To assure the security, heavy monitoring and maintenance procedures for the fatigue of rails are set up by infrastructure managers. In order to go towards a more cost efficient railway system, it is essential to optimize the maintenance of rails: frequency of monitoring, rail replacement strategy, grinding policy...

To progress in this area, a numerical modeling tool has been developed thanks to a long-term collaboration between railway organizations (SNCF, RFF, RATP), rail producer (Tata Steel) and research institutes and universities (INRETS, LMS, MECAMIX, INSA) within the IDR2 consortium (Initiative for Development and for Research on Rail). This modelling starts with a dynamical simulation of the vehicle rolling on a track, from which the cyclic mechanical state of the rail is calculated by means of a 3D finite element simulation and an original and time-cost efficient direct stationary algorithm. Finally, a fatigue analysis of the rail is performed with the Dang Van criterion.

The modeling tool has been recently completed with the simulation of the crack propagation in the rails. A two-scale frictional contact fatigue crack model developed within the X-FEM framework is used to solve the crack problem. Using this approach, contact and friction between the crack faces is taken into account in the simulation. Realistic residual stresses, coming from dedicated software developed by SNCF are introduced in the propagation simulation via projection of the asymptotic mechanical fields. 2D Crack growth is performed taking into account the residual stresses. The results highlight their influence on the crack growth rate. Finally 3D preliminary results are introduced.

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Keynotes: Fatigue crack growth simulation; Rolling-sliding contact fatigue of rails; XFEM
1. Introduction

Due to the repeated passages of the wheels, rolling contact fatigue cracks can appear in the surface of the rails. These defects, such as squats and head-checks, can propagate and lead to the rail fracture and potentially to a derailment. Hence, when a rail break is detected, the traffic can be stopped or a circulation speed limitation is imposed until a rail replacement is carried out. To avoid such a nuisance, heavy monitoring and maintenance procedures for the fatigue of rails are set up by infrastructure managers. Quite important costs are generated and they may increase with the continuous traffic intensification and the global commercial running speed rise. In order to go towards a more cost efficient railway system, it is essential to optimize the maintenance of rails: frequency of monitoring, rail replacement strategy, grinding policy... One key to reach this target is to have a better understanding of the physical phenomena occurring within a fatigue crack propagation.

To progress in this area, a numerical modeling tool has been developed thanks to a long-term collaboration between railway organizations (SNCF, RFF, RATP), rail producer (Tata Steel) and research institutes and universities (INRETS, LMS, MECAMIX, INSA) within the IDR2 consortium (Initiative for Development and for Research on Rail). This modeling (see Fig. 1) starts with a dynamical simulation of the vehicle rolling on a track, from which the cyclic mechanical state of the rail is calculated by means of a 3D finite element simulation and an original and time-cost efficient direct stationary algorithm. Finally, a fatigue analysis of the rail is performed with the Dang Van criterion.

![Fig. 1. the numerical modelling process for the rail rolling contact fatigue assessment](image)

The final step consists of modeling the crack growth. This paper focuses on this last point: fatigue crack growth in the rails taking into account frictional contact between the crack faces, mixed-mode propagation and realistic residual stresses. Numerical fatigue crack growth is a large research topic which involves the understanding and the modeling of numerous local phenomena like confined plasticity or interfacial frictional contact. Contact with friction between the crack faces notably occurs in rolling contact fatigue problems. These possible time-dependent, multi-axial, non proportional loadings may lead to a crack, up to the development of very complex 3D crack network.

Modeling the crack propagation under rolling contact fatigue (RCF) requires to take into account different phenomena acting on different scales. At the structure scale, the wheel-rail contact imposes a very high gradient close to the wheel-rail contact area and leads to a multi-axial non proportional loading of the cracks. Moreover the repeated traffic of the wheel over the rail leads to residual stresses in the rail that will influence the crack propagation. All these solicitations generate complex sequences of opening, sticking and sliding conditions at the crack scale (Fig. 2).
Previous works about fatigue crack growth in the rails are available in the literature. Some authors studied the role of liquid entrapment using FEM analysis [1,2,3] or BEM analysis [4,5] in the crack growth mechanism. This effect is not considered in this work. Other works studied the influence of different parameters such as elastic foundation [6], the crack initial geometry [6,7,8,9] or the crack face friction coefficient [6,8,9,10] on the stress intensity factors (SIFs). In this paper we present 2D results and 3D preliminary results of a fatigue crack growth in the rails taking into account realistic residual stresses using a two-scale X-FEM/LATIN crack model with interfacial frictional contact.

2. Two-scale crack model with contact and friction between the crack faces and xfem discretization

We consider a cracked body \( \Omega \in \mathbb{R}^3 \) where contact and friction can occur along the crack faces \( \Gamma^+_c \) and \( \Gamma^-_c \). Under small displacement and small strain assumptions, we assume the interface \( \Gamma_c = \Gamma^+_c \cup \Gamma^-_c \) as an autonomous entity with its own nonlinear behavior. This fracture problem is divided in a global problem (structure scale \( \Omega \setminus \Gamma_c \)) and a local problem (crack scale \( \Gamma_c \)) (see Fig. 3).

Using the principle of virtual work we can write the work of the uncracked body, the interface problem and the coupling work and obtain a weak field formulation of the problem. Details can be found in [11,12]. At the local scale governing equations are the unilateral contact and Coulomb’s law.

The eXtended Finite Element Method [13] is used to model the crack propagation. In this method no explicit representation of the crack is needed. The crack is modeled using function enrichments. The crack discontinuity is introduced as a Heaviside step function. In addition, branch functions are introduced for all elements containing the
crack front. Hence, the mesh does not necessarily conform to the crack and both field interpolation and remeshing are not required during the possible crack propagation.

3. Introduction of realistic residual stresses

The evaluation of the mechanical state in the rail due to the contact stress induced by the train traffic is crucial for the modeling of the rail resistance: plastic deformations occur in the region near the contact zone due to repeated rolling–sliding contacts between the wheels and the rail. To be realistic, it is necessary to take into account this phenomenon which may be very significant for crack initiation and propagation in the rail head. It is well known that under repeated rolling contacts, different asymptotic mechanical states could occur in the structure: elasticity, elastic shakedown, plastic shakedown or ratcheting.

Determination of the stabilized state in the rail is performed by using sequentially VOCOLIN software and the stationary algorithm [14]. First, the contact between wheel and rail is evaluated by means of VOCOLIN. Its characteristics, which are number and dimensions of contact areas, normal and tangential pressure, can be Hertzian or non-Hertzian (Fig. 4(a)). Then, using the stationary algorithm, the stabilized mechanical state (residual stresses and plastic strain distribution) is computed. An elastic shakedown is obtained (Fig. 4(b)); all components of the plastic deformation tensor are constant along all the streamlines of the gauge corner. As a consequence, high cycle fatigue is likely to occur [15].

![Fig. 4. (a) Examples of normal contact pressure distribution (MPa) (wheel-rail contact) obtained by simulation (VOCOLIN). (b) Stabilized longitudinal plastic strain distribution [18].](image)

The meshes used for the computation of the asymptotic mechanical fields and the one used for the crack propagation are different (Fig. 5). Indeed, for the crack propagation, fine elements are required in the area where the crack propagates and the mesh can be coarser in the depth of the rail. Therefore the asymptotic mechanical fields are projected on the mesh used to model the crack propagation (see Fig. 5). Those fields are considered as the initial state of the propagation simulation. This state is permanent and non-uniform. No redistribution of residual stresses are considered throughout the crack growth. Since only elastic shakedown is considered, the fields after projection do not required to be re-balanced.
4. Crack growth under procedure rolling contact fatigue

A wheel passage on the crack corresponds to one loading. In this paper, wheel-rail contact is simply modeled as a fully sliding Hertzian load. Each cycle is divided in time steps corresponding to the position of the wheel with respect to the crack. For each position of the wheel, the crack body problem is solved and SIFs are computed using integral methods. At the end of a simulated cycle, the history of SIFs throughout the cycle is determined. Using this history, the crack growth path (direction) is predicted according to Hourlier and Pineau’s criterion, which has already given good agreements for fretting-fatigue experiments [16]. This criterion assumes that a crack follows the easiest path available that is the path along the direction \( \theta \) that maximizes the growth rate at the tip of an infinitesimal part \( s \) of the kinked crack \( \frac{da}{dN}(s, \theta) \) (Fig. 6.). Experimental crack growth rates under characteristic RCF load sequence on one hand, and on the other hand theoretical expressions of \( k_1 \) and \( k_2 \), the local stress intensity factors at the tip of the infinitesimal part, \( s \), inclined at an angle \( \theta \) of the original crack must be combined to use this criterion.

Finally a dedicated mixed-mode propagation law for RCF is used to predict the crack growth rate [9]. In the end of the cycle the new crack is created and the corresponding jump cycle is computed thanks to the propagation law. The procedure is repeated until no more cycles are required.

5. 2D results with realistic residual stresses

2D parametric studies have been performed to study and quantify the influence of the friction coefficient between the crack faces, the initial crack length, the crack orientation with the upper rail surface and the introduction of the residual stresses. Parameters are defined on Fig. 7.
The corresponding stress intensity factors with and without residual stresses are computed along the cycle and shown on Fig. 8. The results without residual stresses have been compared with [9] and show very good agreements. Introducing results stressed in such a configuration leads to a crack always closed since the crack tip is in an area where high compressive stresses occur (Fig. 8). This can be seen with KI always equal to 0. We can also see the residual stress effect on KII. It can be observe a translation of the KII values. The sliding between the crack faces is increased in one direction and decreased in the other one. This effect will influence the ratio KI/KII leading to a different crack growth rate.

We can compare on Fig. 9, the two crack growth path with and without residual stresses (free on Fig. 8). It first must be pointed out that in this case the crack growth path is mainly driven by the direction of the tangential loading. This is the reason why the crack tends to propagates upwards on Fig.9. The two crack growth path are very similar, but to reach the same length, it takes five times more cycle for the crack propagating in the residual stress field.
Different initial crack lengths with a fixed angle \( \theta \) have also been investigated to compute the evolution of the crack growth rate with the crack length (Fig. 10). Once more we can see (Fig. 10) that the introduction of residual stresses decreases the crack growth rate.

\[
\frac{da}{dN} (nm / cycle)
\]

![Graph showing crack growth rate with and without residual stresses for different initial crack lengths.](image)

6. 3D preliminary results

The development of the whole strategy implemented in CAST3M has been done for 2D and 3D crack growth. Only 3D preliminary results are available. A semi circular initial crack (diameter = 8mm) inclined of 90° has been considered. Fig. 11 illustrates the 3d crack behavior for the considered case.

![3D mesh used for the crack propagation, KII along the front and traction field between the crack faces for a given time step.](image)
7. Conclusions and prospects

This paper aims at predicting fatigue crack growth and branch conditions under RCF. A two-dimensional linear elastic numerical model for fatigue crack growth has been presented including contact with friction at crack interface. The model is based on a weak field formulation using X-FEM and an iterative scheme dedicated to non-linear interface problems adapted from the LATIN method. Using the tools already developed by SNCF to solve the wheel-rail contact problem and to compute the asymptotic stresses in the rail, realistic residual stresses have been introduced in the propagation model assuming elastic shakedown for the rail. 2D parametric studies are easily performed using this strategy. The same mesh is used for all the propagation simulations. 2D quantitative results are already available and emphasize the role of residual stresses in the crack growth rate.

Some short prospects for this work are to reach quantitative results for 3D crack growth, add the contribution of the rail bending by coupling the model with a macro model dedicated to the simulation of the rail bending. In the future, those results will be compared to the measurements coming from the railway network.

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