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On analytical study of holographic superconductors with Born–Infeld electrodynamics

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ABSTRACT

Based on the Sturm–Liouville eigenvalue problem, Banerjee et al. proposed a perturbative approach to analytically investigate the properties of the $(2+1)$-dimensional superconductor with Born–Infeld electrodynamics (Banerjee et al., 2013) [29]. By introducing an iterative procedure, we will further improve the analytical results and the consistency with the numerical findings, and can easily extend the analytical study to the higher-dimensional superconductor with Born–Infeld electrodynamics. We observe that the higher Born–Infeld corrections make it harder for the condensation to form but do not affect the critical phenomena of the system. Our analytical results can be used to back up the numerical computations for the holographic superconductors with various condensates in Born–Infeld electrodynamics.

1. Introduction

As one of the most significant developments in fundamental physics in the last one decade, the anti-de Sitter/conformal field theories (AdS/CFT) correspondence [1–3] allows to describe the strongly coupled conformal field theories through a weakly coupled dual gravitational description. A recent interesting application of such a holography is constructing of a model of a high T_c superconductor, for reviews, see Refs. [4–7] and references therein. It was found that the instability of the bulk black hole corresponds to a second order phase transition from normal state to superconducting state which brings the spontaneous U(1) symmetry breaking [8], and the properties of a $(2+1)$-dimensional superconductor can indeed be reproduced in the $(3+1)$-dimensional holographic dual model based on the framework of usual Maxwell electrodynamics [9]. In order to understand the influences of the $1/N$ or $1/\lambda$ (λ is the 't Hooft coupling) corrections on the holographic dual models, it is of great interest to consider the holographic superconductor models with the nonlinear electrodynamics since the nonlinear electrodynamics essentially implies the higher derivative corrections of the gauge field [10]. Jing and Chen introduced the first holographic superconductor model in Born–Infeld electrodynamics and observed that the nonlinear Born–Infeld corrections will make it harder for the scalar condensation to form [11]. Along this line, there have been accumulated interest to study various holographic dual models with the nonlinear electrodynamics [12–24].

In most cases, the holographic dual models were studied numerically. In order to back up numerical results and gain more insights into the properties of the holographic superconductors, Siopsis et al. developed the variational method for the Sturm–Liouville (S–L) eigenvalue problem to analytically calculate the critical exponent near the critical temperature and found that the analytical results obtained by this way are in good agreement with the numerical findings [25,26]. Generalized to study the holographic insulator/superconductor phase transition [27], this method can clearly present the condensation and critical phenomena of the system at the critical point in AdS soliton background.

More recently, Gangopadhyay and Roychowdhury extended the S–L method to investigate the properties of the $(2+1)$-dimensional superconductor with Born–Infeld electrodynamics by introducing a perturbative technique, and observed that the analytical results agree well with the existing numerical results for the condensation operator $\langle\mathcal{O}_-\rangle$ [28]. For the operator $\langle\mathcal{O}_+\rangle$, Banerjee et al. improved the perturbative approach and explored the effect of the

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Born–Infeld electrodynamics on the $(2+1)$-dimensional superconductor [29]. However, comparing with the case of $\langle\mathcal{O}_-\rangle$ [28], we find that for the operator $\langle\mathcal{O}_+\rangle$ the agreement of the analytical result with the numerical calculation is not so good, for example in the case of the Born–Infeld parameter $b=0.3$ [29], the difference between the analytical and numerical values is 22.1%! Furthermore, this perturbative approach is not very valid to study the higher-dimensional superconductor with Born–Infeld electrodynamics. Thus, the motivation for completing this work is two fold. On one level, it is worthwhile to reduce the disparity between the analytical and numerical results for the operator $\langle\mathcal{O}_+\rangle$, and further improve the analytical results and the consistency with the numerical findings. On another more speculative level, it would be important to develop a more general analytical technique which can be used to study systematically the d-dimensional superconductors with Born–Infeld electrodynamics and see some general features for the effects of the higher derivative corrections to the gauge field on the holographic dual models. In order to avoid the complex computation, in this work we will concentrate on the probe limit where the backreaction of matter fields on the spacetime metric is neglected.

The plan of the work is the following. In Section 2 we will introduce the holographic superconductor models with Born–Infeld electrodynamics in the $(d+1)$-dimensional AdS black hole background. In Section 3 we will improve the perturbative approach proposed in [29] and give an analytical investigation of the holographic superconductors with Born–Infeld electrodynamics by using the S–L method. We will conclude in the last section with our main results.

2. Holographic superconductors with Born–Infeld electrodynamics

We begin with the background of the $(d+1)$-dimensional planar Schwarzschild–AdS black hole

$$ds^2=-r^2f(r)dt^2+\frac{dr^2}{r^2f(r)}+r^2\sum_{i=1}^{d-1}dx_i^2, \tag{1}$$

where $f(r)=1-r_+^d/r^d$ with the radius of the event horizon r_+. For convenience, we have set the AdS radius $L=1$. The Hawking temperature of the black hole is determined by

$$T=\frac{dr_+}{4\pi}, \tag{2}$$

which will be interpreted as the temperature of the CFT.

Working in the probe limit, we consider the Born–Infeld electrodynamics and the charged complex scalar field coupled via the action

$$S=\int d^{d+1}x\sqrt{-g}\left[\frac{1}{b}\left(1-\sqrt{1+\frac{1}{2}bF^2}\right)\right.$$
$$\left.-|\nabla\psi-iA\psi|^2-m^2|\psi|^2\right], \tag{3}$$

with the quadratic term $F^2=F_{\mu\nu}F^{\mu\nu}$. When the Born–Infeld parameter $b\to 0$, the model (3) reduces to the standard holographic superconductors investigated in [9,30].

With the ansatz of the matter fields as $\psi=|\psi|$, $A_t=\phi$ where ψ and ϕ are both real functions of r only, we can arrive at the following equations of motion for the scalar field ψ and the gauge field ϕ

$$\psi''+\left(\frac{1+d}{r}+\frac{f'}{f}\right)\psi'+\left(\frac{\phi^2}{r^4f^2}-\frac{m^2}{r^2f}\right)\psi=0, \tag{4}$$

$$\phi''+\frac{d-1}{r}\left(1-b\phi'^2\right)\phi'-\frac{2\psi^2}{r^2f}\left(1-b\phi'^2\right)^{3/2}\phi=0, \tag{5}$$

where the prime denotes the derivative with respect to r.

Applying the S–L method to analytically study the properties of the holographic superconductors with Born–Infeld electrodynamics, we will introduce a new variable $z=r_+/r$ and rewrite the equations of motion (4) and (5) into

$$\psi''+\left(\frac{1-d}{z}+\frac{f'}{f}\right)\psi'+\left(\frac{\phi^2}{r_+^2f^2}-\frac{m^2}{z^2f}\right)\psi=0, \tag{6}$$

$$\phi''+\frac{1}{z}\left[(3-d)+\frac{b(d-1)z^4}{r_+^2}\phi'^2\right]\phi'$$
$$-\frac{2\psi^2}{z^2f}\left(1-\frac{bz^4}{r_+^2}\phi'^2\right)^{3/2}\phi=0, \tag{7}$$

with $f=1-z^d$. Here and hereafter the prime denotes the derivative with respect to z.

In order to get the solutions in the superconducting phase, we have to impose the appropriate boundary conditions for ψ and ϕ. At the event horizon $z=1$ of the black hole, the regularity gives the boundary conditions

$$\psi(1)=-\frac{d}{m^2}\psi'(1)\,,\quad \phi(1)=0\,. \tag{8}$$

Near the AdS boundary $z\to 0$, the asymptotic behaviors of the solutions are

$$\psi=\frac{\psi_-}{r_+^{\Delta_-}}z^{\Delta_-}+\frac{\psi_+}{r_+^{\Delta_+}}z^{\Delta_+}\,,\qquad \phi=\mu-\frac{\rho}{r_+^{d-2}}z^{d-2}\,, \tag{9}$$

where $\Delta_\pm=(d\pm\sqrt{d^2+4m^2})/2$ is the conformal dimension of the scalar operator dual to the bulk scalar field, μ and ρ are interpreted as the chemical potential and charge density in the dual field theory respectively. It should be pointed out that, provided Δ_- is larger than the unitarity bound, both ψ_- and ψ_+ can be normalizable and they can be used to define operators in the dual field theory according to the AdS/CFT correspondence, $\psi_-=\langle\mathcal{O}_-\rangle$ and $\psi_+=\langle\mathcal{O}_+\rangle$, respectively. Just as in Refs. [9,30], we will impose boundary condition that either ψ_- or ψ_+ vanishes. In this work, we impose boundary condition $\psi_-=0$ since we concentrate on the condensate for the operator $\langle\mathcal{O}_+\rangle$. For clarity, we set $\langle\mathcal{O}\rangle=\langle\mathcal{O}_+\rangle$ and $\Delta=\Delta_+$ in the following discussion.

3. Analytical study of holographic superconductors with Born–Infeld electrodynamics

Here we will improve the perturbative approach proposed in [29] and use the S–L method [25] to analytically discuss the properties of the d-dimensional superconductor phase transition with Born–Infeld electrodynamics. We will investigate the relation between critical temperature and charge density as well as the critical exponent of condensation operators, and examine the effect of the Born–Infeld parameter.

3.1. Critical temperature

At the critical temperature T_c, the scalar field $\psi=0$. Thus, near the critical point the equation of motion (7) for the gauge field ϕ becomes

$$\phi''+\frac{1}{z}\left[(3-d)+\frac{b(d-1)z^4}{r_{+c}^2}\phi'^2\right]\phi'=0, \tag{10}$$

where r_{+c} is the radius of the horizon at the critical point. Defining $\xi(z) = \phi'(z)$, we can obtain

$$\xi' + \frac{3-d}{z}\xi = \frac{b(1-d)z^3}{r_{+c}^2}\xi^3, \tag{11}$$

which is the special case of Bernoulli's Equation $y'(x) + f(x)y = g(x)y^n$ [31] for $n = 3$. Considering that the boundary condition (9) for ϕ, we can get the solution to Eq. (11)

$$\xi(z) = \phi'(z) = -\frac{\lambda r_{+c}(d-2)z^{d-3}}{\sqrt{1+(d-2)^2 b\lambda^2 z^{2(d-1)}}}, \tag{12}$$

which leads to the expression

$$\phi(z) = \lambda r_{+c}\zeta(z), \tag{13}$$

with

$$\zeta(z) = \int_z^1 \frac{(d-2)\tilde{z}^{d-3}}{\sqrt{1+(d-2)^2 b\lambda^2 \tilde{z}^{2(d-1)}}} d\tilde{z}, \tag{14}$$

where we have set $\lambda = \rho / r_{+c}^{d-1}$ and used the fact that $\phi(1) = 0$.

Obviously, the integral in (14) is not doable exactly. Just as in Refs. [28,29], we will perform a perturbative expansion of $(d-2)^2 b\lambda^2$. In order to simplify the following calculation, we will express the Born–Infeld parameter b as

$$b_n = n\Delta b, \qquad n = 0, 1, 2, \ldots, \tag{15}$$

where $\Delta b = b_{n+1} - b_n$ is the step size of our iterative procedure. Considering the fact that

$$\begin{aligned}(d-2)^2 b\lambda^2 &= (d-2)^2 b_n \lambda^2 \\ &= (d-2)^2 b_n (\lambda^2|_{b_{n-1}}) + 0[(\Delta b)^2],\end{aligned} \tag{16}$$

where we have set $b_{-1} = 0$ and $\lambda^2|_{b_{-1}} = 0$, we will discuss the following two cases (note that the variable z has a range $0 \le z \le 1$):

Case 1. If $(d-2)^2 b_n(\lambda^2|_{b_{n-1}}) < 1$, we have

$$\begin{aligned}\zeta(z) &= \zeta_1(z) \\ &\approx \int_z^1 (d-2)\tilde{z}^{d-3}\left[1 - \frac{(d-2)^2 b_n(\lambda^2|_{b_{n-1}})\tilde{z}^{2(d-1)}}{2}\right]d\tilde{z} \\ &= (1-z^{d-2}) + \frac{(d-2)^3 b_n(\lambda^2|_{b_{n-1}})}{2(4-3d)}(1-z^{3d-4}).\end{aligned} \tag{17}$$

Case 2. If $(d-2)^2 b_n(\lambda^2|_{b_{n-1}}) > 1$, we set $(d-2)^2 b_n(\lambda^2|_{b_{n-1}}) \times \Lambda^{2(d-1)} = 1$ for $z = \Lambda$. Obviously, we find that $(d-2)^2 b_n(\lambda^2|_{b_{n-1}}) \times z^{2(d-1)} < 1$ for $z < \Lambda < 1$, which results in

$$\begin{aligned}\zeta(z) &= \zeta_{2A}(z) \\ &\approx \int_z^\Lambda (d-2)\tilde{z}^{d-3}\left[1 - \frac{(d-2)^2 b_n(\lambda^2|_{b_{n-1}})\tilde{z}^{2(d-1)}}{2}\right]d\tilde{z} \\ &\quad + \int_\Lambda^1 \frac{1}{\sqrt{b_n}(\lambda|_{b_{n-1}})\tilde{z}^2}\left[1 - \frac{1}{2(d-2)^2 b_n(\lambda^2|_{b_{n-1}})\tilde{z}^{2(d-1)}}\right]d\tilde{z} \\ &= -z^{d-2} + \frac{(d-2)z^{3d-4}}{2(3d-4)\Lambda^{2(d-1)}} \\ &\quad + \frac{3(d-1)[6+d(4d-9)]}{2(2d-1)(3d-4)}\Lambda^{d-2} \\ &\quad + (d-2)\Lambda^{d-1}\left[\frac{\Lambda^{2(d-1)}}{2(2d-1)} - 1\right],\end{aligned} \tag{18}$$

and $(d-2)^2 b_n(\lambda^2|_{b_{n-1}})z^{2(d-1)} > 1$ for $\Lambda < z \le 1$, which leads to

$$\begin{aligned}\zeta(z) &= \zeta_{2B}(z) \\ &\approx \int_z^1 \frac{1}{\sqrt{b_n}(\lambda|_{b_{n-1}})\tilde{z}^2}\left[1 - \frac{1}{2(d-2)^2 b_n(\lambda^2|_{b_{n-1}})\tilde{z}^{2(d-1)}}\right]d\tilde{z} \\ &= (d-2)\Lambda^{d-1}\left[\frac{\Lambda^{2(d-1)}}{2(2d-1)}\left(1 - z^{1-2d}\right) + \frac{1}{z} - 1\right].\end{aligned} \tag{19}$$

It should be noted that in both cases we observe that $\zeta(1) = 0$ from (17) and (19), which is consistent with the boundary condition $\phi(1) = 0$ given in (8).

Introducing a trial function $F(z)$ near the boundary $z = 0$ as

$$\psi(z) \sim \frac{\langle \mathcal{O} \rangle}{r_+^\Delta} z^\Delta F(z), \tag{20}$$

with the boundary conditions $F(0) = 1$ and $F'(0) = 0$, from Eq. (6) we can obtain the equation of motion for $F(z)$

$$(TF')' + T\left(P + \lambda^2 Q\zeta^2\right)F = 0, \tag{21}$$

with

$$\begin{aligned}&T = z^{1+2\Delta-d}(1-z^d), \qquad P = \frac{\Delta(\Delta-d)}{z^2} + \frac{\Delta f'}{zf} - \frac{m^2}{z^2 f}, \\ &Q = \frac{1}{f^2}.\end{aligned} \tag{22}$$

According to the S–L eigenvalue problem [32], we deduce the eigenvalue λ minimizes the expression

$$\lambda^2 = \frac{\int_0^1 T\left(F'^2 - PF^2\right)dz}{\int_0^1 TQ\zeta_1^2 F^2 dz}, \quad \text{for } (d-2)^2 b_n(\lambda^2|_{b_{n-1}}) < 1, \tag{23}$$

and

$$\lambda^2 = \frac{\int_0^1 T\left(F'^2 - PF^2\right)dz}{\int_0^\Lambda TQ\zeta_{2A}^2 F^2 dz + \int_\Lambda^1 TQ\zeta_{2B}^2 F^2 dz},$$
$$\text{for } (d-2)^2 b_n(\lambda^2|_{b_{n-1}}) > 1. \tag{24}$$

Using Eqs. (23) and (24) to compute the minimum eigenvalue of λ^2, we can obtain the critical temperature T_c for different Born–Infeld parameter b, spacetime dimension d and mass of the scalar field m from the following relation

$$T_c = \frac{d}{4\pi}\left(\frac{\rho}{\lambda_{min}}\right)^{\frac{1}{d-1}}. \tag{25}$$

In the following calculation, we will assume the trial function to be $F(z) = 1 - az^2$ with a constant a.

As an example, we will study the case for $d = 3$ and $m^2L^2 = -2$ with the chosen values of the Born–Infeld parameter b. Setting $\Delta b = 0.1$, for $b_0 = 0$ we use Eq. (23) and get

$$\lambda^2 = \frac{4(15 - 20a + 12a^2)}{10(9 - \sqrt{3}\pi - 3\ln 3) + 10(13 - 12\ln 3)a + (10\sqrt{3}\pi - 21 - 30\ln 3)a^2}, \tag{26}$$

whose minimum is $\lambda^2|_{b_0} = 17.31$ at $a = 0.6016$. According to Eq. (25), we can easily obtain the critical temperature $T_c = 0.1170\rho^{1/2}$, which is in good agreement with the numerical result

$T_c = 0.1184\rho^{1/2}$ [9]. For $b_1 = 0.1$, we can easily have $b_1(\lambda^2|_{b_0}) > 1$ and $\Lambda = [b_1(\lambda^2|_{b_0})]^{-1/4} = 0.8718$. Using Eq. (24) we arrive at

$$\lambda^2 = \frac{1 - \frac{4a}{3} + \frac{4a^2}{5}}{0.02060 - 0.01199a + 0.002659a^2}, \tag{27}$$

whose minimum is $\lambda^2|_{b_1} = 33.84$ at $a = 0.6532$. So the critical temperature $T_c = 0.09898\rho^{1/2}$, which also agrees well with the numerical finding $T_c = 0.1007\rho^{1/2}$ [11]. For $b_1 = 0.2$, we still have $b_2(\lambda^2|_{b_1}) > 1$ and $\Lambda = [b_2(\lambda^2|_{b_1})]^{-1/4} = 0.6200$. With the help of Eq. (24) we obtain

$$\lambda^2 = \frac{1 - \frac{4a}{3} + \frac{4a^2}{5}}{0.01176 - 0.006582a + 0.001450a^2}, \tag{28}$$

whose minimum is $\lambda^2|_{b_2} = 58.19$ at $a = 0.6640$. Therefore the critical temperature $T_c = 0.08644\rho^{1/2}$, which is again consistent with the numerical result $T_c = 0.08566\rho^{1/2}$ [11]. For other values of b, the similar iterative procedure can be applied to give the analytical result for the critical temperature.

In Table 1, we provide the critical temperature T_c of the chosen parameter b with the scalar operator $\langle\mathcal{O}\rangle = \langle\mathcal{O}_+\rangle$ for the $(2+1)$-dimensional superconductor if we fix the mass of the scalar field by $m^2L^2 = -2$ and the step size by $\Delta b = 0.1$. From Table 1, we observe that the differences between the analytical and numerical values are within 4.1%. Compared with the analytical results given in Table 1 of Ref. [29], the iterative procedure can further improve our analytical results and improve the consistency with the numerical findings.

Extending the investigation to the $(3+1)$-dimensional superconductor, in Table 2 we also give the critical temperature T_c for the scalar operator $\langle\mathcal{O}\rangle = \langle\mathcal{O}_+\rangle$ when we fix the mass of the scalar field $m^2L^2 = -3$ for different Born–Infeld parameter b by choosing the step size $\Delta b = 0.05$ and 0.025, respectively. Obviously, for the case of $\Delta b = 0.025$ the agreement of the analytical results derived from S–L method with the numerical calculation is impressive. Thus, we argue that, even in the higher dimension, the analytical results derived from the S–L method are in very good agreement with the numerical calculation. Furthermore, reducing the step size Δb reasonably, we can improve the analytical result and get the critical temperature more consistent with the numerical result.

From Tables 1 and 2, we point out that the critical temperature T_c decreases as the Born–Infeld parameter b increases for the fixed scalar field mass and spacetime dimension, which supports the numerical computation found in Refs. [11,12,21]. It is shown that the higher Born–Infeld electrodynamics corrections will make the scalar hair more difficult to be developed. On the other hand, the consistency between the analytical and numerical results indicates that the S–L method is a powerful analytical way to investigate the holographic superconductor with various condensates even when we take the Born–Infeld electrodynamics into account.

3.2. Critical phenomena

Since the condensation for the scalar operator $\langle\mathcal{O}\rangle$ is so small when $T \to T_c$, we can expand $\phi(z)$ in $\langle\mathcal{O}\rangle$ near the boundary $z = 0$ as

$$\frac{\phi(z)}{r_+} = \lambda\zeta(z) + \frac{\langle\mathcal{O}\rangle^2}{r_+^{2\Delta}}\chi(z) + \cdots, \tag{29}$$

with the boundary conditions $\chi(1) = 0$ and $\chi'(1) = 0$ [25,33,34]. Thus, substituting the functions (20) and (29) into (7), we keep terms up to $0(b)$ [29] to get the equation of motion for $\chi(z)$

$$(U\chi')' = \frac{2\lambda z^{1+2\Delta-d}F^2\zeta}{f}, \tag{30}$$

where we have introduced a new function

$$U(z) = \frac{e^{3b\lambda^2 z^4\zeta'^2/2}}{z^{d-3}}. \tag{31}$$

Making integration of both sides of Eq. (30), we have

$$\left[\frac{\chi'(z)}{z^{d-3}}\right]\Bigg|_{z\to 0} = \begin{cases} -\lambda\alpha_1, & \text{for } (d-2)^2 b_n(\lambda^2|_{b_{n-1}}) < 1, \\ -\lambda(\alpha_{2A}+\alpha_{2B}), & \text{for } (d-2)^2 b_n(\lambda^2|_{b_{n-1}}) > 1, \end{cases} \tag{32}$$

with

$$\alpha_1 = \int_0^1 \frac{2z^{1+2\Delta-d}F^2\zeta_1}{f}dz, \qquad \alpha_{2A} = \int_0^\Lambda \frac{2z^{1+2\Delta-d}F^2\zeta_{2A}}{f}dz,$$
$$\alpha_{2B} = \int_\Lambda^1 \frac{2z^{1+2\Delta-d}F^2\zeta_{2B}}{f}dz. \tag{33}$$

For clarity, we will fix the spacetime dimension d in the following discussion. Considering the case of $d = 3$ and the asymptotic behavior (9), for example, near $z \to 0$ we can arrive at

$$\frac{\rho}{r_+^2}(1-z) = \lambda\zeta(z) + \frac{\langle\mathcal{O}\rangle^2}{r_+^{2\Delta}}\left[\chi(0) + \chi'(0)z + \cdots\right]. \tag{34}$$

From the coefficients of the z^1 terms in both sides of the above formula, we can obtain

$$\frac{\rho}{r_+^2} = \lambda - \frac{\langle\mathcal{O}\rangle^2}{r_+^{2\Delta}}\chi'(0), \tag{35}$$

where $\chi'(0)$ can be easily calculated by using Eq. (32). Therefore we will know that

$$\langle\mathcal{O}\rangle = \beta T_c^\Delta\left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}, \tag{36}$$

where the coefficient β is given by

$$\beta = \begin{cases} \left(\frac{4\pi}{3}\right)^\Delta\sqrt{\frac{2}{\alpha_1}}, & \text{for } b_n(\lambda^2|_{b_{n-1}}) < 1, \\ \left(\frac{4\pi}{3}\right)^\Delta\sqrt{\frac{2}{\alpha_{2A}+\alpha_{2B}}}, & \text{for } b_n(\lambda^2|_{b_{n-1}}) > 1. \end{cases} \tag{37}$$

Obviously, the expression (36) is valid for different values of the Born–Infeld parameter and scalar field mass in the case of the $(2+1)$-dimensional superconductor. For concreteness, we will focus on the case for the mass of the scalar field $m^2L^2 = -2$ and the step size $\Delta b = 0.1$. Since in Ref. [11] the scalar operator is given by $\langle\mathcal{O}_+\rangle = \sqrt{2}\psi_+$ which is different from $\langle\mathcal{O}_+\rangle = \psi_+$ in this work, we present the condensation value $\gamma = \sqrt{2}\beta$ obtained by the analytical S–L method and from numerical calculation with the chosen values of the Born–Infeld parameter b for the $(2+1)$-dimensional superconductor in Table 3. We see that the condensation value γ increases as the Born–Infeld parameter b increases for the fixed scalar field mass and spacetime dimension, which indicates the consistent picture shown in T_c that the higher Born–Infeld electrodynamics corrections make the condensation to be formed harder. On the other hand, comparing with the analytical results shown in Table II of Ref. [29], we find that the iterative procedure indeed reduces the disparity between the analytical and numerical results.

Table 1
The critical temperature T_c obtained by the analytical S–L method and from numerical calculation [11] for the chosen values of the Born–Infeld parameter b in the case of 4-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2L^2 = -2$ and the step size by $\Delta b = 0.1$.

b	0	0.1	0.2	0.3
Analytical	$0.1170\rho^{1/2}$	$0.09898\rho^{1/2}$	$0.08644\rho^{1/2}$	$0.07586\rho^{1/2}$
Numerical	$0.1184\rho^{1/2}$	$0.1007\rho^{1/2}$	$0.08566\rho^{1/2}$	$0.07292\rho^{1/2}$

Table 2
The critical temperature T_c with the chosen values of the Born–Infeld parameter b and the step size Δb in the case of 5-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2L^2 = -3$.

b	0	0.1	0.2	0.3
Analytical ($\Delta b = 0.05$)	$0.1962\rho^{1/3}$	$0.1460\rho^{1/3}$	$0.1091\rho^{1/3}$	$0.07866\rho^{1/3}$
Analytical ($\Delta b = 0.025$)	$0.1962\rho^{1/3}$	$0.1329\rho^{1/3}$	$0.08754\rho^{1/3}$	$0.05195\rho^{1/3}$
Numerical	$0.1980\rho^{1/3}$	$0.1275\rho^{1/3}$	$0.08298\rho^{1/3}$	$0.05292\rho^{1/3}$

Table 3
The condensation value $\gamma = \sqrt{2}\beta$ obtained by the analytical S–L method and from numerical calculation [11] with the chosen values of the Born–Infeld parameter b in the case of 4-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2L^2 = -2$ and the step size by $\Delta b = 0.1$.

b	0	0.1	0.2	0.3
Analytical	92.80	117.92	137.22	161.14
Numerical	139.24	207.36	302.76	432.64

Table 4
The condensation value β obtained by the analytical S–L method with the chosen values of the Born–Infeld parameter b and step size Δb in the case of 5-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2L^2 = -3$.

b	0	0.1	0.2	0.3
$\Delta b = 0.05$	238.91	418.95	697.64	1195.56
$\Delta b = 0.025$	238.91	496.06	1005.38	2303.28

As another example, let us move on to the case of $d = 4$. From the asymptotic behavior (9), we can expand ϕ when $z \to 0$ as

$$\frac{\rho}{r_+^3}(1 - z^2) = \lambda\zeta(z) + \frac{\langle\mathcal{O}\rangle^2}{r_+^{2\Delta}}\left[\chi(0) + \chi'(0)z + \frac{1}{2}\chi''(0)z^2 + \cdots\right]. \quad (38)$$

Considering the coefficients of z^1 terms in above equation, we observe that $\chi'(0) \to 0$ if $z \to 0$, which is consistent with Eq. (32). Comparing the coefficients of the z^2 terms, we have

$$\frac{\rho}{r_+^3} = \lambda - \frac{\langle\mathcal{O}\rangle^2}{2r_+^{2\Delta}}\chi''(0), \quad (39)$$

where $\chi''(0)$ can be computed by using Eq. (32). So we can deduce the same relation (36) for the $(3+1)$-dimensional superconductor with the different condensation coefficient

$$\beta = \begin{cases} \pi^{\Delta}\sqrt{\frac{6}{\alpha_1}}, & \text{for } 4b_n(\lambda^2|_{b_{n-1}}) < 1, \\ \pi^{\Delta}\sqrt{\frac{6}{\alpha_{2A}+\alpha_{2B}}}, & \text{for } 4b_n(\lambda^2|_{b_{n-1}}) > 1. \end{cases} \quad (40)$$

In Table 4, we give the condensation value β obtained by the analytical S–L method with the chosen values of the Born–Infeld parameter b and step size Δb for the $(3+1)$-dimensional superconductor. In both cases we find again that, for the fixed scalar field mass and spacetime dimension, the condensation value β increases as the Born–Infeld parameter b increases, just as the observation obtained in the $(2+1)$-dimensional superconductor with Born–Infeld electrodynamics.

It should be noted that one can easily extend our discussion to the higher-dimensional superconductor and get our expression (36), although the coefficient β is different. Thus, near the critical point, the scalar operator $\langle\mathcal{O}\rangle$ will satisfy

$$\langle\mathcal{O}\rangle \sim (1 - T/T_c)^{1/2}, \quad (41)$$

which holds for various values of the Born–Infeld parameter b, spacetime dimension d and mass of the scalar field m. It shows that the phase transition is of the second order and the critical exponent of the system always takes the mean-field value 1/2. The Born–Infeld electrodynamics will not influence the result.

4. Conclusions

We have generalized the variational method for the S–L eigenvalue problem to analytically investigate the condensation and critical phenomena of the d-dimensional superconductors with Born–Infeld electrodynamics, which may help to understand the influences of the $1/N$ or $1/\lambda$ corrections on the holographic superconductor models. We found that the S–L method is still powerful to disclose the properties of the holographic superconductor with various condensates even when we take the Born–Infeld electrodynamics into account. Using the iterative procedure in the perturbative approach proposed by Banerjee et al. [29], we further improved the analytical results and the consistency with the numerical findings for the $(2+1)$-dimensional superconductor. Furthermore, extending the investigation to the higher-dimensional superconductor with Born–Infeld electrodynamics, we observed again that the analytical results derived from this method with a reasonable step size are in very good agreement with those obtained from numerical calculation. Our analytical result shows that the Born–Infeld parameter makes the critical temperature of the superconductor decrease, which can be used to back up the numerical findings as shown in the existing literatures that the higher Born–Infeld electrodynamics corrections can hinder the condensation to be formed. Moreover, with the help of this analytical method, we interestingly noted that the Born–Infeld electrodynamics, spacetime dimension and scalar mass cannot modify the critical phenomena, and found that the holographic superconductor phase transition belongs to the second order and the critical exponent of the system always takes the mean-field value. It should be noted that one can easily extend our technique to the holographic superconductor models with the logarithmic form [20] and exponential form [21] of nonlinear electrodynamics. More recently, a model of p-wave holographic superconductors from charged Born–Infeld black holes [35] via a Maxwell complex vector field model [36–38] was studied numerically. It would be of interest to generalize our study to this p-wave model and analytically discuss the effect of the Born–Infeld electrodynamics on the system. We will leave it for further study.

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