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$m_u + m_d$ from isovector pseudoscalar sum rules

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Abstract

We revisit the isovector pseudoscalar sum rule determination of $m_u + m_d$, using families of finite energy sum rules known to be very accurately satisfied in the isovector vector channel. The sum rule constraints are sufficiently strong to allow a determination of both $m_u + m_d$ and the excited resonance decay constants. The corresponding Borel transformed sum rules are also very well satisfied, providing a non-trivial consistency check on the treatment of direct instanton contributions. We obtain $[m_u + m_d](2 \text{ GeV}) = 7.8 \pm 1.1 \text{ MeV}$ (in the $\overline{\text{MS}}$ scheme), only marginally compatible with the most recent sum rule determinations, but in good agreement with recent unquenched lattice extractions.

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1. Introduction

Because of the Ward identity $\partial_\mu A_{ud}^\mu = (m_u + m_d) \times \bar{u}i\gamma_5 d$, a study of the correlator

$$\begin{aligned} \Pi_{ud}(q^2) &= i \int d^4x e^{iq \cdot x} \langle 0 | T(\partial_\mu A_{ud}^\mu(x) \partial_\nu A_{ud}^{\nu \dagger}(0)) | 0 \rangle \quad (1) \end{aligned}$$

allows one, in principle, to determine $m_u + m_d$ [1]. A number of such studies have been performed [1–4], the most recent (Refs. [3] (BPR) and [4] (P98)) employing, respectively, 3- and 4-loop expressions for the dominant $D = 0$ OPE contribution. We concentrate on the results of P98 (which updates BPR) in what fol-

lows. P98 quotes,² for $m_u + m_d$

$$[m_u + m_d](2 \text{ GeV}) = 9.8 \pm 1.9 \text{ MeV}. \quad (2)$$

Recent unquenched lattice simulations, in contrast, yield [5,6]

$$\begin{aligned} [m_u + m_d](2 \text{ GeV}) &= 6.88_{-0.44}^{+0.28} \text{ MeV} \quad (\text{CP-PACS}), \\ [m_u + m_d](2 \text{ GeV}) &= 7.0 \pm 0.4 \text{ MeV} \quad (\text{QCDSF-UKQCD}), \quad (3) \end{aligned}$$

where the errors do not reflect the uncertainty involved in using perturbative versions of the renormalization constants. Because the consistency of the lattice and sum rule determinations is not particularly good, we revisit the sum rule treatment of Π_{ud} .

In this Letter we study Π_{ud} using Borel transformed and finite energy sum rules (BSRs and FESRs). The

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² All quark masses, here and in what follows, are in the $\overline{\text{MS}}$ scheme.

BSRs have the form [7]

$$M^6 \mathcal{B}[\Pi''_{ud}](M^2) = \int_0^\infty ds e^{-s/M^2} \rho_{ud}(s) \quad (4)$$

$$\simeq \int_0^{s_0} ds e^{-s/M^2} \rho_{ud}(s) + \int_{s_0}^\infty ds e^{-s/M^2} \rho_{ud}^{\text{OPE}}(s), \quad (5)$$

where M is the Borel mass, s_0 the “continuum threshold”, and $\mathcal{B}[\Pi''_{ud}](M^2)$ the Borel transform of $\Pi''_{ud}(Q^2) \equiv d^2 \Pi_{ud}(Q^2)/(dQ^2)^2$. The FESRs have the form

$$\frac{-1}{2\pi i} \oint_{|s|=s_0} ds w(s) \Pi_{ud}(s) = \int_0^{s_0} ds w(s) \rho_{ud}(s), \quad (6)$$

where s_0 is arbitrary and $w(s)$ is any function analytic in the region of the contour.

The π contribution to ρ_{ud} , $[\rho_{ud}(s)]_\pi = 2f_\pi^2 m_\pi^4 \delta(s - m_\pi^2)$, with $f_\pi = 92.4$ MeV, is very accurately known. The decay constants of the $\pi(1300)$ and $\pi(1800)$, needed to describe the remaining contributions to ρ_{ud} below $s \sim 4$ GeV², are not known, and need to be determined as part of the sum rule analysis.

The LHS of Eq. (4) can be evaluated using the OPE, provided that M is sufficiently large compared to the QCD scale. The condition that s_0 be similarly large, though necessary, is not sufficient for the OPE to be employed reliably on the LHS of Eq. (6) since, except for extremely large s_0 , the OPE is expected to break down near the timelike real axis [8]. For the isovector vector (IVV) channel, this breakdown can be seen explicitly using the very precise spectral data available from hadronic τ decay [9]: FESRs involving $w(s) = s^k$ with $k = 0, 1, 2, 3$ (which fail to suppress contributions from the region of the circle $|s| = s_0$ near the timelike real axis) are rather poorly satisfied at scales $2 \text{ GeV}^2 < s_0 < m_\tau^2$ [10]. The breakdown of the OPE, however, turns out to be very closely localized to the vicinity of the timelike axis: FESRs based on weights having even a single zero at $s = s_0$ are very accurately satisfied over this whole range [10]. Thus at the scales $2 \text{ GeV}^2 < s_0 < 4 \text{ GeV}^2$ of interest to us, the supplementary constraint $w(s_0) = 0$ must be imposed in order to obtain reliable FESRs. We call such FESRs “pinch-weighted”, or pFESRs.

The OPE representation of $\Pi_{ud}(Q^2)$ is known up to dimension $D = 6$, with the dominant $D = 0$ perturbative contribution known to 4-loop order [11,12]. Working with $\Pi''(Q^2)$, which allows logarithms to be summed via the scale choice $\mu^2 = Q^2$, one has [11,12]

$$[\Pi''_{ud}(Q^2)]_{D=0} = \frac{3}{8\pi^2} \frac{(\bar{m}_u + \bar{m}_d)^2}{Q^2} \times \left(1 + \frac{11}{3} \bar{a} + 14.1793 \bar{a}^2 + 77.3683 \bar{a}^3 \right), \quad (7)$$

$$[\Pi''_{ud}(Q^2)]_{D=4} = \frac{(\bar{m}_u + \bar{m}_d)^2}{Q^6} \times \left(\frac{1}{4} \Omega_4 + \frac{4}{9} \bar{a} \Omega_3^{ss} - \left[1 + \frac{26}{3} \bar{a} \right] (m_u + m_d) \langle \bar{u}u \rangle - \frac{3}{28\pi^2} \bar{m}_s^4 \right), \quad (8)$$

$$[\Pi''_{ud}(Q^2)]_{D=6} = \frac{(\bar{m}_u + \bar{m}_d)^2}{Q^8} \times \left(-3[\langle m_u g \bar{d} \sigma \cdot G d + m_d g \bar{u} \sigma \cdot G u \rangle] - \frac{32}{9} \pi^2 a \rho_{\text{VSA}} [\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2 - 9 \langle \bar{u}u \rangle \langle \bar{d}d \rangle] \right), \quad (9)$$

where $\bar{a} \equiv a(Q^2) = \alpha_s(Q^2)/\pi$, $\bar{m}_k \equiv m_k(Q^2)$, with $\alpha_s(Q^2)$ and $m(Q^2)$ the running coupling and running mass at scale $\mu^2 = Q^2$ in the $\overline{\text{MS}}$ scheme, Ω_4 and Ω_3^{ss} are the RG invariant modifications of $\langle aG^2 \rangle$ and $\langle m_s \bar{s}s \rangle$ defined in Ref. [11], and ρ_{VSA} in Eq. (9) describes the deviation of the four-quark condensates from their vacuum saturation values. We have dropped $D = 2$ contributions, which are suppressed by two additional powers of $m_{u,d}$, and additional $D = 4$ contributions proportional to $[m_u + m_d]^2 m_{u,d}^4$. The Borel transforms of the above expressions are well known, and may be found in Refs. [11,12].

In scalar and pseudoscalar channels, direct instanton contributions are potentially important, but are not incorporated in the OPE representation of Π_{ud} [13,14]. We estimate their size using the instanton liquid model [15]. ILM contributions to the theoretical side of the Π_{ud} BSR are given by

$$\frac{3\rho_I^2(m_u + m_d)^2 M^6}{8\pi^2} [K_0(\rho_I^2 M^2/2) + K_1(\rho_I^2 M^2/2)], \quad (10)$$

where $\rho_I \simeq (1/0.6 \text{ GeV})$ is the average instanton size and K_i are the MacDonald functions. ILM contributions play only a small (few percent) role in the BSR analysis at scales $M^2 > 2 \text{ GeV}^2$, but are important for FESR analyses. For polynomial weights, ILM FESR contributions follow from [16]

$$\begin{aligned} & \frac{-1}{2\pi i} \oint_{|s|=s_0} ds s^k [\Pi_{ud}(s)]_{\text{ILM}} \\ &= \frac{-3[m_u + m_d]^2}{4\pi} \int_0^{s_0} ds s^{k+1} J_1(\rho_I \sqrt{s}) Y_1(\rho_I \sqrt{s}). \end{aligned} \quad (11)$$

We employ the following values for OPE/ILM input: $\rho_I = 1/(0.6 \text{ GeV})$ [14,15]; $\alpha_s(m_\tau^2) = 0.334 \pm 0.022$ [9]; $\langle \alpha_s G^2 \rangle = (0.07 \pm 0.01) \text{ GeV}^4$ [17]; $(m_u + m_d) \langle \bar{u}u \rangle = -f_\pi^2 m_\pi^2$; $0.7 < \langle \bar{s}s \rangle / \langle \bar{u}u \rangle \equiv r_c < 1$ [11,12]; $\langle g\bar{q}\sigma Fq \rangle = (0.8 \pm 0.2 \text{ GeV}^2) \langle \bar{q}q \rangle$ [18]; and $\rho_{\text{VSA}} = 0 \rightarrow 10$. The $D = 0$ and 4 OPE contributions are evaluated via the contour-improvement prescription [19], using the analytic solutions for $\alpha_s(Q^2)$ and $m(Q^2)$ obtained from the 4-loop-truncated versions of the β [20] and γ [21] functions.

2. Potential problems with the existing sum rule treatment and the updated pFESR analysis

The BPR and P98 analyses employ FESRs with $w(s) = 1, s$. The global normalization of the resonance contributions is fixed by assuming resonance dominance of ρ_{ud} at 3π threshold and normalizing the tails of the resonance contributions to the known ChPT threshold expression. Direct instanton contributions are neglected. The relative strengths of the two resonance contributions are constrained by optimizing a “duality” match between OPE and spectral ansatz versions of the ratio of the $w(s) = 1$ - and s -weighted FESRs.³ $m_u + m_d$ is then extracted from the optimized duality matching region of the $(w(s) = 1)$ -weighted FESR. The result of Eq. (2) corresponds to $s_0 \sim 2 \text{ GeV}^2$. The determination is, in fact, not stable with respect to s_0 , falling roughly linearly from

9.8 MeV at $s_0 \simeq 2 \text{ GeV}^2$ to 7.5 MeV at $s_0 \simeq 4 \text{ GeV}^2$ (see Fig. 2 of [4]).

Two potential problems with this analysis are (1) the use of non-pinched-weighted FESRs at scales for which they are poorly satisfied in the IVV channel, and (2) neglect of direct instanton contributions. In addition, the P98 result,

$$\frac{[m_u + m_d](1 \text{ GeV})}{[m_u + m_d](2 \text{ GeV})} = 1.31,$$

corresponds (using 4-loop running) to $\alpha_s(m_\tau^2) = 0.307$, significantly lower than the recent ALEPH determination. Since this will produce an overestimate of $m_u + m_d$, an update of OPE input is also in order.

Concerning the first problem, one could, of course, be lucky: the scale at which the OPE can be safely used right down to the timelike axis might turn out to be lower in the isovector pseudoscalar than in the IVV channel. If so, however, pFESRs employing the same spectral ansatz and same value of $m_u + m_d$ should also be well satisfied at the scales used in P98. We test this possibility using pFESRs based on $w_N(y, A) = (1 - y)(1 + Ay)$ and $w_D(y, A) = (1 - y)^2(1 + Ay)$ (where $y = s/s_0$ and A is a free parameter), which are known to be well satisfied in the IVV channel. The resulting OPE/spectral integral match (corresponding to the above values for the OPE input and, as in P98, neglect of ILM contributions) is shown for the w_N case in Fig. 1, and is obviously quite poor. (The quality of the w_D match is even worse.)

We have reanalyzed Π_{ud} , using the P98 spectral ansatz as input, but fixing $m_u + m_d$ via a combined w_N, w_D pFESR analysis. If we do not include ILM contributions, the optimized OPE/spectral integral match remains poor. Including ILM contributions produces a reasonable optimized match. The corresponding value of $m_u + m_d$ is

$$[m_u + m_d](2 \text{ GeV}) = 6.8 \text{ MeV}. \quad (12)$$

The quality of this match is shown in Fig. 2 for the w_N family and in Fig. 3 for the w_D family. The result of Eq. (12) is compatible with the P98 results corresponding to $s_0 \simeq 4 \text{ GeV}^2$ but significantly smaller than that corresponding to $s_0 = 2 \text{ GeV}^2$. Since, in spite of optimization, the match for w_N is best where that for w_D is worst, and vice versa, it appears that some modification of the P98 spectral ansatz is also required.

³ The duality matching is equivalent to imposing the local duality (OPE) version of the spectral function, up to an overall multiplicative constant, over the whole of the matching window.

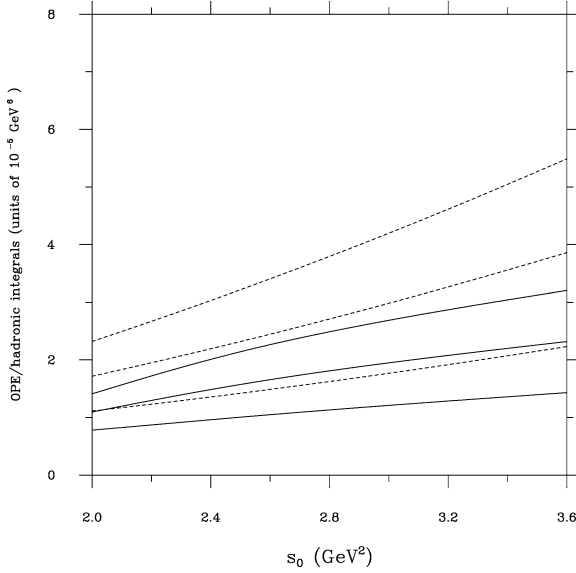


Fig. 1. The w_N OPE/spectral integral match corresponding to central values of all OPE input, the quoted P98 value $[m_u + m_d](1 \text{ GeV}) = 12.8 \text{ MeV}$ and the P98 spectral ansatz. The solid (dashed) lines represent the spectral (OPE) integrals. The lower, middle and upper lines for each case correspond to $A = 0, 2$ and 4 , respectively.

In this work, we aim to determine simultaneously the excited resonance decay constants, $f_{\pi(1300)} \equiv f_1$ and $f_{\pi(1800)} \equiv f_2$, which characterize the modifications of the spectral ansatz, and $m_u + m_d$. To this end, we perform a combined w_N and w_D pFESR analysis.⁴ Our spectral ansatz is

$$\rho_{ud}(s) = 2f_{\pi}^2 m_{\pi}^4 \delta(s - m_{\pi}^2) + 2f_1^2 m_{\pi(1300)}^4 B_1(s) + 2f_2^2 m_{\pi(1800)}^4 B_2(s), \quad (13)$$

where $B_{1,2}(s)$ are standard Breit–Wigner forms for the $\pi(1300)$ and $\pi(1800)$. We employ PDG2000 [23] values for the masses and widths. This ansatz can be used sensibly only up to $s_0 \simeq (m_{\pi(1800)} + \Gamma_{\pi(1800)})^2 \simeq 4 \text{ GeV}^2$. To maintain good convergence of the OPE we also require $s_0 \geq 3 \text{ GeV}^2$. For $3 \text{ GeV}^2 \leq s_0 \leq 4 \text{ GeV}^2$, the integrated $D = 0$ OPE series converges well for all $A \geq 0$. Larger A produces larger relative contributions from the resonance region, and hence aids in the extraction of f_1 and f_2 . The results of this

⁴ The same treatment of the IVV channel results in a determination of f_{ρ} accurate to within the experimental error [22].

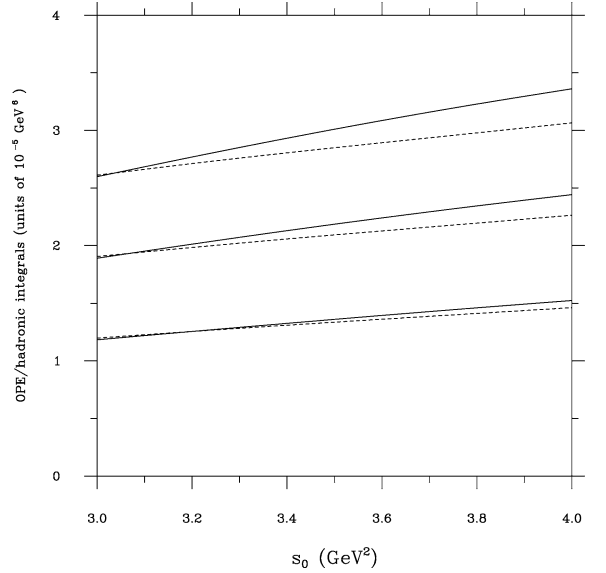


Fig. 2. The w_N OPE+ILM/spectral integral match corresponding to central values of all OPE input, the ILM estimate of direct instanton contributions, and use of the P98 spectral ansatz. The OPE+ILM curves employ the optimized value, $[m_u + m_d](2 \text{ GeV}) = 6.8 \text{ MeV}$, obtained in a combined w_N , w_D pFESR fit. The conventions for identifying spectral and OPE+ILM integrals, and the $A = 0, 2$ and 4 cases, are as for Fig. 1 above.

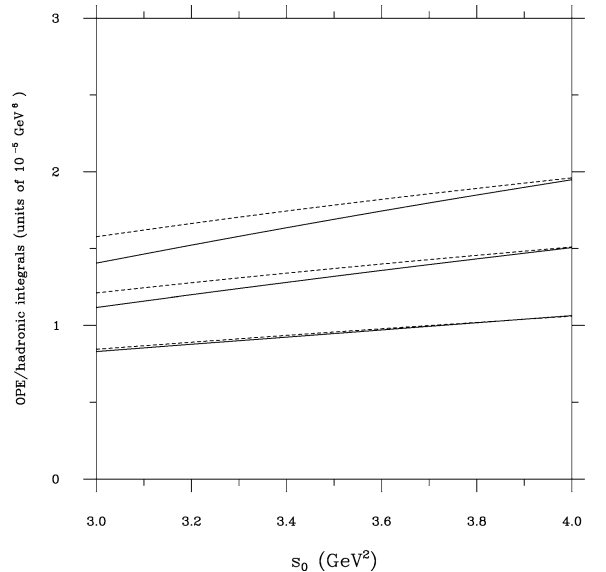


Fig. 3. The OPE+ILM/spectral integral match, as in Fig. 2, except for the w_D rather than w_N weight family.

analysis are

$$[m_u + m_d](2 \text{ GeV}) = 7.8 \pm 0.8_{\Gamma} \pm 0.5_{\text{theory}} \pm 0.4_{\text{method}} \text{ MeV}, \quad (14)$$

$$f_1 = 2.20 \pm 0.39_{\Gamma} \pm 0.18_{\text{theory}} \pm 0.18_{\text{method}} \text{ MeV}, \quad (15)$$

$$0 < f_2 < 0.37 \text{ MeV}. \quad (16)$$

The errors labelled “ Γ ” result from varying the input resonance parameters within the PDG2000 errors, and are due essentially entirely to the (large) uncertainty on the $\pi(1300)$ width. Those labelled “theory” reflect uncertainties in the OPE input and our estimate of the error associated with truncating the $D = 0$ series at $O(a^3)$. Those labelled “method” are obtained by studying the impact of employing different analysis windows in s_0 and A , and performing separate w_N and w_D analyses. Further details of the analysis, and a breakdown of the separate error contributions will be given elsewhere [24]. The OPE+ILM/spectral integral match corresponding to these results, shown in Fig. 4 for the w_N family and Fig. 5 for the w_D family, is obviously excellent.

As noted above, the ILM contributions play a non-negligible role in the pFESR analysis. In fact, if one removes ILM contributions, an equally good OPE/spectral integral match is obtained, but now corresponding to $[m_u + m_d](2 \text{ GeV}) = 9.9 \pm 1.2_{\Gamma} \pm 1.0_{\text{theory}} \pm 0.5_{\text{method}} \text{ MeV}$, $f_1 = 2.41 \pm 0.50_{\Gamma} \pm 0.21_{\text{theory}} \pm 0.27_{\text{method}} \text{ MeV}$ and $f_2 = 1.36 \pm 0.16_{\Gamma} \pm 0.09_{\text{theory}} \pm 0.11_{\text{method}} \text{ MeV}$. The pFESR analysis alone thus provides no evidence either for or against including ILM contributions. Fortunately, the requirement of consistency between BSR and pFESR analyses places non-trivial constraints on the ILM representation. This works as follows. The pFESR analysis provides a determination of $m_u + m_d$ and $f_{1,2}$ which is sensitive to whether or not ILM contributions are included. The output $f_{1,2}$, together with f_π , determine the low- s part of ρ_{ud} , and hence can be used as input to a BSR analysis. The high- s part is, as usual, approximated by the continuum ansatz, with the continuum threshold, s_0 , determined by optimizing stability of the BSR output (in this case $m_u + m_d$) with respect to M . The BSR and pFESR output values for $m_u + m_d$ should be compatible if the ILM representation is reasonable. Errors associated with uncertainties in input

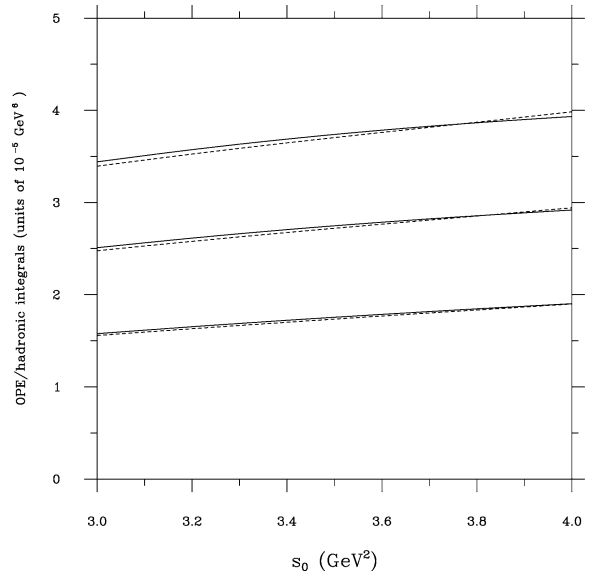


Fig. 4. The optimized OPE+ILM/spectral integral match for the w_N pFESR family, with $m_u + m_d$, f_1 and f_2 given by the central values of Eqs. (14), (15) and (16). The labelling of the hadronic integrals, OPE integrals and the $A = 0, 2$ and 4 cases, is as for Fig. 1 above.

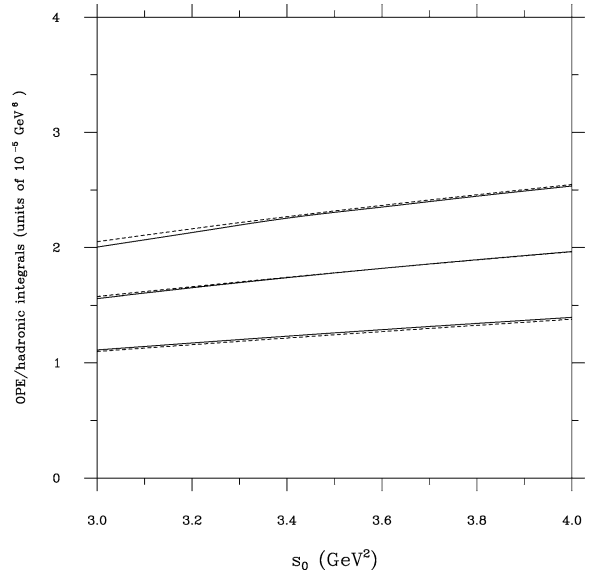


Fig. 5. The optimized OPE+ILM/spectral integral match, as in Fig. 4, except for the w_D rather than w_N weight family.

OPE and resonance parameter values are common to the pFESR and BSR analyses and strongly correlated. Additional errors are present for the BSR analysis as

a result of the crudeness of the continuum approximation and the uncertainties in the criterion for fixing s_0 . We assign a 20% error to continuum spectral contributions, and allow s_0 to vary by $\pm 0.5 \text{ GeV}^2$ about the optimal stability value. We work in a window of Borel masses $2 \text{ GeV}^2 \leq M^2 \leq 3 \text{ GeV}^2$ for which continuum contributions are $< 50\%$ of the dominant $D = 0$ OPE contribution and convergence of the Borel transformed $D = 0$ OPE series is still good. For the case that the ILM contributions are included in the pFESR analysis we obtain, quoting only the *additional* errors present for the BSR analysis,

$$[m_u + m_d](2 \text{ GeV}) = 7.5 \pm 0.9 \text{ MeV}, \quad (17)$$

to be compared to the *central* value given in Eq. (14). The agreement is excellent. The stability of the BSR analysis, shown in Fig. 6, is also extremely good. In contrast, if ILM contributions are omitted from the pFESR analysis, the BSR result becomes $[m_u + m_d] \times (2 \text{ GeV}) = 8.8 \pm 0.6 \text{ MeV}$, incompatible with the pFESR determination. If one performs a pFESR optimization of $f_{1,2}$ separately for each $m_u + m_d$, one finds that, with no ILM contributions, the pFESR and BSR values remain inconsistent, within the additional

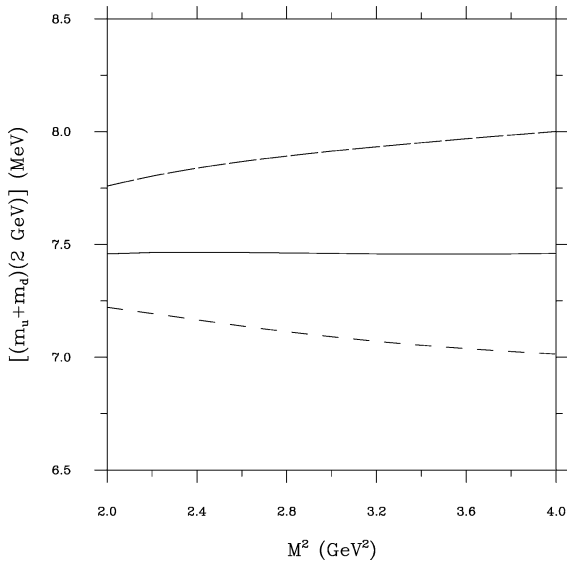


Fig. 6. $[m_u + m_d](2 \text{ GeV})$, as a function of M^2 for the BSR analysis described in the text. The solid line corresponds to $s_0 = 3.7 \text{ GeV}^2$, which produces optimal stability for $m_u + m_d$ with respect to M^2 in the window $2 \text{ GeV}^2 \leq M^2 \leq 3 \text{ GeV}^2$. The lower (short) dashed line corresponds to $s_0 = 4.2 \text{ GeV}^2$ and the upper (long) dashed line to $s_0 = 3.2 \text{ GeV}^2$. Note the compressed vertical scale.

BSR errors, unless the pFESR input, $[m_u + m_d] \times (2 \text{ GeV})$, is $< 8.1 \text{ MeV}$. The corresponding optimized value for f_1 for this marginal case turns out to be consistent within errors with that quoted in Eq. (15), though the OPE/spectral integral match is significantly worse than that obtained for the optimized fit, including ILM contributions. The low value for $m_u + m_d$ thus appears to be an unavoidable feature of the combined analysis of Π_{ud} .

3. Summary and discussion

Combining the 0.3 MeV difference of pFESR and BSR central values in quadrature with all other sources of error, we obtain, for our final result,

$$[m_u + m_d](2 \text{ GeV}) = 7.8 \pm 1.1 \text{ MeV}. \quad (18)$$

This is compatible, within errors, with the unquenched lattice determinations of Refs. [5,6], and with the result obtained by combining the ChPT determination $R \equiv 2m_s/[m_u + m_d] = 24.4 \pm 1.5$ [25] with recent determinations of m_s using hadronic τ data [26]. A recent summary [27] gives $83 \text{ MeV} < m_s(2 \text{ GeV}) < 130 \text{ MeV}$, which corresponds to $6.8 < [m_u + m_d] \times (2 \text{ GeV}) < 10.7 \text{ MeV}$. Our analysis, in fact, favors values of m_s in the lower part of this range, in good agreement with recent unquenched lattice results for m_s .

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References

- [1] A.I. Vainshtein et al., Sov. J. Nucl. Phys. 27 (1978) 274.
- [2] N. Hubschmid, S. Mallik, Nucl. Phys. B 193 (1981) 368; T.N. Truong, Phys. Lett. B 117 (1982) 109;

- J. Gasser, H. Leutwyler, *Phys. Rep.* 87 (1982) 77;
 S. Narison, N. Paver, D. Treleani, *Nuovo Cimento A* 74 (1983) 347;
 A.L. Kataev, N.V. Krasnikov, A.A. Pivovarov, *Phys. Lett. B* 123 (1983) 93;
 S.G. Gorishny, A.L. Kataev, S.A. Larin, *Phys. Lett. B* 135 (1984) 457;
 C.A. Dominguez, *Z. Phys. C* 26 (1984) 269;
 C.A. Dominguez, E. de Rafael, *Ann. Phys.* 174 (1987) 372;
 S. Narison, *Phys. Lett. B* 216 (1989) 191;
 T.G. Steele et al., *Nucl. Phys. A* 624 (1997) 517.
- [3] J. Bijnens, J. Prades, E. de Rafael, *Phys. Lett. B* 348 (1995) 226.
- [4] J. Prades, *Nucl. Phys. Proc. Suppl.* 64 (1998) 253.
- [5] A. Ali Khan et al., CP-PACS Collaboration, *Phys. Rev. Lett.* 85 (2000) 4674, hep-lat/0105015.
- [6] D. Pleiter, for the QCDSF and UKQCD Collaborations, *Nucl. Phys. Proc. Suppl.* 94 (2001) 265.
- [7] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys. B* 147 (1979) 385;
 M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys. B* 147 (1979) 448.
- [8] E. Poggio, H. Quinn, S. Weinberg, *Phys. Rev. D* 13 (1976) 1958.
- [9] R. Barate et al., ALEPH Collaboration, *Z. Phys. C* 76 (1997) 379;
 R. Barate et al., ALEPH Collaboration, *Eur. Phys. J. C* 4 (1998) 409;
 G. Abbiendi et al., OPAL Collaboration, *Phys. Lett. B* 447 (1999) 134;
 G. Abbiendi et al., OPAL Collaboration, *Eur. Phys. J. C* 7 (1999) 571.
- [10] K. Maltman, *Phys. Lett. B* 440 (1998) 367.
- [11] M. Jamin, M. Münz, *Z. Phys. C* 66 (1995) 633.
- [12] K.G. Chetyrkin, D. Pirjol, K. Schilcher, *Phys. Lett. B* 404 (1997) 337.
- [13] V.A. Novikov et al., *Nucl. Phys. B* 191 (1981) 301.
- [14] E.V. Shuryak, *Nucl. Phys. B* 203 (1982) 93;
 E.V. Shuryak, *Nucl. Phys. B* 203 (1982) 116;
 E.V. Shuryak, *Nucl. Phys. B* 214 (1983) 237.
- [15] E.V. Shuryak, *Phys. Rep.* 115 (1984) 151;
 E.V. Shuryak, J.J.M. Verbaarschot, *Nucl. Phys. B* 410 (1993) 55;
 E.V. Shuryak, *Rev. Mod. Phys.* 65 (1993) 1;
 T. Schafer, E.V. Shuryak, *Rev. Mod. Phys.* 70 (1998) 323.
- [16] A.E. Dorokhov, S.V. Esaibegian, N.I. Kochelev, N.G. Stefanis, *J. Phys. G* 23 (1997) 643;
 V. Elias, F. Shi, T.G. Steele, *J. Phys. G* 24 (1998) 267.
- [17] S. Narison, *Nucl. Phys. Proc. Suppl. A* 54 (1997) 238.
- [18] A.A. Ovchinnikov, A.A. Pivovarov, *Sov. J. Nucl. Phys.* 48 (1988) 721.
- [19] A.A. Pivovarov, *Sov. J. Nucl. Phys.* 54 (1991) 676;
 A.A. Pivovarov, *Z. Phys. C* 53 (1992) 461;
 F. Le Diberder, A. Pich, *Phys. Lett. B* 286 (1992) 147;
 F. Le Diberder, A. Pich, *Phys. Lett. B* 289 (1992) 165.
- [20] T. van Ritbergen, J.A.M. Vermaseren, S.A. Larin, *Phys. Lett. B* 400 (1997) 179.
- [21] K.G. Chetyrkin, *Phys. Lett. B* 404 (1997) 161;
 T. Van Ritbergen, J.A.M. Vermaseren, S.A. Larin, *Phys. Lett. B* 405 (1997) 327.
- [22] K. Maltman, *Phys. Lett. B* 462 (1999) 14;
 K. Maltman, *Nucl. Phys. A* 680 (2000) 171.
- [23] Particle Data Group, D.E. Groom et al., *Eur. Phys. J. C* 15 (2000) 1.
- [24] K. Maltman, J. Kambor, Decay constants, light quark masses and mass bounds from light quark pseudoscalar sum rules, YU-PP-I/E-KM-3-01, in preparation.
- [25] H. Leutwyler, *Phys. Lett. B* 374 (1996) 163;
 H. Leutwyler, *Phys. Lett. B* 378 (1996) 313;
 H. Leutwyler, hep-ph/0011044.
- [26] K.G. Chetyrkin, J.H. Kühn, A.A. Pivovarov, *Nucl. Phys. B* 533 (1998) 473;
 R. Barate et al., ALEPH Collaboration, *Eur. Phys. J. C* 11 (1999) 599;
 A. Pich, J. Prades, *JHEP* 10 (1999) 004;
 J. Kambor, K. Maltman, *Phys. Rev. D* 62 (2000) 093023;
 J. Kambor, K. Maltman, *Nucl. Phys. Proc. Suppl.* 98 (2001) 314;
 J.G. Körner, F. Krajewski, A.A. Pivovarov, hep-ph/0003165;
 A. Pich, *Nucl. Phys. Proc. Suppl.* 98 (2001) 385;
 S. Chen et al., hep-ph/0105253.
- [27] R. Gupta, K. Maltman, hep-ph/0101132;
 R. Gupta, K. Maltman, Status report on lattice QCD and QCD sum rule determinations of the light quark masses, in preparation.