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# High-energy string scatterings of compactified open string

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## Abstract

We calculate high-energy massive string scattering amplitudes of compactified open string. We derive infinite linear relations, or stringy symmetries, among soft high-energy string scattering amplitudes of different string states in the Gross kinematic regime (GR). In addition, we systematically analyze all hard power-law and soft exponential fall-off regimes of high-energy compactified open string scatterings by comparing the scatterings with their 26D noncompactified counterparts. In particular, we discover the existence of a power-law regime at fixed angle and an exponential fall-off regime at small angle for high-energy compactified open string scatterings. The linear relations break down as expected in all power-law regimes. The analysis can be extended to the high-energy scatterings of the compactified closed string, which corrects and extends the previous results in [J.C. Lee, Y. Yang, Linear relations and their breakdown in high energy massive string scatterings in compact spaces, Nucl. Phys. B 784 (2007) 22].

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## 1. Introduction

There are three fundamental characteristics of high energy, fixed angle string scattering amplitudes [1–3], which are not shared by the field theory scatterings. These are the softer exponential fall-off behavior [1,4] (in contrast to the hard power-law behavior of field theory scatterings [5]), the infinite Regge-pole structure [6,7] of the form factor and the existence of infinite number of linear relations [6–17], or stringy symmetries, discovered recently among string scattering am-

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plitudes of different string states in the high-energy string scattering amplitudes. An important new ingredient to derive these linear relations is the zero-norm states (ZNS) [18–20] in the old covariant first-quantized (OCFQ) string spectrum. Other approaches related to this development can be found in [21–26].

It was believed [27,28] that the newly discovered linear relations are responsible for the softer exponential fall-off behavior of high-energy string scatterings. One way to justify this conjecture is to find more hard power-law high-energy string scatterings, which simultaneously show the breakdown of the above linear relations. However, it is well known that the genetic high-energy, fixed angle behavior of string scatterings is soft exponential fall-off rather than hard power-law. Recently, following an old suggestion of Mende [29], two of the present authors [28] calculated high-energy massive scattering amplitudes of closed bosonic string with some coordinates compactified on the torus. They obtained infinite linear relations among high-energy scattering amplitudes of different string states in the Gross kinematic regime (GR). This result is reminiscent of the existence of an infinite number of massive ZNS in the compactified closed string spectrum constructed in [30]. In addition, they discovered that, for some kinematic regime, these infinite linear relations break down and, simultaneously, the string amplitudes enhance to hard power-law behavior instead of the usual soft exponential fall-off behavior at high energies.

To further understand the relationship of the infinite linear relations and the softer exponential fall-off behavior of high-energy, fixed angle string scatterings, it is crucial to find more examples of high-energy string scatterings, which show the unusual power-law behavior and, simultaneously, give the breakdown of the infinite linear relations. In this paper, we calculate high-energy massive string scattering amplitudes of open bosonic string with some coordinates compactified on the torus. As in the case of compactified closed string, we obtain infinite linear relations among soft high-energy scattering amplitudes of different string states in the GR. This result is reminiscent of the existence of an infinite number of massive ZNS in the compactified open string spectrum constructed in [31]. More importantly, we analyze all possible hard power-law and soft exponential fall-off regimes of high-energy compactified open string scatterings by comparing the scatterings with their 26D noncompactified counterparts. In particular, we discover the existence of a power-law regime at fixed angle and an exponential fall-off regime at small angle for high-energy compactified open string scatterings. These new phenomena never happen in the 26D string scatterings. The linear relations break down as expected in all power-law regimes. The analysis can be extended to the high-energy scatterings of the compactified closed string, which corrects and extends the previous results in [28]. In particular, we correct the “Mende regime” discussed in [28], which is indeed exponential fall-off behaved rather than power-law claimed in [28]. As an example, we derive a hard power-law regime at fixed angle for high-energy compactified closed string scatterings. This paper is organized as follows. In Section 2 we calculated high-energy massive scattering amplitudes of compactified open string. In Section 3 we classify all kinematic regimes of the amplitudes and extend our results to the closed string case. A brief conclusion is given in Section 4.

## 2. High-energy scatterings

We consider 26D open bosonic string with one coordinate compactified on  $S^1$  with radius  $R$ . As we will see later, it is straightforward to generalize our calculation to more compactified

coordinates. The mode expansion of the compactified coordinate is

$$X^{25}(\sigma, \tau) = x^{25} + K^{25}\tau + i \sum_{k \neq 0} \frac{\alpha_k^{25}}{k} e^{-ik\tau} \cos n\sigma, \tag{1}$$

where  $K^{25}$  is the canonical momentum in the  $X^{25}$  direction

$$K^{25} = \frac{2\pi l - \theta_j + \theta_i}{2\pi R}. \tag{2}$$

Note that  $l$  is the quantized momentum and we have included a nontrivial Wilson line with  $U(n)$  Chan–Paton factors,  $i, j = 1, 2, \dots, n$ , which will be important in the later discussion. The mass spectrum of the theory is

$$M^2 = (K^{25})^2 + 2(N - 1) \equiv \left( \frac{2\pi l - \theta_j + \theta_i}{2\pi R} \right)^2 + \hat{M}^2, \tag{3}$$

where we have defined level mass as  $\hat{M}^2 = 2(N - 1)$  and  $N = \sum_{k \neq 0} \alpha_{-k}^{25} \alpha_k^{25} + \alpha_{-k}^\mu \alpha_k^\mu$ ,  $\mu = 0, 1, 2, \dots, 24$ . We are going to consider 4-point correlation function in this paper. In the center of momentum frame, the kinematics can be set up to be [28]

$$k_1 = \left( +\sqrt{p^2 + M_1^2}, -p, 0, -K_1^{25} \right), \tag{4}$$

$$k_2 = \left( +\sqrt{p^2 + M_2^2}, +p, 0, +K_2^{25} \right), \tag{5}$$

$$k_3 = \left( -\sqrt{q^2 + M_3^2}, -q \cos \phi, -q \sin \phi, -K_3^{25} \right), \tag{6}$$

$$k_4 = \left( -\sqrt{q^2 + M_4^2}, +q \cos \phi, +q \sin \phi, +K_4^{25} \right), \tag{7}$$

where  $p$  is the incoming momentum,  $q$  is the outgoing momentum and  $\phi$  is the center of momentum scattering angle. In the high-energy limit, one includes only momenta on the scattering plane, and we have included the fourth component for the compactified direction as the internal momentum. The conservation of the fourth component of the momenta implies

$$K_1^{25} - K_2^{25} + K_3^{25} - K_4^{25} = 0. \tag{8}$$

Note that

$$k_i^2 = K_i^2 - M_i^2 = -\hat{M}_i^2. \tag{9}$$

The center of mass energy  $E$  is defined as (for large  $p, q$ )

$$E = \frac{1}{2} \left( \sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2} \right) = \frac{1}{2} \left( \sqrt{q^2 + M_3^2} + \sqrt{q^2 + M_4^2} \right). \tag{10}$$

We have

$$\begin{aligned} -k_1 \cdot k_2 &= \sqrt{p^2 + M_1^2} \cdot \sqrt{p^2 + M_2^2} + p^2 + K_1^{25} K_2^{25} \\ &= \frac{1}{2} (s + k_1^2 + k_2^2) = \frac{1}{2} s - \frac{1}{2} (\hat{M}_1^2 + \hat{M}_2^2), \end{aligned} \tag{11}$$

$$\begin{aligned}
 -k_2 \cdot k_3 &= -\sqrt{p^2 + M_2^2} \cdot \sqrt{q^2 + M_3^2} + pq \cos \phi + K_2^{25} K_3^{25} \\
 &= \frac{1}{2}(t + k_2^2 + k_3^2) = \frac{1}{2}t - \frac{1}{2}(\hat{M}_2^2 + \hat{M}_3^2),
 \end{aligned}
 \tag{12}$$

$$\begin{aligned}
 -k_1 \cdot k_3 &= -\sqrt{p^2 + M_1^2} \cdot \sqrt{q^2 + M_3^2} - pq \cos \phi - K_1^{25} K_3^{25} \\
 &= \frac{1}{2}(u + k_1^2 + k_3^2) = \frac{1}{2}u - \frac{1}{2}(\hat{M}_1^2 + \hat{M}_3^2),
 \end{aligned}
 \tag{13}$$

where  $s$ ,  $t$  and  $u$  are the Mandelstam variables with

$$s + t + u = \sum_i \hat{M}_i^2 \sim 2(N - 4).
 \tag{14}$$

Note that the Mandelstam variables defined above are not the usual 25-dimensional Mandelstam variables in the scattering process since we have included the internal momentum  $K_i^{25}$  in the definition of  $k_i$ . We are now ready to calculate the high-energy scattering amplitudes. In the high-energy limit, we define the polarizations on the scattering plane to be

$$e^P = \frac{1}{M_2} \left( \sqrt{p^2 + M_2^2}, p, 0, 0 \right),
 \tag{15}$$

$$e^L = \frac{1}{M_2} \left( p, \sqrt{p^2 + M_2^2}, 0, 0 \right),
 \tag{16}$$

$$e^T = (0, 0, 1, 0),
 \tag{17}$$

where the fourth component refers to the compactified direction. It is easy to calculate the following relations:

$$e^P \cdot k_1 = -\frac{1}{M_2} \left( \sqrt{p^2 + M_1^2} \sqrt{p^2 + M_2^2} + p^2 \right),
 \tag{18}$$

$$e^P \cdot k_3 = \frac{1}{M_2} \left( \sqrt{q^2 + M_3^2} \sqrt{p^2 + M_2^2} - pq \cos \phi \right),
 \tag{19}$$

$$e^L \cdot k_1 = -\frac{p}{M_2} \left( \sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2} \right),
 \tag{20}$$

$$e^L \cdot k_3 = \frac{1}{M_2} \left( p \sqrt{q^2 + M_3^2} - q \sqrt{p^2 + M_2^2} \cos \phi \right),
 \tag{21}$$

$$e^T \cdot k_1 = 0, \quad e^T \cdot k_3 = -q \sin \phi.
 \tag{22}$$

In this paper, we will consider the case of a tensor state [28]

$$(\alpha_{-1}^T)^{N-2r} (\alpha_{-2}^L)^r |k_2, l_2, i, j\rangle
 \tag{23}$$

at a general mass level  $\hat{M}_2^2 = 2(N - 1)$  scattered with three ‘‘tachyon’’ states (with  $\hat{M}_1^2 = \hat{M}_3^2 = \hat{M}_4^2 = -2$ ). In general, we could have considered the more general high-energy state

$$(\alpha_{-1}^T)^{N-2r-2m-\sum_n n s_n} (\alpha_{-1}^L)^{2m} (\alpha_{-2}^L)^r \prod_n (\alpha_{-n}^{25})^{s_n} |k_2, l_2, i, j\rangle.
 \tag{24}$$

However, for our purpose here and for simplicity, we will not consider the general vertex in this paper. The  $s$ - $t$  channel of the high-energy scattering amplitude can be calculated to be (we will ignore the trace factor due to Chan–Paton in the scattering amplitude calculation; this does not

affect our final results in this paper)

$$\begin{aligned}
 A &= \int d^4x \langle e^{ik_1 X}(x_1) (\partial X^T)^{N-2r} (i\partial^2 X^L)^r e^{ik_2 X}(x_2) e^{ik_3 X}(x_3) e^{ik_4 X}(x_4) \rangle \\
 &= \int d^4x \cdot \prod_{i < j} (x_i - x_j)^{k_i \cdot k_j} \\
 &\quad \times \left[ \frac{ie^T \cdot k_1}{x_1 - x_2} + \frac{ie^T \cdot k_3}{x_3 - x_2} + \frac{ie^T \cdot k_4}{x_4 - x_2} \right]^{N-2r} \\
 &\quad \times \left[ \frac{e^L \cdot k_1}{(x_1 - x_2)^2} + \frac{e^L \cdot k_3}{(x_3 - x_2)^2} + \frac{e^L \cdot k_4}{(x_4 - x_2)^2} \right]^r. \tag{25}
 \end{aligned}$$

After fixing the  $SL(2, R)$  gauge and using the kinematic relations derived previously, we have

$$\begin{aligned}
 A &= i^N (-1)^{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3} (q \sin \phi)^{N-2r} \left( \frac{1}{M_2} \right)^r \int_0^1 dx x^{k_1 \cdot k_2} (1-x)^{k_2 \cdot k_3} \left[ \frac{1}{1-x} \right]^{N-2r} \\
 &\quad \times \left[ \frac{\sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2}}{x^2} - \frac{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi}{(1-x)^2} \right]^r \\
 &= (-1)^{k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3} (iq \sin \phi)^N \left( -\frac{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi}{M_2 q^2 \sin^2 \phi} \right)^r \\
 &\quad \times \sum_{i=0}^r \binom{r}{i} \left[ -\frac{\sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2}}{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi} \right]^i \\
 &\quad \times \int_0^1 dx \cdot x^{k_1 \cdot k_2 - 2i} (1-x)^{k_2 \cdot k_3 - N + 2i} \\
 &= (-iq \sin \phi)^N \left( -\frac{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi}{M_2 q^2 \sin^2 \phi} \right)^r \\
 &\quad \times \sum_{i=0}^r \binom{r}{i} \left[ -\frac{\sqrt{p^2 + M_1^2} \sqrt{p^2 + M_2^2} + p^2}{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi} \right]^i \\
 &\quad \times B \left( -\frac{1}{2}s + N - 2i - 1, -\frac{1}{2}t + 2i - 1 \right), \tag{26}
 \end{aligned}$$

where  $B(u, v)$  is the Euler beta function. We can do the high-energy approximation of the gamma function  $\Gamma(x)$  then do the summation, and end up with

$$\begin{aligned}
 A &= (-iq \sin \phi)^N \left( -\frac{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi}{M_2 q^2 \sin^2 \phi} \right)^r \\
 &\quad \times \sum_{i=0}^r \binom{r}{i} \left[ -\frac{\sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2}}{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi} \right]^i \\
 &\quad \times \frac{\Gamma(-1 - \frac{1}{2}s + N - 2i)\Gamma(-1 - \frac{1}{2}t + 2i)}{\Gamma(2 + \frac{1}{2}u)} \\
 &\simeq (-iq \sin \phi)^N \left( -\frac{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi}{M_2 q^2 \sin^2 \phi} \right)^r \\
 &\quad \times \sum_{i=0}^r \binom{r}{i} \left[ -\frac{\sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2}}{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi} \right]^i \\
 &\quad \times B\left(-1 - \frac{1}{2}s, -1 - \frac{1}{2}t\right) \left(-1 - \frac{1}{2}s\right)^{N-2i} \left(-1 - \frac{1}{2}t\right)^{2i} \left(2 + \frac{1}{2}u\right)^{-N} \\
 &= \left(-iq \frac{\sin \frac{\phi}{2}}{\cos \frac{\phi}{2}}\right)^N \left(-\frac{1}{M_2}\right)^r B\left(-1 - \frac{1}{2}s, -1 - \frac{1}{2}t\right) \\
 &\quad \times \left[ \frac{p\sqrt{q^2 + M_3^2} - q\sqrt{p^2 + M_2^2} \cos \phi}{q^2 \sin^2 \phi} - \frac{\sqrt{p^2 + M_1^2} + \sqrt{p^2 + M_2^2}}{q^2 \sin^2 \phi} \left(\frac{t}{s}\right)^2 \right]^r.
 \end{aligned} \tag{27}$$

### 3. Classification of compactified string scatterings

It is well known that there are two kinematic regimes for the high-energy string scatterings in 26D open bosonic string theory. The UV behavior of the finite and fixed angle scatterings in the GR is soft exponential fall-off. Moreover, there exist infinite linear relations among scatterings of different string states in this regime [6,8–16]. On the other hand, the UV behavior of the small angle scatterings in the Regge regime is hard power-law. The linear relations break down in the Regge regime. As we will see soon, the UV structure of the compactified open string scatterings is more richer. In the following, we will systematically analyze all possible hard power-law regimes of high-energy compactified open string scatterings by comparing the scatterings with their noncompactified counterparts. In particular, we show that all hard power-law regimes of high-energy compactified open string scatterings can be traced back to the Regge regime of the 26D high-energy string scatterings. The linear relations break down as expected in all power-law regimes. The analysis can be extended to the high-energy scatterings of the compactified closed string, which corrects and extends the previous results in [28].

3.1. *Gross regime—linear relations*

In the Gross regime,  $p^2 \simeq q^2 \gg K_i^2$  and  $p^2 \simeq q^2 \gg N$ , Eq. (27) reduces to

$$A \simeq \left(-iE \frac{\sin \frac{\phi}{2}}{\cos \frac{\phi}{2}}\right)^N \left(-\frac{1}{2M_2}\right)^r B\left(-1 - \frac{1}{2}s, -1 - \frac{1}{2}t\right). \tag{28}$$

For each fixed mass level  $N$ , we have the linear relation for the scattering amplitudes

$$\frac{\mathcal{T}^{(n,r)}}{\mathcal{T}^{(n,0)}} = \left(-\frac{1}{2M_2}\right)^r \tag{29}$$

with coefficients consistent with our previous results [6,8–16]. Note that in Eq. (28) there is an exponential fall-off factor in the high-energy expansion of the beta function. The infinite linear relation in Eq. (29) “soften” the high-energy behavior of string scatterings in the GR.

3.2. *Classification of compactified open string*

We first discuss the open string case. Since our definitions of the Mandelstam variables  $s, t$  and  $u$  in Eqs. (11)–(13) include the compactified coordinates, we can analyze the UV structure of the compactified string scatterings by comparing the scatterings with their simpler 26D counterparts. We introduce the space part of the momentum vectors

$$\mathbf{k}_1 = (-p, 0, -K_1^{25}), \tag{30}$$

$$\mathbf{k}_2 = (+p, 0, +K_2^{25}), \tag{31}$$

$$\mathbf{k}_3 = (-q \cos \phi, -q \sin \phi, -K_3^{25}), \tag{32}$$

$$\mathbf{k}_4 = (+q \cos \phi, +q \sin \phi, +K_4^{25}), \tag{33}$$

and define the “26D scattering angle”  $\tilde{\phi}$  as follows:

$$\mathbf{k}_1 \cdot \mathbf{k}_3 = |\mathbf{k}_1| |\mathbf{k}_3| \cos \tilde{\phi}. \tag{34}$$

It is then easy to see that the UV behavior of the compactified string scatterings is power-law if and only if  $\tilde{\phi}$  is small. This criterion can be used to classify all possible power-law and exponential fall-off kinematic regimes of high-energy compactified open string scatterings. We first consider the high-energy scatterings with one coordinate compactified.

3.2.1. *Compactified 25D scatterings*

I. For the case of  $\phi = \text{finite}$ , the only choice to achieve UV power-law behavior is to require (we choose  $K_1^{25} \simeq K_2^{25} \simeq K_3^{25} \simeq K_4^{25}$  and  $p \simeq q$  in the following discussion)

$$(K_i^{25})^2 \gg p^2 \simeq q^2 \gg N. \tag{35}$$

By the criterion of Eq. (34), this is a power-law regime. To explicitly show that this choice of kinematic regime does lead to UV power-law behavior, we will show that it implies

$$s = \text{const} \tag{36}$$

in the open string scattering amplitudes, which in turn gives the desired power-law behavior of high-energy compactified open string scattering in Eq. (27). On the other hand, it can be shown

that the linear relations break down as expected in this regime. For the choice of kinematic regime in Eq. (35), Eqs. (11) and (36) imply

$$\lim_{p \rightarrow \infty} \frac{\sqrt{p^2 + M_1^2} \cdot \sqrt{p^2 + M_2^2} + p^2}{K_1^{25} K_2^{25}} = \lim_{p \rightarrow \infty} \frac{\sqrt{p^2 + M_1^2} \cdot \sqrt{p^2 + M_2^2} + p^2}{\frac{2\pi l_1 - \theta_{j,1} + \theta_{i,1}}{2\pi R} \frac{2\pi l_2 - \theta_{j,2} + \theta_{i,2}}{2\pi R}} = -1. \quad (37)$$

For finite momenta  $l_1$  and  $l_2$ , Eq. (37) can be achieved by scattering of string states with “super-highly” winding nontrivial Wilson lines

$$(\theta_{i,1} - \theta_{j,1}) \rightarrow \infty, \quad (\theta_{i,2} - \theta_{j,2}) \rightarrow -\infty. \quad (38)$$

A careful analysis for this choice gives

$$(\lambda_1 + \lambda_2)^2 = 0, \quad (39)$$

where signs of  $\lambda_1 = \frac{p}{K_1^{25}}$  and  $\lambda_2 = -\frac{p}{K_2^{25}}$  are chosen to be the same. It can be seen now that the kinematic regime in Eq. (35) does solve Eq. (39).

We now consider the second possible regime for the case of  $\phi = \text{finite}$ , namely

$$(K_i^{25})^2 \simeq p^2 \simeq q^2 \gg N. \quad (40)$$

By the criterion of Eq. (34), this is an exponential fall-off regime. To explicitly show that this choice of kinematic regime does lead to UV exponential fall-off behavior, we see that, for this regime, it is impossible to achieve Eq. (36) since Eq. (39) has no nontrivial solution. Note that there are no linear relations in this regime. Although  $\phi = \text{finite}$  in this regime, it is different from the GR in the 26D scatterings since  $K_i^{25}$  is as big as the scattering energy  $p$ . *In conclusion, we have discovered a  $\phi = \text{finite}$  regime with UV power-law behavior for the high-energy compactified open string scatterings.* This new phenomenon never happens in the 26D string scatterings. *The linear relations break down as expected in this regime.*

II. For the case of small angle  $\phi \simeq 0$  scattering, we consider the first power-law regime

$$qK_1^{25} = -pK_3^{25} \quad \text{and} \quad (K_i^{25})^2 \simeq p^2 \simeq q^2 \gg N. \quad (41)$$

By the criterion of Eq. (34), this is a power-law regime. To explicitly show that this choice of kinematic regime does lead to UV power-law behavior, we will show that it implies

$$u = \text{const} \quad (42)$$

in the open string scattering amplitudes, which in turn gives the desire power-law behavior of high-energy compactified open string scattering in Eq. (27). For this choice of kinematic regime, Eqs. (13) and (41) imply

$$\lim_{p \rightarrow \infty} \frac{\sqrt{p^2 + M_1^2} \cdot \sqrt{q^2 + M_3^2} + pq}{K_1^{25} K_3^{25}} = -1. \quad (43)$$

By choosing different sign for  $K_1^{25}$  and  $K_3^{25}$ , Eq. (43) can be solved for any real number  $\lambda \equiv \frac{p}{K_1^{25}} = -\frac{q}{K_3^{25}}$ .

The second choice for the power-law regime is the same as Eq. (35) in the  $\phi = \text{finite}$  regime. The proof to show that it is indeed a power-law regime is similar to the proof in Section 1.



Table 1

$\phi$	$\tilde{\phi}$	UV behavior	Examples of the kinematic regimes	Linear relations
Finite	Finite	Exponential fall-off	$\vec{K}_i^2 \ll p^2 \simeq q^2 \gg N$	Yes
			$\vec{K}_i^2 \simeq p^2 \simeq q^2 \gg N$	No
$\phi \simeq 0$	$\tilde{\phi} \simeq 0$	Power-law	$\vec{K}_i^2 \gg p^2 \simeq q^2 \gg N$ and $\cos \delta \neq 0$	No
	$\tilde{\phi} \simeq 0$	Power-law	$\vec{K}_i^2 \gg p^2 \simeq q^2 \gg N$ and $\cos \delta = 0$	
$\phi \simeq 0$	Finite	Exponential fall-off	$\vec{K}_i^2 \ll p^2 \simeq q^2 \gg N$	No
			$\vec{K}_i^2 \simeq p^2 \simeq q^2 \gg N$ and $q\vec{K}_1 = -p\vec{K}_3$	
			$\vec{K}_i^2 \gg p^2 \simeq q^2 \gg N$ and $\cos \delta = 0$	
			$\vec{K}_i^2 \simeq p^2 \simeq q^2 \gg N$ and $q\vec{K}_1 \neq -p\vec{K}_3$	
			$\vec{K}_i^2 \gg p^2 \simeq q^2 \gg N$ and $\cos \delta \neq 0$	

The last choice for the power-law regime is

$$(K_i^{25})^2 \ll p^2 \simeq q^2 \gg N. \tag{44}$$

It is easy to show that this is indeed a power-law regime.

The last kinematic regime for the case of small angle  $\phi \simeq 0$  scattering is

$$qK_1^{25} \neq -pK_3^{25} \quad \text{and} \quad (K_i^{25})^2 \simeq p^2 \simeq q^2 \gg N. \tag{45}$$

By the criterion of Eq. (34), this is an exponential fall-off regime. We give one example here. Let us choose  $\lambda \equiv \frac{p}{K_1^{25}} \neq -\frac{q}{K_3^{25}} = 2\lambda$ . By choosing different sign for  $K_1^{25}$  and  $K_3^{25}$ , Eq. (43) reduces to

$$\lambda^2 = 0, \tag{46}$$

which has no nontrivial solution for  $\lambda$ , and one cannot achieve the power-law condition Eq. (42). So this is an exponential fall-off regime. *In conclusion, we have discovered a  $\phi \simeq 0$  regime with UV exponential fall-off behavior for the high-energy compactified open string scatterings.* This new phenomenon never happens in the 26D string scatterings. This completes the classification of all kinematic regimes for compactified 25D scatterings.

### 3.2.2. Compactified 24D (or less) scatterings

For this case, we need to introduce another parameter to classify the UV behavior of high-energy scatterings, namely the angle  $\delta$  between  $\vec{K}_1$  and  $\vec{K}_2$ ,  $\vec{K}_1 \cdot \vec{K}_2 = |K_1||K_2| \cos \delta$ . Similar results can be easily derived through the same method used in the compactified 25D scatterings. The classification is independent of the details of the moduli space of the compact spaces. We summarize the results in Table 1.

### 3.3. Classification of compactified closed string

The classification for the high-energy compactified open string scatterings can be easily extended to the case of compactified closed string [28]. For illustration, we discuss the case of  $\phi =$  finite regime with UV power-law behavior for the high-energy 25D compactified closed string scatterings. The kinematic set up for the compactified closed string is

$$k_{1L,R} = \left( +\sqrt{p^2 + M_1^2}, -p, 0, -K_{1L,R} \right), \tag{47}$$

$$k_{2L,R} = \left( +\sqrt{p^2 + M_2^2}, +p, 0, +K_{2L,R} \right), \tag{48}$$

$$k_{3L,R} = \left( -\sqrt{q^2 + M_3^2}, -q \cos \phi, -q \sin \phi, -K_{3L,R} \right), \tag{49}$$

$$k_{4L,R} = \left( -\sqrt{q^2 + M_4^2}, +q \cos \phi, +q \sin \phi, +K_{4L,R} \right), \tag{50}$$

where the left and right compactified momenta are defined to be

$$K_{L,R} = K \pm L = \frac{m}{R} \pm \frac{1}{2}nR, \tag{51}$$

where  $m$  is the quantized momentum and  $n$  is the winding number. The “super-highly” winding nontrivial Wilson line for the open string in Eq. (38) is replaced by “super-highly” closed string winding  $n_i$ . The condition to achieve power-law behavior for the compactified open string scatterings, Eq. (36), is replaced by

$$s_L = \text{const}, \quad s_R = \text{const} \tag{52}$$

for the compactified closed string scatterings. The left and the right Mandelstam variables are defined to be

$$s_{L,R} \equiv -(k_{1L,R} + k_{2L,R})^2, \tag{53}$$

$$t_{L,R} \equiv -(k_{2L,R} + k_{3L,R})^2, \tag{54}$$

$$u_{L,R} \equiv -(k_{1L,R} + k_{3L,R})^2, \tag{55}$$

with

$$s_{L,R} + t_{L,R} + u_{L,R} = \sum_i M_{iL,R}^2. \tag{56}$$

One can easily calculate the following kinematic relations:

$$\begin{aligned} -k_{1L,R} \cdot k_{2L,R} &= \sqrt{p^2 + M_1^2} \cdot \sqrt{p^2 + M_2^2} + p^2 + K_{1L,R}K_{2L,R} \\ &= \frac{1}{2}(s_{L,R} + k_{1L,R}^2 + k_{2L,R}^2) = \frac{1}{2}s_{L,R} - \frac{1}{2}(M_{1L,R}^2 + M_{2L,R}^2), \end{aligned} \tag{57}$$

$$\begin{aligned} -k_{2L,R} \cdot k_{3L,R} &= -\sqrt{p^2 + M_2^2} \cdot \sqrt{q^2 + M_3^2} + pq \cos \phi + K_{2L,R}K_{3L,R} \\ &= \frac{1}{2}(t_{L,R} + k_{2L,R}^2 + k_{3L,R}^2) = \frac{1}{2}t_{L,R} - \frac{1}{2}(M_{2L,R}^2 + M_{3L,R}^2), \end{aligned} \tag{58}$$

$$\begin{aligned} -k_{1L,R} \cdot k_{3L,R} &= -\sqrt{p^2 + M_1^2} \cdot \sqrt{q^2 + M_3^2} - pq \cos \phi - K_{1L,R}K_{3L,R} \\ &= \frac{1}{2}(u_{L,R} + k_{1L,R}^2 + k_{3L,R}^2) = \frac{1}{2}u_{L,R} - \frac{1}{2}(M_{1L,R}^2 + M_{3L,R}^2). \end{aligned} \tag{59}$$

It was calculated in [28] that the beta function part of the high-energy closed string scattering amplitude can be written as

$$\begin{aligned} A_{\text{closed}} &\sim B\left(-1 - \frac{t_R}{2}, -1 - \frac{u_R}{2}\right) B\left(-1 - \frac{t_L}{2}, -1 - \frac{u_L}{2}\right) \\ &= \frac{\Gamma(-\frac{t_R}{2} - 1)\Gamma(-\frac{u_R}{2} - 1)}{\Gamma(\frac{s_R}{2} + 2)} \frac{\Gamma(-\frac{t_L}{2} - 1)\Gamma(-\frac{u_L}{2} - 1)}{\Gamma(\frac{s_L}{2} + 2)}. \end{aligned} \tag{60}$$

It is easy to see that the condition (52) leads to the power-law behavior of the compactified closed string scattering amplitudes. To satisfy the condition in (52), we define the following “super-highly” winding kinematic regime

$$n_i^2 \gg p^2 \simeq q^2 \gg N_R + N_L. \quad (61)$$

Note that all  $m_i$  were chosen to vanish in order to satisfy the conservations of compactified momentum and winding number respectively [28]. For the choice of the kinematic regime in Eq. (61), Eqs. (57) and (52) imply

$$\lim_{p \rightarrow \infty} \frac{\sqrt{p^2 + M_1^2} \cdot \sqrt{p^2 + M_2^2} + p^2}{2(K_1 K_2 + L_1 L_2)} = \lim_{p \rightarrow \infty} \frac{\sqrt{p^2 + M_1^2} \cdot \sqrt{p^2 + M_2^2} + p^2}{2(\frac{m_1 m_2}{R^2} + \frac{1}{4} n_1 n_2 R^2)} = -1. \quad (62)$$

Note that since we have set  $m_i = 0$ , Eq. (62) is similar to Eq. (37) for the compactified open string case, and one can get nontrivial solution for Eq. (39) with signs of  $\lambda_1 = \frac{2p}{n_1 R}$  and  $\lambda_2 = -\frac{2p}{n_2 R}$  the same. This completes the discussion of power-law regime at fixed angle for high-energy compactified closed string scatterings. The “super-highly” winding regime derived in this subsection is to correct the “Mende regime”

$$E^2 \simeq M^2 \gg N_R + N_L \quad (63)$$

discussed in [28]. The regime defined in Eq. (63) is indeed exponential fall-off behavior rather than power-law claimed in [28].

#### 4. Conclusion

In this paper, we calculate high-energy massive string scattering amplitudes of compactified open string. We derive infinite linear relations among soft high-energy string scattering amplitudes of different string states at arbitrary but fixed mass level in the Gross kinematic regime (GR). We then systematically analyze all hard power-law and soft exponential fall-off regimes of high-energy compactified open string scatterings by comparing the scatterings with their 26D noncompactified counterparts. We classify all kinematic regimes for high-energy compactified open string scatterings. In particular, we discover the existence of a power-law regime at fixed angle and an exponential fall-off regime at small angle for high-energy compactified open string scatterings. These two new phenomena do not exist for high-energy 26D noncompactified open string scatterings. It is a stringy effect due to string compactification. The linear relations break down as expected in all power-law regimes. Finally, we have extended the analysis to the high-energy scatterings of the compactified closed string, which corrects and extends the previous results in [28].

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