intersect. Then, the boundary elements \( i+1, i+2, \ldots, j-1 \) are eliminated (they are shown by grey line in Fig. 4) and the initial point of boundary element \( j \) shifts to the intersection point 1. A portion of workpiece surface boundary upon treating the topological change is shown by black line in Fig. 4d.

In the course of machining, the sizes of boundary elements can vary significantly due to the nonuniform rate of anodic dissolution over the workpiece surface, the topological changes, etc. An increase in the boundary element length reduces the accuracy of calculated current density and a decrease in the boundary element length raises the volume of computation. To provide the required accuracy of calculations at the admissible computational time, the adaptive boundary remeshing was performed by the following simple, but well-proven algorithm:

- when the boundary element length increased by more than 50% of its initial length, the boundary element was divided into two elements of equal length;
- when the boundary element length decreased by more than 50% of its initial length, the boundary element was removed (its initial point was eliminated).

The treatment of topological changes, the adaptive boundary remeshing, the reducing of the computational region size, and the application of nonuniform mesh of boundary elements provide a compromise between the accuracy of modeling and the volume of computation.

4. Results and discussion

The following values of parameters were taken in the modeling: the dimensionless steady-state frontal interelectrode gap of 0.05 to 0.2; the dimensionless time step was chosen so that the stability and accuracy of numerical solution were provided (from 0.002 to 0.02). From 20 to 100 linear boundary elements were prescribed on the initial workpiece and TE surfaces. In the course of modeling, the distance between the nodes of boundary element mesh varied: it increased on the convex areas of workpiece surface and decreased on the concave areas. To provide a sufficient accuracy of numerical solution and reduce the volume of computation, the adaptive boundary remeshing was performed in the course of modeling.

At the first stage, we performed the modeling of ECM with the linearly moving wire TE with variously shaped cross-section: round, triangular, square and rectangular (Fig. 5). The starting and final positions of TE are shown by solid grey cross-sections, and the intermediate positions are shown by
the dashed lines. Black bold line shows the machined surface, and other lines correspond to the workpiece surface at various instances of time during the WECMM. The following values of process parameters were taken as the initial data: 

\[ \eta = 1, S_p = 0.1, V_w = 1, \mathcal{U}_s = 1, \mathcal{U}_{w, a} = \mathcal{U}_{w, s} = 0, \Delta \tau = \pi / 160 . \]

The elements of different width form on the workpiece surface as a result of machining. The largest width \( H = 1.904 \) is obtained in the case of TE with round cross-section (Fig. 5a); the smallest width is obtained in the case of TE with rectangular \( H = 1.513 \) (Fig. 5d) and triangular \( H = 1.580 \) (Fig. 5b) cross-sections.

![Fig. 5. The results of modeling of WECMM in the case of linear motion of TE with variously shaped cross-sections at \( S_p = 0.1 \): (a) round; (b) triangular; (c) square; (d) rectangular](image)

Within the linear-one-dimensional approximation, when a plane TE moves along the workpiece surface, the width of formed element can be determined by the following equation:

\[ H = H_{te} + 2 \sqrt{S_p^2 + 2 S_p B_{te}} , \]  

(4)

Taking into account that, for round TE, \( H_{te} = B_{te} = 1 \), using equation (4), we obtain \( H = 2.095 \), which differs by 10% from the value obtained by the numerical modeling \( H = 1.904 \). For the rectangular TE, \( H_{te} = 0.4 \) and \( B_{te} = \sqrt{1 - 0.4^2} = 0.917 . \) Using equation (4), we obtain \( H = 1.465 \), which differs by 3.2% from the result of numerical solution \( H = 1.513 \). The above analysis shows that the locally one-dimensional approximation model can be used for preliminary estimation of the width of slits, which form in the case of linear motion of TE. However, in the cases of more complex motions of TE, for example, the linear motion and rotation of tool-electrode (Fig. 6), the application of simplified models can lead to the erroneous results.

As it is seen from the above results (Figs. 5 and 6), only rather simple-shaped slits can be formed using the linear motion of TE along the workpiece surface and its rotation.

The application of transverse oscillation with amplitude \( A_v \) and angular frequency \( \Omega_v \) onto the linear motion of TE provides additional means for the electrochemical formation of variously shaped elements (Figs. 7 and 8). In this case, equation, which describes the motion of TE center along the \( Y \) coordinate, takes the following form:

\[ \frac{dY_c}{dt} = V_{te}(\tau) + A_v \sin(\Omega_v \tau) \]  

(5)

The calculations (Fig. 8) were performed at the following values of transverse oscillation parameters: \( \Omega_v = 1, A_v = 0.2 \). The obtained results show that the proposed models and methods of simulation can be used for the design and optimization of the processes of WECMM for fabricating the surfaces with complex-shaped micro- and nano-slits.

![Fig. 6. The results of modeling of WECMM in the case of linear motion and rotation of TE with variously shaped cross-sections at \( S_p = 0.1, \Omega_v = 0.5 \): (a) triangular; (b) square; (c) rectangular](image)

![Fig. 7. The results of modeling of WECMM in the case of linear motion with transverse oscillation of TE with variously shaped cross-sections at \( S_p = 0.1, \Omega_v = 1, A_v = 0.2 \): (a) round; (b) triangular; (c) square; (d) rectangular](image)

![Fig. 8. The results of modeling of WECMM in the case of linear motion and rotation with transverse oscillation of TE with variously shaped cross-sections at \( S_p = 0.1, \Omega_v = 0.5, \Omega_v = 1, A_v = 0.2 \): (a) triangular; (b) square; (c) rectangular](image)
Then, the effect of the machining conditions and the shape of TE on the accuracy of machining of the openings with square cross-section was studied (Fig. 9). The initial (round) and machined openings are shown with bold lines. The tool-electrode trajectory is shown dashed. The tool-electrode just before machining is shown as gray square and triangle. As well as in the machining of straight slits, the shape of cross-section, trajectory of motion, and rotation of TE have an effect on the geometry of machined workplace surface. In the absence of TE rotation, square openings form; they have virtually plane side surfaces, which conjugate by the curves near the corner points of TE trajectory (Figs 9a and 9c).

From the above results, it follows that the process reaches the steady state rather quickly. The sizes of square opening can be estimated by the length of straight portions of TE trajectory and the cut width in the machining of straight slits (Figs. 5 and 6). In the zone near the starting point of TE trajectory, an error of the shape is observed; it can be eliminated by correcting the TE trajectory or changing the machining mode.

![Fig. 9. The results of modeling of WECMM of square opening by TE with variously shaped cross-sections at $S_1 = 0.1$: (a, b) square; (c, d) triangular; (a, c) without rotation of TE; (b, d) with rotation of TE ($Q_{TE} = 0.125$)](image)

The obtained results show that the proposed models and methods of modeling can be used for design and optimization of anodic machining in the formation of complex-shaped surfaces.

5. Conclusions

The developed scheme of numerical modeling of wire electrochemical micromachining of surfaces with moving tool-electrode enables one to predict the sizes and shape of machined surface. It is shown that the proposed scheme of modeling can be used for various schemes of machining, including the cases of topological changes of workplace surface. The regularities of electrochemical machining with wire tool-electrodes, which execute a complex motion involving the motion along the workplace surface with transverse oscillation and electrode rotation around its center, were studied. It is shown that various shapes of TE and types of its motion can be used to extend the capabilities of wire electrochemical micromachining. The obtained results can be used for optimization of WECMM.

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