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Constraints on neutrino-photon interactions from rare Z decays

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Abstract

It is shown that the rare decays $Z \to v\bar{v}\gamma$ and $Z \to v\bar{v}\gamma\gamma$ are useful to put model-independent bounds on neutrino-onephoton and neutrino-two-photon interactions. The results are then used to constrain the τ neutrino magnetic moment $\mu_{\nu_{\tau}}$ and the double radiative decay $v_j \to v_i \gamma\gamma$. It is found that the decay $Z \to v\bar{v}\gamma\gamma$ gives a more stringent bound on $\mu_{\nu_{\tau}}$ than that obtained from $Z \to v\bar{v}\gamma\gamma$; the latter decay in turn gives limits on the neutrino-two-photon interaction that are less stringent than those obtained for a sterile neutrino v_s from the analysis of $v_{\mu}N \to v_sN$ conversion.

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The behavior recently observed of atmospheric [1] and solar [2] neutrinos provides rather strong evidence that neutrinos have mass. This fact has renewed the interest in neutrino electromagnetic properties, which have received considerable attention as they may shed light on some physics issues. In particular, neutrinoone-photon interactions are of interest since they may play a key role in elucidating the solar neutrino puzzle, which can be explained by a large neutrino magnetic moment [3] or a resonant spin flip induced by Majorana neutrinos [4]. In the simplest extension of the standard model (SM), with the presence of massive neutrinos, one-loop radiative corrections induce

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a small magnetic moment proportional to the neutrino mass m_{ν} , i.e., $\mu_{\nu} = 3eG_F m_{\nu}/(8\sqrt{2}\pi^2) = 3 \times$ $10^{-19} m_{\nu} \mu_{\rm B}$ [5], where m_{ν} is to be expressed in KeV and $\mu_{\rm B}$ stands for the Bohr magneton. Several models have been advanced in order to induce neutrino magnetic moments as large as 10^{-11} – $10^{-10} \mu_{\rm B}$ [6], even with neutrino masses compatible with the mass square differences needed by atmospheric [1], solar [2] and the liquid scintillation neutrino detector (LSND) [7] data. As for neutrino-two-photon interactions, they may have direct implications on several low- and highenergy reactions with astrophysical and cosmological interest [8]. For instance, a high annihilation rate of photons into a neutrino pair may explain the observed cooling of stars by neutrino emission [9]. In addition, there are other interesting processes involving neutrino-two-photon interactions, such as $\nu \gamma \rightarrow \nu \gamma$, $\nu\bar{\nu} \rightarrow \gamma\gamma$, and the neutrino double-radiative decay $\nu_i \rightarrow \nu_i \gamma \gamma$. It is important to note that in the SM with massive neutrinos, the decay $v_i \rightarrow v_i \gamma \gamma$ is not

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severely suppressed by the GIM mechanism and can be the main decay channel as long as the v_j mass lies in the range of a few tenths of a MeV [10].

From the experimental side, the L3 Collaboration searched for single-photon events near the Z pole at the CERN LEP collider and set a bound on the rare decay $Z \rightarrow \nu \bar{\nu} \gamma$ [11]. It was shown that the collected data impose a stringent constraint on the τ neutrino magnetic moment [11–13]. In fact, the decay $Z \rightarrow$ $\nu \bar{\nu} \gamma$ can be a valuable tool to search for evidences of new physic since its rate is negligibly small in the SM [14]. By using the experimental bound on $Z \rightarrow$ $\nu \bar{\nu} \gamma$, an analysis in the framework of the effective Lagrangian approach (ELA) was carried out in Refs. [12,15] in order to constrain the operators that induce the couplings $\nu \bar{\nu} \gamma$, $\nu \bar{\nu} Z \gamma$ and $Z Z \gamma$. In regard to the rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$, long ago the L3 and the OPAL Collaborations looked for events with a lepton pair accompanied by a photon pair of large invariant mass [16]. After combining the data of both searches, the OPAL Collaboration set an upper bound on the rate of $Z \to \nu \bar{\nu} \gamma \gamma$: it was found that BR $(Z \to \nu \bar{\nu} \gamma \gamma) \leq$ 3.1×10^{-6} .

In the present Letter we consider the possibility of obtaining indirect bounds on neutrino electromagnetic interactions from the experimental constraints on the decays $Z \rightarrow \nu \bar{\nu} \gamma$ and $Z \rightarrow \nu \bar{\nu} \gamma \gamma$. Our main goal is to study these processes in a model independent way. We will also show that our results can be used to constrain the ν_{τ} magnetic moment and the decay $\nu_i \rightarrow \nu_i \gamma \gamma$.

For the purpose of this Letter we will consider the following effective interaction

$$\mathcal{L}_{\bar{\nu}_i \nu_j \gamma} = \frac{1}{2} \mu_{\nu_i \nu_j} \bar{\nu}_i \sigma_{\mu \nu} \nu_j F^{\mu \nu}, \qquad (1)$$

where $\mu_{\nu_i} \equiv \mu_{\nu_i \nu_i}$ is the ν_i magnetic moment and $\mu_{\nu_i \nu_j}$ ($i \neq j$) is the transition magnetic moment. Although we will focus on Dirac neutrinos here, the discussion can be readily extended to Majorana neutrinos.

As already mentioned, in Ref. [14] it was shown that the SM rate of the decay $Z \rightarrow \nu \bar{\nu} \bar{\nu} \gamma$ is unobservably small. Therefore, it represents an extraordinary mode to look for evidences of new physics arising from neutrino-one-photon interactions at a future e^+e^- linear collider. The search for events with energetic single-photons along with missing energy at LEP was used by the L3 Collaboration to set the bound

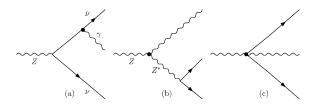


Fig. 1. Feynman diagrams contributing to the decay $Z \rightarrow \nu \bar{\nu} \gamma$ in the effective Lagrangian approach. The dots denote effective couplings.

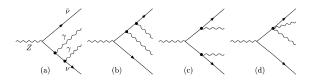


Fig. 2. Feynman diagrams contributing to the decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$. The crossed diagrams are not shown. The dots denote effective couplings.

BR($Z \rightarrow \nu \bar{\nu} \gamma \gamma \leqslant 10^{-6}$ [11]. Within the ELA, the rare decay $Z \rightarrow \nu \bar{\nu} \gamma$ can proceed through the Feynman diagrams shown in Fig. 1. For details of the analysis of these diagrams, we refer the reader to Refs. [12,15]. In particular, the experimental limit on $Z \rightarrow \nu \bar{\nu} \gamma$ gives the following bound on the ν_{τ} magnetic moment

$$\mu_{\nu_{\tau}} \leqslant 2.62 \times 10^{-6} \,\mu_{\rm B}.\tag{2}$$

This bound is in good agreement with that found by the L3 Collaboration [11], and compares favorably with the bounds $\mu_{\nu_{\tau}} < 4 \times 10^{-6} \,\mu_{\rm B}$ [17] and $\mu_{\nu_{\tau}} < 2.7 \times 10^{-6} \,\mu_{\rm B}$ [18]. The former was obtained from low-energy experiments, whereas the latter was derived from the invisible width of the *Z* boson. Furthermore, our bound is close to the one obtained from a beam-dump experiment [19]. It is important to note that the most stringent bounds on the neutrino magnetic moment are obtained from chirality flip in supernova [20].

We now turn to examine the rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$, which also can receive contributions from a neutrino-one-photon interaction through the Feynman diagrams depicted in Fig. 2. We would like to analyze if this decay is useful to bound the neutrino magnetic moment. Given the effective interaction of Eq. (1), the amplitude for the Feynman diagrams of Fig. 2 plus the crossed ones reads

$$\mathcal{M} = \mathcal{M}^{\alpha\beta\mu} \epsilon^*_{\alpha}(k_1) \epsilon^*_{\beta}(k_2) \epsilon_{\mu}(p) + \cdots, \qquad (3)$$

where the ellipsis stands for the crossed diagrams contribution, which can be obtained from the first term after the substitutions $\alpha \leftrightarrow \beta$ and $k_1 \leftrightarrow k_2$. In principle, we must take into account all the neutrino species. However, in order to get an upper bound on $\mu_{
u_{ au}}$, we will make a few assumptions for the sake of simplicity. First of all, we consider that the neutrino magnetic moment matrix is almost flavor diagonal, i.e., $\mu_{\nu_i} \gg \mu_{\nu_i \nu_i}$ $(j \neq i)$. Secondly, we assume that the ν_{τ} magnetic moment dominates over $\mu_{\nu_{e}}$ and $\mu_{\nu_{\mu}}$, i.e., there is the hierarchy $\mu_{\nu_{\tau}} \gg \mu_{\nu_{\mu}} \gg \mu_{\nu_{e}}$. In fact, the most stringent experimental bounds are $\mu_{\nu_e} \leq 1.1 \times 10^{-10} \,\mu_{\rm B}$ [21], $\mu_{\nu_{\mu}} \leq 7.4 \times 10^{-9} \,\mu_{\rm B}$ [21], and $\mu_{\nu_{\tau}} \leq 5.4 \times 10^{-7} \,\mu_{\rm B}$ [19]. Bearing in mind these assumptions, the main contribution to the $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ rate will arise from the $\bar{\nu}_{\tau} \nu_{\tau} \gamma$ vertex. Therefore we can write $\mathcal{M}^{\alpha\beta\mu}$ as

$$\mathcal{M}^{\alpha\beta\mu} = \frac{ig\mu_{\nu_{\tau}}^2}{2c_W} \sum_{k=1}^3 \bar{\nu}_{\tau}(p_2) \Gamma_k^{\alpha\beta\mu} \nu_{\tau}(p_1) k_{1\lambda} k_{2\rho}, \quad (4)$$

with

$$\Gamma_{1}^{\alpha\beta\mu} = \gamma^{\mu} P_{L} \sigma^{\alpha\lambda} (\not p - \not p_{2})^{-1} \sigma^{\beta\rho} (\not p_{1} + \not k_{2})^{-1}, \quad (5)$$

where we have neglected the ν_{τ} mass; $p_1(p_2)$, $k_{1,2}$ and p are the neutrino (antineutrino), photon and Z boson four-momenta; and $P_L = (1 - \gamma^5)/2$ is the left-handed helicity projector.

The transition amplitude can be squared by the usual trace technique. The result is too lengthy to be shown here. The squared amplitude can then be integrated over the four-body phase space with the aid of the Monte Carlo integration method [22]. In order to cross-check our results, we used two different methods for the evaluation of the $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ decay rate. In the first method we squared the amplitude and then used a Monte Carlo event generator to carry out the numerical integration [23]. As far as the second method is concerned, we implemented the $\nu \bar{\nu} \gamma$ and $\nu \bar{\nu} \gamma \gamma$ interactions into the CALCHEP program [24], which automatically generates the respective set of Feynman diagrams, squares the matrix elements, and integrates over the phase space. There was nice agreement between the results obtained by both of these methods.

Under the assumptions discussed above, we can obtain the following estimate for the $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ rate

$$BR(Z \to \nu \bar{\nu} \gamma \gamma) = 1.749 \times 10^{11} \left(\frac{\mu_{\nu_{\tau}}}{1 \,\mu_{B}}\right)^{4}, \tag{8}$$

which along with the experimental bound BR($Z \rightarrow \nu \bar{\nu} \gamma \gamma$) $\leq 3.1 \times 10^{-6}$ yield

$$\mu_{\nu_{\tau}} \leqslant 6.488 \times 10^{-5} \,\mu_{\mathrm{B}} \tag{9}$$

which is just one order of magnitude below than the bound obtained from the three body decay $Z \rightarrow v\bar{\nu}\gamma\gamma$ (cf. Eq. (2)). The rare decay $Z \rightarrow v\bar{\nu}\gamma\gamma\gamma$ is however more sensitive to the value of the neutrino magnetic moment. In this respect, it is interesting if we take a different approach and use the most stringent experimental bound on $\mu_{\nu_{\tau}}$ [19] to constrain the rare decays $Z \rightarrow v\bar{\nu}\gamma\gamma$ and $Z \rightarrow v\bar{\nu}\gamma\gamma$, in which case we are led to

$$BR(Z \to \nu \bar{\nu} \gamma) \leqslant 6.917 \times 10^{-8}, \tag{10}$$

$$BR(Z \to \nu \bar{\nu} \gamma \gamma) \leqslant 1.487 \times 10^{-14}.$$
 (11)

Before proceeding, we would like to stress that the procedure described above can be employed to get bounds on the τ neutrino transition magnetic moments $\mu_{\nu_{\tau}\nu_{i}}$ ($i = e, \mu$), which also have been the source of interest [25]. In that case, we would have to include all the $\mu_{\nu_{\tau}\nu_{i}}$ contributions into the $Z \rightarrow \nu \bar{\nu} \bar{\nu} \gamma$ and $Z \rightarrow \nu \bar{\nu} \bar{\nu} \gamma \gamma$ rates, and the upper bound on each $\mu_{\nu_{\tau}\nu_{i}}$ would be obtained after dropping the remaining contributions. Since the results are insensitive to the neutrino mass, provided that $m_{\nu} \ll m_{Z}$, it follows that the bounds of Eqs. (2) and (9) also apply to $\mu_{\nu_{\tau}\nu_{i}}$. Of course the same is true for any $\mu_{\nu_{i}\nu_{j}}$, but the bounds are very weak for ν_{e} and ν_{μ} , as compared to other results appearing in the literature.

Now we will analyze the impact of the $\nu \bar{\nu} \gamma \gamma$ coupling on the rare decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma \gamma$. While the neutrino-one-photon interaction is strictly vanishing for massless neutrinos, the neutrino-two-photon interaction can have a nonzero value even if neutrinos are massless. Long ago, it was shown that if massless neutrinos interact locally with charged leptons the neutrino-two-photon vertex vanishes [26]. It is possible that neutrinos are kept massless but interact directly with gauge bosons, as in the SM. In that case there is indeed a nonzero neutrino-two-photon coupling, which arises at the one-loop level and is negligibly small, of order $O(G_F^2)$ [27]. Nevertheless, as pointed out in Ref. [28], the introduction of massive neutrinos can enhance dramatically the $\nu \bar{\nu} \gamma$ and $\nu \bar{\nu} \gamma \gamma$ vertices. While in several SM extensions the neutrinotwo-photon interaction is proportional to the neutrino mass, there are some extensions, such as left-right symmetric theories and the Zee model, where it is proportional to a heavy Higgs scalar mass [29]. This fact may give rise to a significant enhancement of the $\nu \bar{\nu} \gamma \gamma$ coupling, which can be parametrized in the following way at the lowest dimension [10,28,29]

$$\mathcal{L}_{\bar{\nu}_i\nu_j\gamma\gamma} = \frac{1}{4\Lambda^3} \bar{\nu}_i \left(\alpha_L^{ij} P_L + \alpha_R^{ij} P_R \right) \nu_j \widetilde{F}^{\mu\nu} F_{\mu\nu}, \quad (12)$$

where $\alpha_{L,R}^{ij}$ are dimensionless coupling constants and Λ is the new physics scale. Given this interaction, the procedure described before can be used to obtain the contribution to the decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ from diagram 2(d) plus that in which the photon pair emerges from the neutrino. We can write the respective transition amplitude as follows

$$\mathcal{M}^{\alpha\beta\mu} = \frac{ig}{4c_W \Lambda^3} \bar{\nu}_i(p_2) \varpi^\mu \nu_j(p_1) \epsilon^{\lambda\rho\alpha\beta} k_{1\lambda} k_{2\rho}, \quad (13)$$

with

$$\varpi^{\mu} = \alpha_L^{ij} P_L (\not p - \not p_1)^{-1} \gamma^{\mu} + \alpha_R^{ij} P_R \gamma^{\mu} (\not p - \not p_2)^{-1}.$$
(14)

Again, we have neglected the neutrino masses since the result is insensitive to them. The squared amplitude can be written in a very short way:

$$|\mathcal{M}|^{2} = \frac{8\alpha\pi(k_{1} \cdot k_{2})^{2}}{3c_{W}^{2}s_{W}^{2}m_{Z}^{2}\Lambda^{6}} \left(\left| \alpha_{L}^{ij} \right|^{2}F(p_{1}) + \left| \alpha_{R}^{ij} \right|^{2}F(p_{2}) \right),$$
(15)

with

$$F(p_1) = \frac{p_1 \cdot p_2}{(p - p_1)^2} \left(m_Z^4 - 4(p \cdot p_1)^2 - \frac{4m_Z^2 p \cdot p_1}{p_1 \cdot p_2} (p - p_1) \cdot p_2 \right).$$
(16)

From the last expressions, the $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ decay rate can be obtained after Monte Carlo integration. Hereafter, we will consider the contributions from the three SM neutrino species. The resulting branching fraction is thus given by

 $\text{BR}(Z \to \nu \bar{\nu} \gamma \gamma)$

$$= 1.092 \times 10^{3} \sum_{i} \sum_{j} \left(|\alpha_{L}^{ij}|^{2} + |\alpha_{R}^{ij}|^{2} \right) \left[\frac{1 \text{ GeV}}{\Lambda} \right]^{\circ},$$
(17)

where the sums run over ν_e , ν_{μ} , and ν_{τ} . After using the experimental bound on $Z \rightarrow \nu \bar{\nu} \gamma \gamma$, we are left with

$$\left[\frac{1 \text{ GeV}}{\Lambda}\right]^{6} \sum_{i} \sum_{j} \left(\left|\alpha_{L}^{ij}\right|^{2} + \left|\alpha_{R}^{ij}\right|^{2}\right) \leqslant 2.85 \times 10^{-9}.$$
(18)

This bound is weaker than that obtained for a sterile neutrino ν_s from the analysis of the Primakoff effect on the process of $\nu_{\mu}N \rightarrow \nu_s N$ conversion in the external Coulomb field of the nucleus N [30].

At this point, we would like to note some interesting features of the photon energy and invariant mass distributions of the decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$. In Fig. 3 we have plotted the distribution of the invariant mass of the photon pair when the contribution from either vertex $\nu \bar{\nu} \gamma$ or $\nu \bar{\nu} \gamma \gamma$ is considered at a time. When only the $\nu \bar{\nu} \gamma$ interaction contributes, the invariant mass peaks around $X_{\nu\nu} = 1/4$; on the other hand, when the $\nu\bar{\nu}\gamma\gamma$ vertex alone contributes, the peak is located around $X_{\gamma\gamma} = 1/2$. A similar situation is observed in Fig. 4, where we have plotted the energy distribution of the photon pair, and in Fig. 5, where it is shown the energy distribution of an isolated photon. We can observe that in both plots the peak of the curve accounting for the $\nu \bar{\nu} \gamma$ contribution is shifted to the left with respect to the curve resulting from the $\nu \bar{\nu} \gamma \gamma$ contribu-

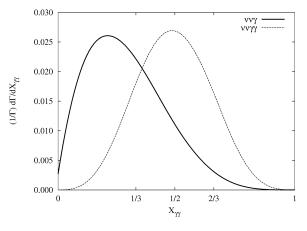


Fig. 3. Invariant mass distribution of the photon pair $(X_{\gamma\gamma} = \sqrt{(k_1 + k_2)^2}/m_Z)$ in the rare decay $Z \to \nu \bar{\nu} \bar{\nu} \gamma \gamma$. The contributions from the vertices $\nu \bar{\nu} \gamma$ and $\nu \bar{\nu} \gamma \gamma$ are shown separately.

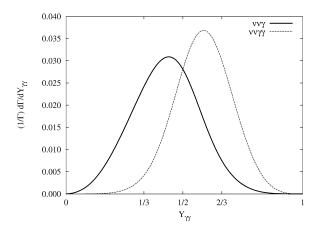


Fig. 4. Energy distribution $(Y_{\gamma\gamma} = (E_1 + E_2)/m_Z)$ of the photon pair in the rare decay $Z \rightarrow v \bar{v} \gamma \gamma$. The contributions from the vertices $v \bar{v} \gamma$ and $v \bar{v} \gamma \gamma$ are shown separately.

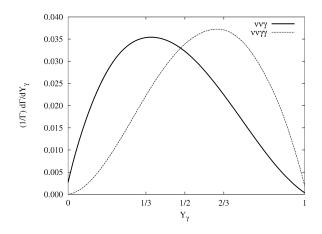


Fig. 5. Energy distribution $(Y_{\gamma} = 2E_1/m_Z)$ of a single photon in the rare decay $Z \rightarrow \nu \bar{\nu} \bar{\nu} \gamma \gamma$. The contributions from the vertices $\nu \bar{\nu} \gamma$ and $\nu \bar{\nu} \gamma \gamma$ are shown separately.

tion. Therefore, in principle a proper set of cuts would allow us to distinguish between the contributions from each vertex. A more comprehensive analysis is however beyond the present Letter.

Finally, we will show that the decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ can also be used to bound the neutrino double radiative decay $\nu_j \rightarrow \nu_i \gamma \gamma$. Neglecting the ν_i mass and after some calculation one can obtain the following expression for the $\nu_j \rightarrow \nu_i \gamma \gamma$ decay width

$$\Gamma_{\nu_{j} \to \nu_{i} \gamma \gamma} = \frac{m_{\nu_{j}}^{\gamma}}{2^{10} \Lambda^{6} \pi^{3}} \Big(|\alpha_{L}^{ij}|^{2} + |\alpha_{R}^{ij}|^{2} \Big)$$

$$\times \int_{0}^{1} \int_{x}^{1} (1-x)x^2 \, dy \, dx, \tag{19}$$

which yields

$$\Gamma_{\nu_j \to \nu_i \gamma \gamma} = 1.59 \times 10^{-3} \left(\left| \alpha_L^{ij} \right|^2 + \left| \alpha_R^{ij} \right|^2 \right) \left[\frac{1 \text{ GeV}}{\Lambda} \right]^6 \\ \times \left[\frac{m_{\nu_j}}{1 \text{ MeV}} \right]^7 \text{ s}^{-1}.$$
(20)

The constraint of Eq. (18) can then be translated into a lower bound on the v_i lifetime

$$\tau_{\nu_j} \ge 1.79 \times 10^{12} \left[\frac{1 \text{ MeV}}{m_{\nu_j}} \right]^7 \text{ s},$$
 (21)

which is true provided that m_{ν_j} lies in the range of a few tenths of a MeV, since in that case $\nu_j \rightarrow \nu_i \gamma \gamma$ is the dominant decay channel [10]. Our bound is one order of magnitude below than the one previously found for the lifetime of a sterile neutrino [30].

In closing we emphasize that the rare decay $Z \rightarrow$ $\nu \bar{\nu} \gamma$ gives rise to a bound on the ν_{τ} magnetic moment that is in excellent agreement with other ones found recently. The bound obtained from the rare decay $Z \rightarrow$ $\nu \bar{\nu} \gamma \gamma$ is just one order of magnitude below. It must be stressed that the current experimental bound on the $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ branching ratio is somewhat weak. In fact, it is of the same order of magnitude than that on $Z \rightarrow$ $\nu \bar{\nu} \gamma$. Some improvement is expected from the data to be collected at a future linear collider. One important feature of the decay $Z \rightarrow \nu \bar{\nu} \gamma \gamma$ is that it can also be used to bound the neutrino-two-photon interaction. In this respect, the resulting bound is weaker than that derived for a sterile neutrino from the process of $\nu_{\mu}N \rightarrow \nu_{s}N$ conversion in the external Coulomb field of the nucleus N. Finally, the bound on the neutrinotwo-photon interaction also allowed us to constrain the width of the decay $\nu_i \rightarrow \nu_i \gamma \gamma$, which in turn can be translated into a constrain on the v_i lifetime as long as m_{ν_i} lies in the range of a few hundreds of KeV. The main advantage of our procedure is that it is modelindependent and relies on a few assumptions.

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