



Finite-volume corrections to the CP-odd nucleon matrix elements of the electromagnetic current from the QCD vacuum angle

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ABSTRACT

Nucleon electric dipole moments originating from strong CP-violation are being calculated by several groups using lattice QCD. We revisit the finite volume corrections to the CP-odd nucleon matrix elements of the electromagnetic current, which can be related to the electric dipole moments in the continuum, in the framework of chiral perturbation theory up to next-to-leading order taking into account the breaking of Lorentz symmetry. A chiral extrapolation of the recent lattice results of both the neutron and proton electric dipole moments is performed, which results in $d_n = (-2.7 \pm 1.2) \times 10^{-16} e\theta_0$ cm and $d_p = (2.1 \pm 1.2) \times 10^{-16} e\theta_0$ cm.

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1. Introduction

The electric dipole moment (EDM) measures the polarity of a system of charged particles. For a hadron or any other elementary particle at rest, it can be expressed as $\vec{d} = d\hat{S}$, where \hat{S} is the direction of the spin of the particle, and the coefficient d refers to the EDM that will be discussed in this paper. The interaction of a dipole with an electric field \vec{E} is described by the Hamiltonian $\mathcal{H}_{\text{edm}} = -\vec{d} \cdot \vec{E}$. Such an interaction changes its sign under both parity transformation (P) or a combined transformation of parity and charge conjugation (C). Thus, the nucleon EDM is a CP-odd quantity. In the Standard Model (SM), the Cabibbo–Kobayashi–Maskawa (CKM) contribution to the neutron EDM is tiny, which was estimated to be $1.4 \times 10^{-33} \leq |d_n| \leq 1.6 \times 10^{-31} e$ cm [1]. Thus, the nucleon EDM serves as a sensitive probe of physics beyond the SM. So far, there has not been an experimental evidence for a non-zero nucleon EDM, and the current experimental upper bound for the neutron EDM is $|d_n| \leq 2.9 \times 10^{-26} e$ cm [2], which is several orders of magnitude larger than the value from the CKM mechanism. For the proton EDM, the experimental upper bound as derived from the EDM of the ¹⁹⁹Hg atom is $|d_p| < 7.9 \times 10^{-25} e$ cm [3].

Within the SM, in addition to the CKM mechanism, there is another source of CP violation, which is from the θ -term of quantum chromodynamics (QCD). Therefore, the experimental information on the nucleon EDMs allows us to constrain both physics beyond the SM (BSM) models and the value of the QCD vacuum angle θ_0 . For a recent review on the EDMs of nucleons, nuclei and atoms both within and beyond the SM, we refer to Ref. [4].

Besides the active and planned experimental activities (for a brief review, see Chapter 7.2 of Ref. [5], and a collection of various experiments can be found on the webpage [6]), the contribution of the θ -term to the nucleon EDMs is being calculated using lattice QCD [7–16]. All these lattice calculations were performed with up and down quark masses larger than their physical values, or equivalently with a pion mass larger than its physical mass. One of the methods used in the lattice calculation is to calculate the CP-odd electric dipole form factor (EDFF) of the nucleon $F_{3,N}(q^2)$ in the space-like region with finite $-q^2$. The definition of the nucleon EDFF in the infinite volume is given by

$$\begin{aligned} \langle N(p', s') | J^\nu | N(p, s) \rangle &= \frac{F_{3,N}(q^2)}{2m_N} \bar{u}(p', s') \sigma^{\mu\nu} q_\mu \gamma_5 u(p, s) + \dots \\ &= i \frac{F_{3,N}(q^2)}{2m_N} (p + p')^\nu \bar{u}(p', s') \gamma_5 u(p, s) + \dots, \end{aligned} \quad (1)$$

with J^ν being the electromagnetic current, p and $p' = p + q$ the momenta of the nucleons and $s^{(\prime)}$ the polarizations. To obtain the

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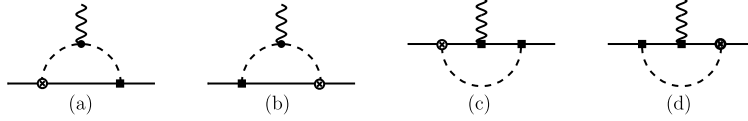


Fig. 1. One-loop contributions to the nucleon EDFFs at NLO. Nucleons and pions are represented by solid and dashed lines, respectively. \otimes , black dots and filled squares denote CP-violating, second order mesonic and first order meson–baryon vertices, respectively.

EDM, one extrapolates the results for finite momentum transfer to the point with $q^2 = 0$,

$$d_N = \frac{F_{3,N}(0)}{2m_N}. \quad (2)$$

In Eq. (1), the CP-conserving parts are not shown, and the axial Gordon identity was used in the second step. The latest results using the form factor method were reported in Ref. [16], where the calculation was performed on a lattice with a volume of $(2.7 \text{ fm})^3$, a lattice spacing of $a = 0.11 \text{ fm}$, and pion masses of 330 MeV and 420 MeV. The smallest momentum transfer is about $-q^2 = 0.2 \text{ GeV}^2$. Various extrapolations or corrections are necessary in order to obtain the result in the physical world: chiral extrapolation to the physical pion mass, finite volume corrections, corrections due to the finite volume spacing, and extrapolation from finite to zero momentum transfer. Chiral perturbation theory (CHPT) is the proper theoretical framework to calculate these corrections. A lot of work on the nucleon EDM in the framework of CHPT has been done, see, e.g., Refs. [17–32].

Finite volume corrections were considered before in Refs. [20, 29]. However, in both works Lorentz invariance was assumed to perform the tensor reduction of loop integrals. This is not legitimate since the Lorentz symmetry is broken to the cubic symmetry on a lattice with periodic boundary conditions, which is a torus. In this paper, we will revisit this issue taking into account the breaking of Lorentz symmetry. In fact, in this case, one cannot define the EDM as in the infinite volume. Instead, we will calculate the finite volume corrections to the CP-violating nucleon matrix elements of the electromagnetic current. The calculations will be presented in Section 2. The chiral extrapolation of the neutron and proton EDMs will be discussed in Section 3, and Section 4 contains a brief summary.

2. Finite volume corrections on a torus

The decomposition of the nucleon matrix element of the electromagnetic current in terms of form factors given in Eq. (1) is based on the Lorentz invariance and gauge symmetry. However, for a torus, the Lorentz invariance is reduced to the cubic symmetry, and the decomposition is not valid any more. In this section, we will evaluate the finite volume corrections to the CP-violating part of the nucleon matrix elements induced by the QCD θ -term. For discussions on the finite volume corrections to the CP-conserving nucleon matrix elements, see, e.g. Refs. [33–35].

Finite volume corrections are a long-distance effect, and are dominated by the degrees of freedom with the longest range. For our case, these corrections are dominated by pion–nucleon loops. The kaon–hyperon loops are suppressed relative to the pion–nucleon loops by a factor of $e^{-(M_K - M_\pi)L}$, and thus will not be considered here. Up to the next-to-leading order (NLO), the one-loop diagrams contributing to the nucleon EDFFs are shown in Fig. 1. Other one-loop diagrams contribute from the next-to-next-to-leading order in the chiral expansion [24,25]. For the neutron, we need to consider the $\{\pi^-, p\}$ loop, and for the proton, the loops of interest are $\{\pi^0, p\}$ and $\{\pi^+, n\}$. The necessary Lagrangians for calculating these diagrams can be found in Refs. [19,24,25,29]. A detailed analysis at the one-loop level in the infinite volume can

be found in these references either, and we will focus on the finite volume corrections here.

We define the finite volume correction to a quantity \mathcal{Q} as

$$\delta_L[\mathcal{Q}] = \mathcal{Q}(L) - \mathcal{Q}(\infty), \quad (3)$$

where $\mathcal{Q}(L)$ and $\mathcal{Q}(\infty)$ denote the quantity in the finite and infinite volumes, respectively. In the infinite volume, the contribution from the pion loops to the CP-violating nucleon matrix element up to NLO is given by

$$\begin{aligned} \epsilon_\mu \langle N(p', s') | J^\mu | N(p, s) \rangle \\ = i \frac{8eV_0^{(2)} \bar{\theta}_0}{F_\pi^4} \epsilon_\mu \bar{u}(p', s') \\ \times [C_{ab}(G_1^\mu(q) + G_2^\mu(q)) + C_{cd}G_3^\mu(q)] \gamma_5 u(p, s), \end{aligned} \quad (4)$$

where e is the electric charge of the proton, $V_0^{(2)}$ is a low-energy constant (LEC) of U(3) chiral perturbation theory [36], ϵ_μ is the polarization vector of the photon, $\bar{\theta}_0$ is related to the measurable vacuum angle θ_0 via [24]

$$\bar{\theta}_0 = \left[1 + \frac{4V_0^{(2)}}{F_\pi^2} \frac{4M_K^2 - M_\pi^2}{M_\pi^2(2M_K^2 - M_\pi^2)} \right]^{-1} \theta_0, \quad (5)$$

and the loop functions $G_1^\mu(q)$, $G_2^\mu(q)$ and $G_3^\mu(q)$ are given by

$$\begin{aligned} G_1^\mu(q) &= i \int \frac{d^4k}{(2\pi)^4} \frac{2k^\mu + q^\mu}{(k^2 - M_\pi^2)[(k+q)^2 - M_\pi^2]}, \\ G_2^\mu(q) &= 2m_N^2 i \int \frac{d^4k}{(2\pi)^4} \\ &\quad \times \frac{2k^\mu + q^\mu}{(k^2 - M_\pi^2)[(k+q)^2 - M_\pi^2][(p-k)^2 - m_N^2]}, \\ G_3^\mu(q) &= -4m_N^2 i \int \frac{d^4k}{(2\pi)^4} \\ &\quad \times \frac{k^\mu}{(k^2 - M_\pi^2)[(p-k)^2 - m_N^2][(p'-k)^2 - m_N^2]}. \end{aligned} \quad (6)$$

For the neutron matrix element, the coefficients C_{ab} and C_{cd} are $2(D+F)(b_D + b_F)$ and $-2(D+F)(b_D + b_F)$, respectively, where D and F are LECs in the leading order Lagrangian of baryon CHPT (the axial coupling constants), and b_D and b_F are LECs in the NLO Lagrangian related to the baryon mass splittings. For the proton, $C_{ab} = -2(D+F)(b_D + b_F)$ originates from the $\{\pi^+, n\}$ loop and $C_{cd} = -(D+F)(b_D + b_F)$ is generated from the $\{\pi^0, p\}$ loop. In the following, we will work in the Lorentz gauge with $\epsilon_\mu q^\mu = 0$. Expressing Eq. (4) in terms of two-point and three-point scalar loop functions in the infinite volume, we get [29]

$$\begin{aligned} C_{ab}(G_1^\mu(q) + G_2^\mu(q)) + C_{cd}G_3^\mu(q) \\ = (p+p')^\mu \left\{ C_{ab} \left[-J_{MM}(q^2) + \left(M_\pi^2 - \frac{q^2}{2} \right) J_{MMm}(q^2, m_N^2) \right] \right. \\ \left. + (C_{ab} + C_{cd}) J_{Mm}(m_N^2) \right\}. \end{aligned} \quad (7)$$

The definitions and the analytic expressions using infrared regularization of the loop functions $J_{MM}(q^2)$, $J_{Mm}(m_N^2)$ and $J_{MMm}(q^2, m_N^2)$ can be found in Appendix B of Ref. [29].

Next let us consider the finite volume corrections, which are due to the quantization of the momentum on a torus. Assuming the temporal direction to be infinite, we can integrate out the temporal component of the loop momentum by means of contour integration. The integration over three-momentum will become a sum, and the finite volume correction to a loop integral is

$$\delta_L[G_i(q)] = \left[\frac{1}{L^3} \sum_{\vec{k}} - \int \frac{d^3\vec{k}}{(2\pi)^3} \right] I_i(k, q), \quad (8)$$

where $I_i(k, q)$ denotes the integrand of the loop function $G_i(q)$. The finite volume corrections to the loops in Eq. (6) can be worked out as

$$\begin{aligned} \delta_L[G_1^\mu(q)] &= -\frac{1}{2} \int_0^1 dx \frac{\partial}{\partial q_{1i}} \mathcal{I}_{1/2}(\Delta_1, \vec{q}_1), \\ \delta_L[G_2^\mu(q)] &= \frac{3}{2} m_N^2 \int_0^1 dx \int_0^1 dy y \left[\frac{1}{3} \frac{\partial}{\partial q_{2i}} \mathcal{I}_{3/2}(\Delta_2, \vec{q}_2) + \bar{y} p^\mu \mathcal{I}_{5/2}(\Delta_2, \vec{q}_2) \right], \\ \delta_L[G_3^\mu(q)] &= \frac{3}{2} m_N^2 \int_0^1 dx \int_0^1 dy y \left[\frac{1}{3} \frac{\partial}{\partial q_{3i}} \mathcal{I}_{3/2}(\Delta_3, \vec{q}_3) + y p'^\mu \mathcal{I}_{5/2}(\Delta_3, \vec{q}_3) \right], \end{aligned} \quad (9)$$

where

$$\begin{aligned} \vec{q}_1 &= x\vec{q}, & \Delta_1 &= M_\pi^2 - x\bar{x}q^2, \\ \vec{q}_2 &= xy\vec{q} - \bar{y}\vec{p}, & \Delta_2 &= \bar{y}^2 m_N^2 + y M_\pi^2 - x\bar{x}y^2 q^2, \\ \vec{q}_3 &= xy\vec{q} - y\vec{p}', & \Delta_3 &= y^2 m_N^2 + \bar{y} M_\pi^2 - x\bar{x}y^2 q^2, \end{aligned} \quad (10)$$

with $\bar{x} = 1 - x$ and $\bar{y} = 1 - y$, see Appendix A for definitions and details. Therefore, the finite volume correction to the CP-violating matrix element of the electromagnetic current for the neutron is given by

$$\begin{aligned} i \frac{16eV_0^{(2)}\bar{\theta}_0}{F_\pi^4} C (\delta_L[G_1^\mu(q)] + \delta_L[G_2^\mu(q)] + \delta_L[G_3^\mu(q)]) \\ \times \bar{u}(p', s') \gamma_5 u(p, s), \end{aligned} \quad (11)$$

with $C = (D + F)(b_D + b_F)$. The correction to the proton matrix element is

$$\begin{aligned} i \frac{16eV_0^{(2)}\bar{\theta}_0}{F_\pi^4} C \left(-\delta_L[G_1^\mu(q)] - \delta_L[G_2^\mu(q)] + \frac{1}{2} \delta_L[G_3^\mu(q)] \right) \\ \times \bar{u}(p', s') \gamma_5 u(p, s). \end{aligned} \quad (12)$$

From Eqs. (11) and (12), it is clear that the matrix elements on a torus cannot be written in the form of Eq. (1) as a consequence of the lack of Lorentz symmetry.

In order to investigate how large the finite volume correction is, we calculate the ratio of the correction over the infinite volume result of the matrix element which can be defined as, see Eq. (1),

$$\tilde{F}_3^\mu(p, q) = \frac{F_3(q^2)}{2m_N} (p + p')^\mu. \quad (13)$$

The infinite volume expressions for both the neutron and proton EDFs were calculated in Refs. [24,25,29], and they are given in Appendix B for completeness. The ratios for the neutron and proton are given by

$$R_n^\mu = \frac{16eV_0^{(2)}\bar{\theta}_0}{F_\pi^4 \tilde{F}_{3,n}^\mu} (\delta_L[G_1^\mu(q)] + \delta_L[G_2^\mu(q)] + \delta_L[G_3^\mu(q)]), \quad (14)$$

and

$$R_p^\mu = \frac{16eV_0^{(2)}\bar{\theta}_0}{F_\pi^4 \tilde{F}_{3,p}^\mu} (2\delta_L[G_1^\mu(q)] + 2\delta_L[G_2^\mu(q)] - \delta_L[G_3^\mu(q)]), \quad (15)$$

respectively.

For numerical values, we take $F_\pi = 92.2$ MeV [37], $V_0^{(2)} \simeq -5 \times 10^{-4}$ GeV⁴ [38], $D = 0.804$ and $F = 0.463$ [39]. From a leading order fitting to the octet baryon mass differences, we get $b_D = 0.068$ GeV⁻¹ and $b_F = -0.209$ GeV⁻¹. The pion mass dependence of the pion decay constant, the nucleon mass and the mass of the eta meson will be neglected since they contribute from the next-to-next-to-leading order. As mentioned before, because the Lorentz symmetry is broken, the finite volume corrections do not only depend on q^2 . Different q^μ with the same q^2 can result in different corrections, as pointed out in, e.g. Ref. [33]. In lattice calculations of form factors, in order to reduce the statistical noise, often the momentum of the sink is set to zero and the momenta of the source and the current take small values. The cubic symmetry of a torus ensures the equivalence of the three spatial directions. Thus, we take $\vec{p} = \{-2\pi/L, 0, 0\}$ and $\vec{q} = \{2\pi/L, 0, 0\}$ to show the numerical results of the ratios defined above. In this case, as can be seen from the expressions, only the temporal and the first spatial components of the matrix elements, both in infinite and finite volumes, are nonvanishing. The ratios for both the neutron and proton are shown in Fig. 2. The solid curves are the results with a physical pion mass, and the dashed ones are for $M_\pi = 330$ MeV, which is the smallest pion mass used in the recent lattice calculation [16]. Notice that in the plots, we have neglected the contribution from the counterterms w_a and w_b and the regularization scale in the infinite volume matrix elements is taken as $\mu = 1$ GeV. That means that we compare the loop contribution in the finite to the one in the infinite volume at a given natural scale (as indicated by the subscript 'loop' in the figure). This is done for better displaying of the corrections as the tree contributions to the nucleon EDM contain some sizeable uncertainties as discussed in the next section. One sees that the correction to the spatial matrix element is larger than that to the temporal one. In the case of a 330 MeV pion mass, the finite volume corrections for $L \geq 2.5$ fm are always smaller than 3% of the loop contributions for both the neutron and the proton. In the case of the physical mass, in order to have a correction less than 10% (5%) for the neutron, L needs to be larger than 4.4 (5.4) fm. The values for the proton are similar.

3. Chiral extrapolation of the neutron and proton EDMs

As stressed in Ref. [29], the baryon octet EDMs depend on two LECs at NLO, which are called w_a and w_b . So far the only knowledge of the values of the LECs $w_a(\mu)$ and $w_b(\mu)$ is from a determination [29] using the lattice data for the neutron and proton EDMs at a large pion mass of 530 MeV [40]. Here we will use the updated lattice results at smaller pion masses of 330 and 420 MeV [16] for a new determination. As shown in the last section, the finite volume corrections to the relevant nucleon matrix elements at $M_\pi = 330$ MeV are smaller than 3%, which is much smaller than the error bars of the lattice data. Those at the larger

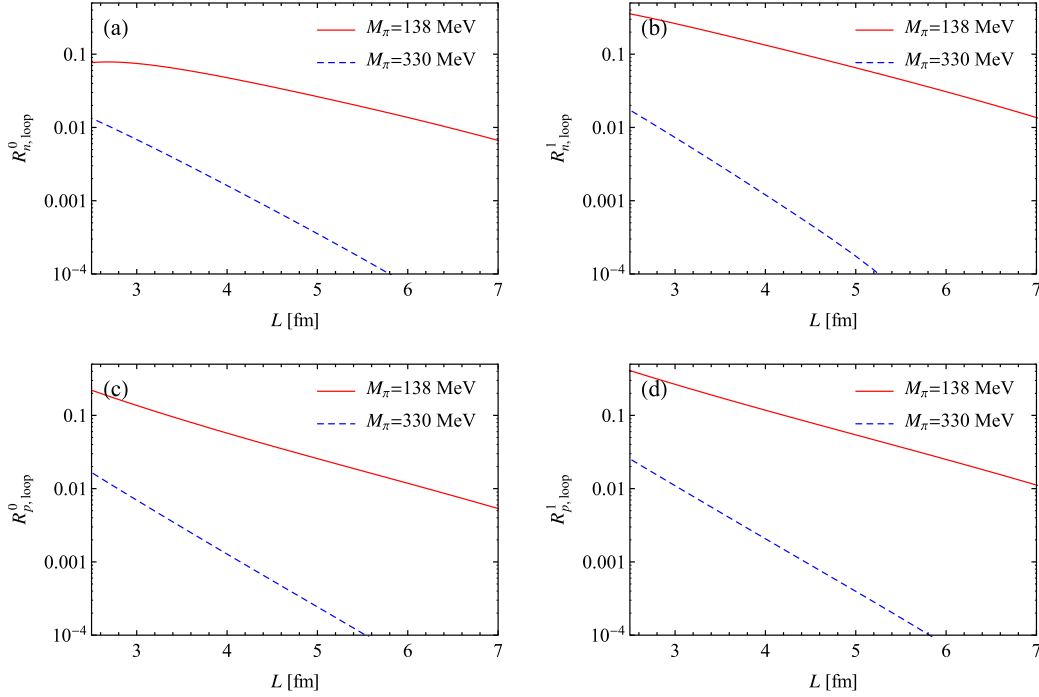


Fig. 2. Ratios of the finite volume corrections to the loop contributions to the CP-violating nucleon matrix elements of the electromagnetic current. The three-momenta are $\vec{p} = \{-2\pi/L, 0, 0\}$ and $\vec{q} = \{2\pi/L, 0, 0\}$. (a) and (b) are for the temporal and first spatial components for the neutron, respectively, while (c) and (d) are for the proton.

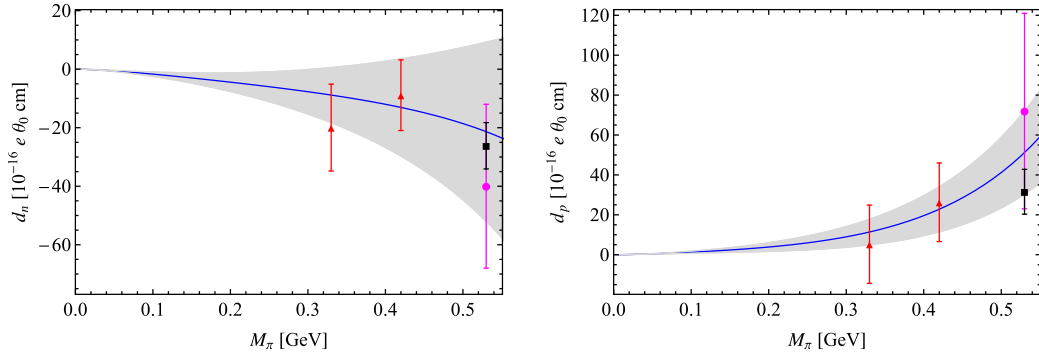


Fig. 3. Fit to the lattice results of the neutron and proton EDMs at $M_\pi = 330$ MeV and 420 MeV [16] (filled triangles with error bars) with the two counterterms $w_a(\mu)$ and $w_b(\mu)$. The filled circle and square with uncertainties at $M_\pi = 530$ MeV are the lattice results reported in Refs. [10] and [40], respectively.

pion mass of 420 MeV are even smaller. Therefore, we can neglect the finite volume effect, and fit to the lattice data at both pion masses using the expressions in Eqs. (24) and (25) with $q^2 = 0$ given in Appendix B. Fig. 3 presents the best fit with the bands reflecting the uncertainties propagated from those of the lattice data. The scale is taken to be $\mu = 1$ GeV. We obtain

$$\begin{aligned} w_a(1 \text{ GeV}) &= (-0.02 \pm 0.04) \text{ GeV}^{-1}, \\ w_b(1 \text{ GeV}) &= (-0.32 \pm 0.05) \text{ GeV}^{-1}. \end{aligned} \quad (16)$$

Using these values, the neutron and proton EDMs at the physical pion mass are predicted to be (in units of $10^{-16}e\theta_0 \text{ cm}$)

$$d_n = -2.7 \pm 0.8 \pm 0.8, \quad d_p = 2.1 \pm 0.6 \pm 1.0, \quad (17)$$

where the first uncertainties are from the counterterms w_a and w_b , and the second are from varying μ between the mass of the rho meson and the mass of the \mathcal{E} . Physical observables do not depend on the choice of the regularization scale, and the scale dependence of the loops is canceled by that of the counterterms. However, when we choose to use the values of the LECs at

a given scale, we may vary the scale in the loops within a certain range. Such a variation is a higher order effect, and we thus use it to estimate the uncertainties due to neglecting higher order contributions. If we only consider the loops, then we obtain [29]

$$d_n^{\text{loop}} = -3.1 \pm 0.8, \quad d_p^{\text{loop}} = 5.6 \pm 1.0 \quad (18)$$

in units of $10^{-16}e\theta_0 \text{ cm}$. Comparing with the values in Eq. (17), one sees that the neutron EDM is dominated by the loops while the counterterms and the loop contribution to the proton EDM are of similar size at $\mu = 1$ GeV.

4. Summary

In this paper, we have calculated the finite volume corrections to the CP-violating nucleon matrix elements of the electromagnetic current on a torus taking into account the breaking of the Lorentz symmetry. The matrix elements do not depend only on q^2 , which is the case in the continuum, and the corrections for different space-time directions differ from each other. We used two

pion masses to investigate the size of the corrections explicitly. For the pion mass of 330 MeV, which is the lowest pion mass employed in the lattice calculation reported in [16], we found that the finite volume corrections are negligible for $L \geq 2.5$ fm in comparison with the other uncertainties of the lattice results. When the pion takes its physical mass, the corrections can be significant. In order to have a correction smaller than 10%, one needs a lattice size $\gtrsim 4$ fm for the neutron and $\gtrsim 6$ fm for the proton (this estimate is based on infinite volume results including both loops and counterterms, and the central values of the counterterms w_a and w_b given in Eq. (16) are used here). We also performed a chiral extrapolation of the neutron and proton EDMs of the lattice results [16], and obtain $d_n = (-2.7 \pm 1.2) \times 10^{-16} e\theta_0$ cm and $d_p = (2.1 \pm 1.2) \times 10^{-16} e\theta_0$ cm. In the future, it might also be interesting to consider finite volume corrections with twisted boundary conditions, which would be helpful to reduce the systematic uncertainties of lattice calculations due to the momentum extrapolation.

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Appendix A. Formulae for finite volume corrections

All the finite volume corrections can be written in terms of corrections to loop integrals. The basic function is given by [34,41] (here, m denotes a quantity with dimension mass)

$$\begin{aligned} \mathcal{I}_\beta(m^2, \vec{A}) &\equiv \left(\frac{1}{L^3} \sum_{\vec{k}} - \int \frac{d^3\vec{k}}{(2\pi)^3} \right) \frac{1}{[(\vec{k} + \vec{A})^2 + m^2]^\beta} \\ &= \frac{1}{(4\pi)^{3/2} \Gamma(\beta)} \int_0^\infty d\tau \tau^{\beta-\frac{5}{2}} e^{-\tau m^2} \\ &\quad \times \left[\prod_{i=1}^3 \vartheta_3\left(\frac{A_i L}{2}, e^{-\frac{L^2}{4\tau}}\right) - 1 \right], \end{aligned} \quad (19)$$

where

$$\vartheta_3(z, q) \equiv \sum_{n=-\infty}^{\infty} q^{(n^2)} e^{i2nz} = 1 + \sum_{n=1}^{\infty} 2q^{(n^2)} \cos(2nz) \quad (20)$$

is the Jacobi elliptic theta function. This function is even in \vec{A} , i.e. $\mathcal{I}_\beta(m, \vec{A}) = \mathcal{I}_\beta(m, -\vec{A})$. For a large value of L , one has the asymptotic form

$$\mathcal{I}_\beta(m^2, \vec{A}) \rightarrow \frac{e^{-mL}}{4\pi m \Gamma(\beta)} \left(\frac{2m}{L}\right)^{2-\beta} \left[\sum_{i=1}^3 \cos(A_i L) \right]. \quad (21)$$

Thus, we see that the finite volume correction is exponentially suppressed.

When there is a factor of momentum in the numerator of the loop integrand, using

$$\frac{\partial}{\partial A^i} [(\vec{k} + \vec{A})^2 + m^2]^{-\beta} = -\frac{2\beta(k^i + A^i)}{[(\vec{k} + \vec{A})^2 + m^2]^{\beta+1}}, \quad (22)$$

one can get the following relations [34,35]

$$\begin{aligned} \mathcal{I}_\beta^i(m^2, \vec{A}) &\equiv \left(\frac{1}{L^3} \sum_{\vec{k}} - \int \frac{d^3\vec{k}}{(2\pi)^3} \right) \frac{k^i}{[(\vec{k} + \vec{A})^2 + m^2]^\beta} \\ &= -A^i \mathcal{I}_\beta(m^2, \vec{A}) - \frac{1}{2(\beta-1)} \frac{\partial}{\partial A_i} \mathcal{I}_{\beta-1}(m^2, \vec{A}), \\ \mathcal{I}_\beta^{ij}(m^2, \vec{A}) &\equiv \left(\frac{1}{L^3} \sum_{\vec{k}} - \int \frac{d^3\vec{k}}{(2\pi)^3} \right) \frac{k^i k^j}{[(\vec{k} + \vec{A})^2 + m^2]^\beta} \\ &= A^i A^j \mathcal{I}_\beta(m^2, \vec{A}) \\ &\quad + \frac{1}{2(\beta-1)} \left(A^i \frac{\partial}{\partial A_j} + A^j \frac{\partial}{\partial A_i} + \delta^{ij} \right) \mathcal{I}_{\beta-1}(m^2, \vec{A}) \\ &\quad + \frac{1}{4(\beta-1)(\beta-2)} \frac{\partial^2}{\partial A_i \partial A_j} \mathcal{I}_{\beta-2}(m^2, \vec{A}). \end{aligned} \quad (23)$$

Appendix B. Nucleon electric dipole form factors in infinite volume

The NLO expressions for the EDFFs of the nucleons in U(3) CHPT has been worked out in Refs. [24,25,29]. They depend on two counterterms $w_a(\mu)$ and $w_b(\mu)$ which are combinations of several LECs in the meson and baryon chiral Lagrangians [29]. The neutron EDFF reads

$$\begin{aligned} \frac{F_{3,n}(q^2)}{2m_N} &= \frac{8}{3} w_a(\mu) e\bar{\theta}_0 + \frac{V_0^{(2)} e\bar{\theta}_0}{\pi^2 F_\pi^4} \left\{ (D+F)(b_D + b_F) I_\pi \right. \\ &\quad \left. - (D-F)(b_D - b_F) I_K + 8(D-F)(b_D - b_F)^2 \right. \\ &\quad \left. \times (M_K^2 - M_\pi^2) \frac{\pi}{\sqrt{-q^2}} \arctan \frac{\sqrt{-q^2}}{2M_K} \right\}, \end{aligned} \quad (24)$$

where

$$\begin{aligned} I_\pi(K) &= 1 - \ln \frac{M_\pi^2(K)}{\mu^2} + \sigma_{\pi(K)} \ln \frac{\sigma_{\pi(K)} - 1}{\sigma_{\pi(K)} + 1} \\ &\quad + \frac{\pi(2M_\pi^2(K) - q^2)}{2m_N \sqrt{-q^2}} \arctan \frac{\sqrt{-q^2}}{2M_\pi(K)} \end{aligned}$$

with $\sigma_{\pi(K)} = \sqrt{1 - 4M_\pi^2(K)/q^2}$, and the proton EDFF is given by

$$\begin{aligned} \frac{F_{3,p}(q^2)}{2m_N} &= -\frac{4}{3} e\bar{\theta}_0 [w_a(\mu) + w_b(\mu)] \\ &\quad - \frac{V_0^{(2)} e\bar{\theta}_0}{6\pi^2 F_\pi^4} \left\{ 6(D+F)(b_D + b_F) \left(I_\pi + \frac{3\pi M_\pi}{2m_N} \right) \right. \\ &\quad \left. + 4(Db_D + 3Fb_F) \left(I_K + \frac{\pi M_K}{m_N} \right) \right. \\ &\quad \left. + 32(M_K^2 - M_\pi^2) \left[F(b_D^2 + 3b_F^2) \right. \right. \\ &\quad \left. \left. - \frac{2}{3} Db_D(b_D - 3b_F) \right] \frac{\pi}{\sqrt{-q^2}} \arctan \frac{\sqrt{-q^2}}{2M_K} \right. \\ &\quad \left. + \frac{\pi}{m_N} [6(D-F)(b_D - b_F) M_K \right. \\ &\quad \left. + (D-3F)(b_D - 3b_F) M_{\eta_8}] \right\}. \end{aligned} \quad (25)$$

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