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Comparing rankings from using TODIM and a fuzzy expert system

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Abstract

TODIM is, in its original formulation, an MCDA method developed to solve ranking problems. As an MCDA method TODIM combines the use of a multi-attribute value function as well as elements of the Outranking Approach, being founded on Prospect Theory. Recent advances in TODIM incorporate concepts from Fuzzy Sets. Although modelling multi-criteria decision problems with Fuzzy Sets has been utilized when the available data are imprecise, their use in MCDA is slightly controversial, because the data fuzzification can invalidate the outcome. Following a mixed qualitative-quantitative research strategy, our aim is to prove that for the ranking problems, TODIM can provide better solutions than Fuzzy Sets. Ranks from TODIM are linear, or strong, in a sense that it has no ties between the alternative solutions. The rank obtained with a Fuzzy Expert System can be weaker, that is, it may be a number of ties. The research strategy extends this result to ranking problems with the occurrence of crisp criteria.

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Keywords: Fuzzy Expert Systems; Ranking Problem; TODIM

1. Introduction

"On any one day people face a plethora of different decisions" [1]. This way, Multi-Criteria Decision Analysis (MCDA) methods have been developed to support decision makers in their decision problems. One reason for different MCDA methods is that there are different decision problems. First classifications of decision problems are Discrete Problem versus Continuous Problem. A Discrete Problem involves a discrete set of alternative solutions. A Continuous Problem involves a case where the number of possible alternatives are infinite [2]. There are four types of Discrete Problem: selecting an alternative solution (Choice Problem),

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grouping alternatives (Sorting Problem), ordering alternatives from best to worst (Ranking Problem), or better descripting alternatives (Description Problem) [3]. This work focuses the Ranking Problem.

The large number of MCDA methods engendered classifications for the methods. American School and European School [4] are, perhaps, the most well-known. These classifications are criticized, not only for xenophobia, but also to difficult developments by international teams [5]. Aggregation Approach and Outranking Approach are better classifications. However, both sets of classification shave often the same result. For instance, Analytic Hierarchy Process (AHP) and Multi-Attribute Utility Theory (MAUT) are MCDA methods for the Aggregation Approach, and they are from American School [6-7]; Elimination et Choix Traduisant la Realité (ELECTRE) and Preference Ranking Organization Method for Enriched Evaluation (PROMETHEE) are MCDA methods for Outranking Approach, and they are from European School [8-9]. An exception is Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), which is a MCDA method for Aggregation Approach, and is from European School [10].

Matter of fact, the choice of an MCDA method shall be based on characteristics from the decision problem, including necessary data and expected results. Nevertheless, the choice for an MCDA method have been a matter of opinion. This way, decision maker chooses to apply a familiar MCDA method to solve a new decision problem. Since decision maker has already applied this method, the new application gains in feasibility. On another way, the decision maker may choose a not familiar method. Or else, a method never applied before, just to expand decision maker's knowledge in MCDA practice.

Different MCDA methods may yield different results when applied to the same problem [11]. Still, a single method application can lead to different ranks. This can be a result from different individuals providing data, or it can be resulted from time-lapses in data collection. This work addresses the divergence between ranks from different MCDA applications with the concepts of rank correlation [12].

Multi-Criteria Interactive Decision-Making (shorted as TODIM, from Portuguese) is an MCDA method [13] developed to solve the Ranking Problem, TODIM combines elements from both Aggregation Approach and Outranking Approach. It first application was to rank projects with environmental impacts. Later, TODIM incorporated elements from Prospect Theory [14-15]. The previous case on environmental impacts was analyzed. The ranks with Prospect Theory diverge from the original rank. However, the ranks have some degree of correlation.

The most well-know TODIM application was to rank residential properties [16]. It was recently revisited, considering criteria interactions [17]. The ranks with criteria interactions and with TODIM also diverge each other. However, the top two alternatives are the same from both applications. That is, the ranks are correlated each other. Recent advances in TODIM incorporate concepts from Fuzzy Sets [18-19].

Fuzzy Sets Theory (FST) was firstly proposed for the Classification Problem [20]. "A fuzzy set is a class of object with a continuum of grades of memberships. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one" [21]. In Classical Sets Theory (CST), sets are crisp. That is, an element belongs to a set or not. Then, when the available data are imprecise, FST is expected to better solve Classification Problem than CST. Therefore, new versions of classical MCDA methods were developed to incorporate FST [22-24].

FST have also been successfully applied to the Ranking Problem [25-26]. However, the use of Fuzzy Sets in MCDA is slightly controversial [27]. When these data are allowed to vary in choice over the values of a scale, as in AHP, these data are themselves already fuzzy [28]. Then, data fuzzification can invalidate the outcome. On the same issue, in real life crisp sets do exist [29]. Therefore, the use of Fuzzy Sets may result in loss of information, when transforming precise data in imprecise information.

This work takes place on the side of questioning the indiscriminate use of FST in MCDA. Our aim is to prove that, for the Ranking Problem, TODIM can provide a better solution than FST. A mixed qualitative-quantitative research strategy [30] was adopted. That is, the goal is not to consider exhaustive cases.

Next section presents concepts on rank correlation, TODIM, and FST. Section 3 presents a case of Ranking Problem from a Brazilian real estate market. This problem involves fifteen alternatives and eight criteria, with a crisp criterion. The rank from TODIM is linear, or strong, in a sense that it has no ties between the alternative solutions. The rank obtained with a FST is weaker, that is, it has a number of ties. The research strategy extends this result to ranking problems with the occurrence of crisp criteria. Section 4 presents some conclusions and proposal for future researches.

2. Theory Background

2.1. Correlation between ranks

As observed in Section 1, different MCDA methods may yield different ranks to the same problem [11]. The rank correlation coefficient is a measure of ranks agreement [12]. For two ranks of n elements, A and B, Kendall coefficient, τ_b, is obtained by Equation 1, when a_{ij} and b_{ij} are score matrices obtained by Equation 2.

$$\tau_b = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} b_{ij}}{n(n-1)} \tag{1}$$

$$a_{ij} = \begin{cases} 1 & \text{object } i \text{ is ranked ahead of object } j \\ -1 & \text{object } i \text{ is ranked behind of object } j \\ 0 & \text{objects } i \text{ and } j \text{ are tied} \end{cases} \tag{2}$$

Two identical ranks will have agreements in all positions, $a_{ij} = b_{ij}$, then $\tau_b = 1$. For opposite ranks, that is two ranks with no agreement on any position, $\tau_b = -1$. For instance, if A = (1, 2, 3, 4), B = (1, 3, 2, 4), and C = (4, 3, 2, 1), we have τ_b (A, B) ≈ 0.67, τ_b (A, C) = –1, and τ_b (B, C) ≈ – 0.67. Ranging from –1 to 1, Kendall coefficient measures the closeness of correspondence between two given ranks. In other words, it measures the compatibility between two ranks [31].

Kendall coefficient performs satisfactory to linear ranks. However, it presents some difficulties with partial and weak ranks. Linear ranks has no ties; weak ranks permits ties; two or more ranks are partial when at least one of them is incomplete, in the sense that each ranker may not necessarily rank all of the objects [32].

Emond-Mason coefficient, τ_x, differs from Kendall coefficient by using a value of one for a_{ij} and b_{ij} to represent ties instead of the value of zero used by Kendall's. By extending this interchange to accommodate ties, τ_x is not flawed as τ_b for weak ranks or for partial ranks [12]. An MCDA application may result weak ranks; a Fuzzy System may result weak ranks, too. Then, τ_x will be adopted in this work, instead of τ_b.

2.2. TODIM method

TODIM has similarities with other MCDA methods as ELECTRE [33] and PROMETHEE [34]. However, while practically all other MCDA methods start from the premise that the decision maker always looks for some maximum overall value, TODIM method makes use of a measurement of overall value calculable according to Prospect Theory.

TODIM application requires numerical values for the evaluation of the alternatives regarding the criteria. For qualitative criteria, alternatives can be evaluated in a verbal scale, but it must be then transformed into a cardinal scale. The numerical evaluation for the alternatives regarding to all the criteria composes the matrix of evaluation. This matrix must be normalized, for each criterion: the value for one alternative must be divided by the sum of values for all the alternatives. This way, a stochastic matrix is obtained, that is, a matrix where all the components are in-between zero to one, and every column sums equal to one. This is the matrix of normalized alternatives' scores against criteria, $\mathbf{P} = p_{nm}$, with n indicating the number of alternatives and m the number of criteria.

The next step is the attribution of weights for the criteria. Usually, weights are attributed by DM using a linear 1 to 5 scale, similar to the Likert scale (Likert, 1932) [35] The decision makers must indicate a criterion r as the reference criterion. The criterion with the highest weight is usually chosen. The vector of weights, $\mathbf{w_r} = w_{rc}$, is composed by the weight of the criterion c divided by the weight of the reference criterion r.

The measurement of dominance $\delta(A_i, A_j)$ of each alternative A_i over each alternative A_j, incorporate concepts of Prospect Theory, according to Equation 3.

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \Phi_c(A_i, A_j), \forall(i, j) \quad (3)$$

Where:

$$\Phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{rc}(P_{ic} - P_{jc})}{\sum_{c=1}^{m} w_{rc}}} & \text{if} \quad (P_{ic} - P_{jc}) > 0 \\ 0 & \text{if} \quad (P_{ic} - P_{jc}) = 0 \\ \dfrac{-1}{\theta}\sqrt{\dfrac{(\sum_{c=1}^{m} w_{rc})\ (P_{jc} - P_{ic})}{w_{rc}}} & \text{if} \quad (P_{ic} - P_{jc}) < 0 \end{cases}$$

The expression $\Phi_c(A_i, A_j)$ is the contribution of criterion c to the dominance of alternative A_i over alternative A_j. If p_{ic} was greater than p_{jc}, it will represent a gain for$\delta(A_i, A_j)$; if p_{ic} and p_{jc} were equal, then a zero will assigned to $\delta(A_i, A_j)$; if p_{ic} was less than p_{jc}, then $\Phi_c(A_i, A_j)$ will be a loss to $\delta(A_i, A_j)$.

The function $\Phi_c(A_i, A_j)$ allows the adjustment of problem data to the Prospect Theory, that is, considering the aversion to risk and the propensity to risk. This function has the shape of an ''S'', as presented in Figure 1.

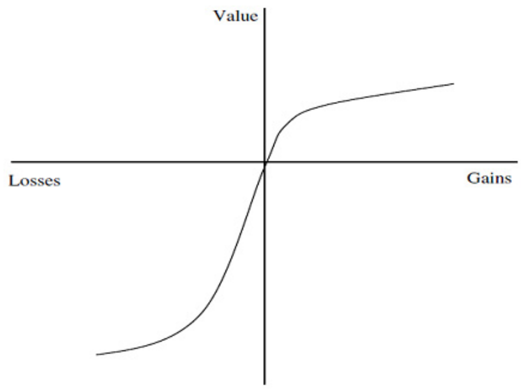

Figure 1. Value function of the TODIM method [15]

Above the horizontal axis, that is for value equal to zero, there is a concave curve representing the gains; below the horizontal axis, there is a convex curve representing the losses. The concave part reflects the aversion to risk in the face of gains and the convex part, in turn, symbolizes the propensity to risk when dealing with losses. θ is the attenuation factor of the losses. Different choices of θ lead to different shapes of the prospect theoretical value function in the negative quadrant.

The overall value for alternative A_i,ξ_i, is obtained with Equation 4.

$$\xi_i = \frac{\sum_{j=1}^{n}\delta(A_i, A_j) - \min\sum_{j=1}^{n}\delta(A_i, A_j)}{\max\sum_{j=1}^{n}\delta(A_i, A_j) - \min\sum_{j=1}^{n}\delta(A_i, A_j)} \quad (4)$$

2.3. Fuzzy expert systems

Expert system is an information system that emulates the decision-making ability of a human expert [36]. An expert system is typically made of three parts: a Knowledge Base, an Inference Engine and a Working Memory [37]. The Working Memory is the stored information gained by the user of the system. The Inference Engine uses the Knowledge Base together with information from the problem to provide an expert solution.

If–Then rules are popular schemes for knowledge representation as "If premise then conclusion". In a Fuzzy Expert System, premises and conclusion are fuzzy propositions, as in "If X is *small* then Y is *large* with a *certainty factor* equal to 0.8" [28], for instance, because Small and Large are fuzzy sets.

Several membership functions can be used in the definition of a fuzzy set. One of the most used is the triangular function [38].As presented in Figure 2, a triangular fuzzy set has a triangular membership function. A triangular fuzzy set is usually represented as a vector, (x_1, x_2, x_3), where $\mu_A(x_1) = \mu_A(x_1) = 0$, and $\mu_A(x_1) = 1$.

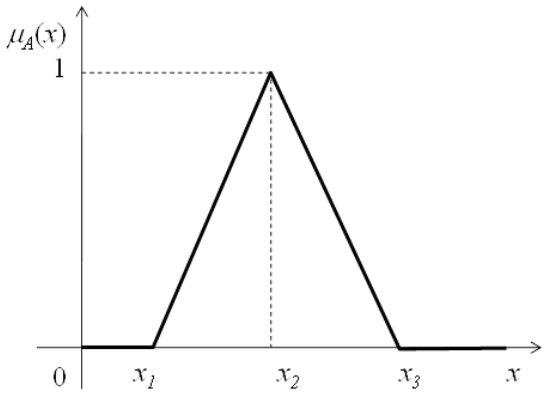

Figure 2. Triangular fuzzy set A = (x_1, x_2, x_3)

One of the most popular Fuzzy Expert System is the Mamdani Model [39]. In a Mamdani Model, If–Then rules may have several clauses as If *A* and *B* and *C*... Then *Z*, where all *A*, *B*, *C*… and *Z* are fuzzy propositions. For every clause in the rule, the matching degree between the current value for the variable and a linguistic label must be computed. The clauses are aggregated, using the minimum fuzzy operator. If more than one rule implies in the same result, the rules must be aggregated, using the maximum fuzzy operator. The overall matching degree can be obtained, also using the minimum fuzzy operator. This degree is referred as alpha-cut, or α-cut [40]. The α-cut level will generate a new fuzzy set, with a trapezoidal membership function, as presented in 错误!未找到引用源。. A real number may be obtained from the centroid of gravity (COG) of the resulting fuzzy set, within a process referred as defuzzification [41].

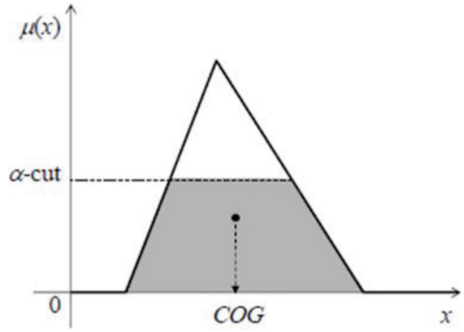

Figure 3. Defuzzification by the centroid of gravity

3. Illustrative Case

3.1. Data collection

Volta Redonda is a Brazilian city situated in the south of the State of Rio de Janeiro. The city has approximately 260,000 inhabitants [42]. There is a large number of properties, residential and commercial, rented or available for rent. The major steel plant installed in the city in the 1940's is a landmark of Brazilian industrialization. Because of this industrial vocation, Volta Redonda was nicknamed Steel City. However, its economy is quite diverse on services as education and transportation, to name a few.

Local real estate agents mentioned eight criteria for the selection of a residential property: Location (C_1), Constructed Area (C_2), Construction Quality (C_3), State of Conservation (C_4), Garage Spaces (C_5), Rooms (C_6), Attractions (C_7), and Security (C_8). Criteria C_2, C_5, C_6, and C_8 are quantitative; C_2 is measured in m^2; C_5 and C_6 are measured in unities of rooms or space garages; and $C_8$is a crisp criterion, since a residential property has security or has not. Tables 1 to 4 present the possible scores to evaluate alternatives according to qualitative criteria.

Table 1. Possible scores to C_1

Location	Score
Periphery	1
Between periphery and an average location	2
Average location	3
Good location	4
Excellent location	5

Table 2. Possible scores to C_2

Construction Quality	Score
Low standard	1
Average standard	2

High standard	3

Table 3. Possible scores to C_3

State of Conservation	Score
Bad	1
Average	2
Good	3
Very good	4

Table 4. Possible scores to C_4

Attractions	Score
Without attractions	0
Backyard or terrace	1
Barbecue	2
Swimming pool	3
Swimming pool, barbecue and others	4

Weights from 1 to 5 must be assigned to the criteria, where 1 goes to the lowest important criterion and 5 to the highest important criterion. Location (C_1) was indicated as the reference criterion. Table 5 presents the assigned weighted and the normalized weights, that is, summing equal to one.

Table 5. Weights of criteria

Criterion	Assigned weight	Normalized weight
Localization (C_1)	5	0.25
Constructed Area (C_2)	3	0.15
Construction Quality(C_3)	2	0.10
State of Conservation (C_4)	4	0.20
Garage Spaces (C_5)	1	0.05
Rooms (C_6)	2	0.10
Attractions (C_7)	1	0.05
Security (C_8)	2	0.10

Fifteen residential properties in different neighborhoods of Volta Redonda were evaluated. These alternatives were simply named as A_1 to A_{15}. Table 6 presents the scores assigned to the alternatives according to the qualitative criteria (C_1, C_3, C_4, and C_7) and real data for the quantitative criteria (C_2, C_5, C_6, and C_8).

Table 6. Data and assigned scores of residential properties

Residential properties	C1	C2	C3	C4	C5	C6	C7	C8
A_1	3	290	3	3	1	6	4	0
A_2	4	180	2	2	1	4	2	0
A_3	3	347	1	2	2	5	1	0
A_4	3	124	2	3	2	5	4	0
A_5	5	360	3	4	4	9	1	1
A_6	2	89	2	3	1	5	1	0
A_7	1	85	1	1	1	4	0	1
A_8	5	80	2	3	1	6	0	1
A_9	2	121	2	3	0	6	0	0
A_{10}	2	120	1	3	1	5	1	0
A_{11}	4	280	2	2	2	7	3	1
A_{12}	1	90	1	1	1	5	2	0

A_{13}	2	160	3	3	2	6	1	1
A_{14}	3	320	3	3	2	8	2	1
A_{15}	4	180	2	4	1	6	1	1

Table 7 presents the normalized score for the residential properties against the criteria.

Table 7. Normalized scores for the residential properties against the criteria

Residential properties	C1	C2	C3	C4	C5	C6	C7	C8
A_1	0.068	0.103	0.100	0.075	0.045	0.069	0.174	0
A_2	0.091	0.064	0.067	0.050	0.045	0.046	0.087	0
A_3	0.068	0.123	0.033	0.050	0.091	0.057	0.043	0
A_4	0.068	0.044	0.067	0.075	0.091	0.057	0.174	0
A_5	0.114	0.127	0.100	0.100	0.182	0.103	0.043	0.143
A_6	0.045	0.031	0.067	0.075	0.045	0.057	0.043	0
A_7	0.023	0.030	0.033	0.025	0.045	0.046	0	0.143
A_8	0.114	0.028	0.067	0.075	0.045	0.069	0	0.143
A_9	0.045	0.043	0.067	0.075	0	0.069	0	0
A_{10}	0.045	0.042	0.033	0.075	0.045	0.057	0.043	0
A_{11}	0.091	0.099	0.067	0.050	0.091	0.080	0.130	0.143
A_{12}	0.023	0.032	0.033	0.025	0.045	0.057	0.087	0
A_{13}	0.045	0.057	0.100	0.075	0.091	0.069	0.043	0.143
A_{14}	0.068	0.113	0.100	0.075	0.091	0.092	0.087	0.143
A_{15}	0.091	0.064	0.067	0.100	0.045	0.069	0.043	0.143

The overall values presented in Table 8 were obtained simply by aggregating the normalized scores for the residential properties (Table 7), weighted by the normalized vector (Table 5). The bolded A_5, A_{11}, and A_{14} have the highest overall values; the stricken throughA_7, A_9, A_{10}, and A_{12} have the lowest values.

Table 8. Overall values for the residential properties

Residential properties	Overall value	Rank
A_1	0.301	6
A_2	0.241	10
A_3	0.245	9
A_4	0.257	8
A_5	**0.454**	**1**
A_6	0.192	11
~~A_7~~	~~0.159~~	~~14~~
A_8	0.311	5
~~A_9~~	~~0.185~~	~~12~~
~~A_{10}~~	~~0.185~~	~~12~~
A_{11}	**0.351**	**3**
~~A_{12}~~	~~0.125~~	~~15~~
A_{13}	0.291	7
A_{14}	**0.366**	**2**
A_{15}	0.338	4

3.2. TODIM application

As in many other TODIM applications [43], $\theta = 1$ is adopted. To illustrate TODIM computation, from Table 2, let us consider the pair A_2 and A_4:

For C_1, $p_{21}>p_{41}$, then $\Phi_1 = \sqrt{\frac{1(0.091-0.068)}{4}} \cong 0.075$

For C_2, $p_{22}>p_{42}$, then $\Phi_2 = 0.054$

For C_3, $p_{23} = p_{43}$, then $\Phi_3 = 0$
For C_4, $p_{24} < p_{44}$, then $\Phi_4 = \frac{-1}{1}\sqrt{\frac{4(0.075-0.050)}{0.8}} \cong -0.353$
For C_5, $p_{25} < p_{45}$, then $\Phi_5 = -0.959$
For C_6, $p_{26} < p_{46}$, then $\Phi_6 = -0.331$
For C_7, $p_{27} < p_{47}$, then $\Phi_7 = -1.319$
For C_8, $p_{28} = p_{48}$, then $\Phi_8 = 0$

Then, substituting values in Equation 1, $\delta(A_2, A_4) \cong -2.833$. In analogy, all other $\delta(A_2,A_j)$ can be found, and adding then, $\sum_{j=1}^{n} \delta(A_1, A_j) = -27.02$. The minimum and maximum sums are $\sum_{j=1}^{n} \delta(A_7, A_j) = -44.23$ and $\sum_{j=1}^{n} \delta(A_5, A_j) = 0.343$. By substituting in Equation 4, one gets $\xi_2 = \frac{-27.02+44.23}{0.343+44.23} \cong 0.386$.

Table 9 presents he overall values for the residential properties with TODIM. It can be noted in Tables 8 and 9 that the incorporation of Prospect Theory implies in a different rank. However, the top three (the bolded A_5, A_{11} and A_{14}) and the bottom four (A_7, A_9, A_{10}, and A_{12}) will be the same. The Emond-Mason coefficient computed for these ranks is $\tau_x \approx 0.733$, which indicates a positive correlation.

Table 9. Overall values for the residential properties with TODIM

Residential properties	Overall value	Rank
A_1	0.692	5
A_2	0.386	10
A_3	0.399	9
A_4	0.621	7
A_5	**1**	**1**
A_6	0.286	11
~~A_7~~	~~0~~	~~15~~
A_8	0.441	8
~~A_9~~	~~0.020~~	~~14~~
~~A_{10}~~	~~0.213~~	~~12~~
A_{11}	**0.858**	**3**
~~A_{12}~~	~~0.107~~	~~13~~
A_{13}	0.719	4
A_{14}	**0.937**	**2**
A_{15}	0.673	6

3.3. Fuzzy expert system application

A fuzzy expert system was developed, as presented in Figure 4. To facilitate the implementation of the expert system, the Software FuzzyTECH [44] was selected. The choice for FuzzyTECH was mainly because of it has been successfully applied in MCDA practice [25].

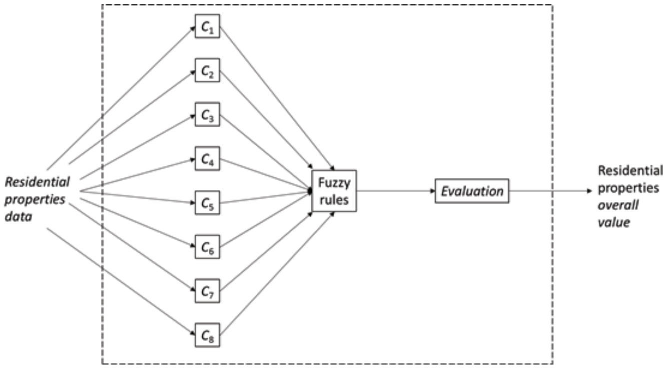

Figure 4. Fuzzy expert system to evaluate residential properties

Three triangular fuzzy sets were defined for each qualitative criteria (C_1, C_3, C_4 and C_7): Bad (2, 2, 3), Average (2, 3, 4), and Good (3, 4, 4). For the quantitative criteria (C_2, C_5, C_6 and C_8), only one set was defined: Good. For Evaluation, two sets were defined: Bad (0.25, 0.75, 0.75) and Good (0.25, 0.25, 0.75). Figures 5 to 7 present the triangular fuzzy sets for Location, the fuzzy set for Constructed Area, and the triangular fuzzy sets for Evaluation.

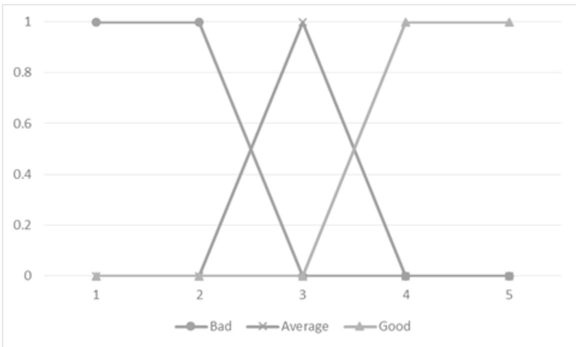

Figure 5. Triangular fuzzy sets for C_1

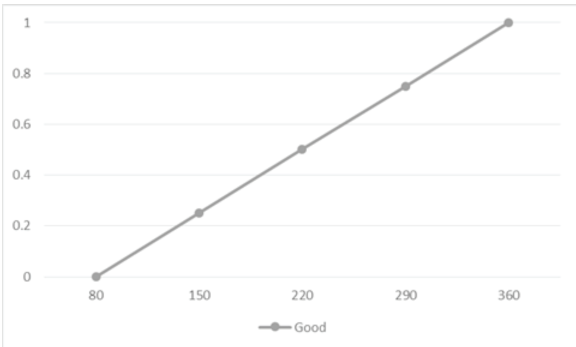

Figure 6. Triangular fuzzy set for C_2

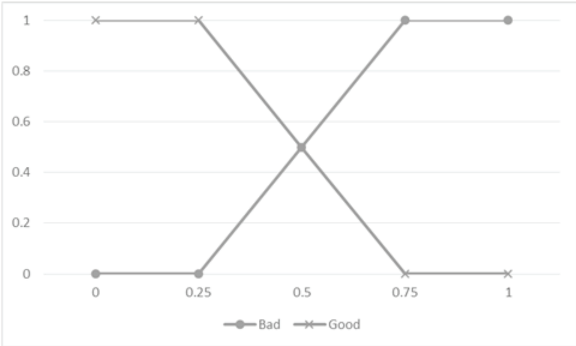

Figure 7. Triangular fuzzy sets for Evaluation

The fuzzy expert system is accomplished in $3^4 = 81$ rules. In FuzzyTECH, these rules were created in groups of three rules, inserted in blocks of three groups. Table 10 presents the first three and the latest three groups. When there is, at least, one Bad set, the rule output will be Evaluation equals to Bad. Otherwise, the output will be Evaluation equals to Good.

Table 10. Fuzzy rules

	Input				Output
Rule	Location	Construction quality	State of conservation	Attractions	Evaluation
1	Bad	Bad	Bad	Bad	Bad
2	Bad	Bad	Bad	Average	Bad
3	Bad	Bad	Bad	Good	Bad
…	…	…	…	…	…
79	Good	Good	Good	Bad	Bad
80	Good	Good	Good	Average	Good
81	Good	Good	Good	Good	Good

Table 11 presents the overall value obtained with defuzzification by COG. The first observation from Table 11 is that for 10 of 15 residential properties, Evaluation resulted overall values equal to zero. This is due to the fuzzy system design. Residential properties with the lowest value according to a quantitative criterion received a α-cut equal to zero.

Table 11. Overall values for the residential properties with fuzzy expert system

Residential properties	Overall value	Rank
~~A_1~~	~~0~~	~~6~~
~~A_2~~	~~0~~	~~6~~
~~A_3~~	~~0~~	~~6~~
~~A_4~~	~~0~~	~~6~~
A_5	**0.259**	**3**
~~A_6~~	~~0~~	~~6~~
~~A_7~~	~~0~~	~~6~~
~~A_8~~	~~0~~	~~6~~
~~A_9~~	~~0~~	~~6~~
~~A_{10}~~	~~0~~	~~6~~
A_{11}	**0.452**	**2**
~~A_{12}~~	~~0~~	~~6~~
A_{13}	0.259	3

A_{14}	**0.741**	**1**
A_{15}	0.259	3

This situation was mainly originated because C_8, is a crisp criterion. This way, A_1, A_2, A_3, A_4, A_6, A_9, A_{10}, and A_{12} have overall values equal to zero. Nevertheless, it seems to be plausible, since they are residential properties without security system, and, Security is a major issue in Brazil, particularly in Rio de Janeiro.

4. Discussion and Conclusions

Comparing the rank from the fuzzy expert system to the one resulted with TODIM application, both resulted A_5, A_{11}, and A_{14} as top residential properties; and A_7, A_9, A_{10}, and A_{12} in the bottom. Therefore, besides the differences in the overall values, the ranks from both applications can be considered as compatible each other, in qualitative terms. However, the Emond-Mason coefficient computed for these ranks is $\tau_x \approx 0.408$, which indicates a fragile correlation.

From ordering theory, the rank obtained with TODIM is classified as a linear rank, since it has no ties. Now, the rank from fuzzy expert system is a weak rank, since there are ten alternatives tied in the last 6^{th} position. Surprisingly, the fuzzy expert system's rank is crisp and TODIM's rank is fuzzy. That is, in this presented case the use of an MCDA method was superior to fuzzy systems. The fuzzy expert system's fragility exposed in this case was due the necessity of a crisp evaluation on Security.

This work satisfied its objective presenting a comparison between TODIM's rank with the rank from a fuzzy expert system. The comparison follows a mixed qualitative-quantitative research strategy, that is, our findings are based in a single case. Then, new researches must be conducted comparing ranks from TODIM with ranks from other MCDA methods and mainly comparing fuzzy system ranks with ranks from other decision techniques. With multiple examples, that is with quantitative researches, the fuzzy system's fragility observed in this work can be demonstrated. Another interesting subject for future investigation is the use of Emond-Mason coefficient to compare ranks previously obtained with MCDA methods in an *ex-post facto* research approach.

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