Identify Resonant Frequencies
In AC Distribution Networks – A Numerical Example
Part II – The State Matrix Method

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Abstract

One of the most important issues regarding the operation of electrical distribution networks which is harmonic polluted is to avoid parallel resonance frequencies of relatively high harmonic currents, that are present in the network. For this it is absolutely necessary to know the frequency response of the network or its harmonic impedance, more exactly, the frequencies which can produce parallel harmonic resonance. This article is the second part of the paper which presents two methods of calculating the parallel resonance frequencies corresponding to peaks (poles) harmonic impedance seen in the buses: harmonic nodal admittance matrix method (in the first part), respectively that state matrix method (in the second part). The results obtained by the two methods are compared and a qualitative and quantitative analysis is done.

1. Introduction

To avoid parallel harmonic resonances that may occur in a network harmonic polluted, it is necessary to know the network status, namely the operating conditions, including knowledge of the frequency for which harmonic resonance phenomena may occur in the network (Dugan et al., 2002; Arrillaga & Watson, 2003). Solving the problem of determining the resonance frequencies in a network can be done by classical methods and modern analytical techniques or artificial intelligence (Wang et al., 2004; Bergen & Vittal, 2000). In the first part of the paper, the application of the classical method based on harmonic nodal admittance matrix construction was presented. The
method applied in this second part of the paper is considered a modern one that involves to the techniques which are specific of automated systems. It is known that the automatic behavior analysis can be done by means of the input-output characteristics. A more efficient way of analyzing the behavior of these systems lies in their description in the abstract space of state variables (Sano & Furuta, 1988; Meliopoulus et al., 1994; Martinon et al., 1996; Buta et al., 2003). State variables (or state quantities) are a group of sizes that completely define the system state at a particular time. For a system they are not unique, but must set out, so as to allow, starting from a certain state (initial) known to know the future state of the system. Applying this method in the field of electrical networks makes it possible to analyze in the frequency domain the behavior of the network, providing parallel and series harmonic resonance frequency values, without the knowledge of harmonic impedance values seen in buses (Martinon et al., 1996).

2. Application of the state matrix method to determine the resonance frequencies of an electric network

In the mathematical model of the state variable method, for describing the system behavior, the state matrix, control matrix, output matrix, and respectively the state vector, output vector and control vector are used (Martinon et al., 1996). If the system considered is associated to a distribution network with linear circuit elements, the choice of state variables is performed taking into account that the harmonic resonant frequencies are determined by the values of equivalent capacities and inductances of the component elements. It follows that inductance currents and voltages across capacitors are the state variables. Control variables are harmonic currents injected into each bus of the network and like output variables are considered harmonic voltages results in each bus of the network (Martinon et al., 1996). Construction of the state matrix of the system can be obtained by writing the corresponding equations applying Kirchhoff’s Laws in the network considered. For example, for the network area related to buses i and j from Fig 1, equations (1 ÷ 3) can be written, which are rewritten as in equation (4 ÷ 6) in order to express the state variables. The meaning of notations used in Fig 1 and in the expressions (1 ÷ 6) is obvious.

\begin{align}
  u_i &= -L_i \frac{di_i}{dt} \\
  u_i - u_j &= L_{ij} \frac{di_{ij}}{dt} + R_{ij} \cdot i_{ij} \\
  i_i + j_i &= C_i \frac{du_i}{dt} + \frac{u_i}{R_i} + i_{ij}
\end{align}

or

\begin{align}
  \frac{di_i}{dt} &= -\frac{1}{L_i} u_i \\
  \frac{di_{ij}}{dt} &= -\frac{R_{ij}}{L_{ij}} \cdot i_{ij} + \frac{1}{L_{ij}} u_i - \frac{1}{L_{ij}} u_j \\
  \frac{du_i}{dt} &= \frac{1}{C_i} i_i - \frac{1}{C_i} i_{ij} - \frac{u_i}{C_i R_i} + \frac{1}{C_i} j_i
\end{align}

Fig. 1 The equivalent schema for a network area between buses i and j.

After writing all the state equations, in the order shown above (harmonic currents through the transversal inductances, harmonic currents through the longitudinal inductances, harmonic voltages at the capacitor terminals) state matrix is build, whose general form is:
In expression (7) \( R, L, C \) are resistive, inductive respectively capacitive equivalent circuit components, \( n_l \) - is the number of equivalent longitudinal and shunt inductances and \( m \) - the number of output variables (represented by harmonic voltages in network buses).

The matrix \( A \) is an \( n \times n \) elements matrix, \( n \) being the number of state variables (represented by harmonic currents flowing through the equivalent longitudinal and shunt inductances and by the harmonic voltages at the capacitor terminals).

It is mathematically proved that the poles of the network correspond to the eigenvalues of the matrix \( A \) and the zeros of the network seen from bus \( p \), the eigenvalues of the matrix \( A_p \), obtained from \( A \) by deleting the row and column index \( n_1+p \) (Sano & Furuta, 1988).

Therefore, the determination of series and parallel resonance frequencies is limited to determining the eigenvalues of the matrix \( A \) respectively of the \( m \) matrices \( A_p \).

For the method to be applied correctly the equivalent circuit must satisfy the following conditions (Martinon et al., 1996):
- the connection between two buses must be inductive (it can’t be considered series capacities);
- in each bus must be considered a shunt capacity (which can be even zero);
- in every bus must be considered a harmonic current source injected in the network (which can be null).

Total number of poles of the network is equal to the number of parallel capacity (equal to the number of buses).

Not all the network buses show the same poles and zeros, and sometimes it is possible that some buses, some zeros overlap with poles, thus compensating each other.

In practical problems, in order to avoid parallel harmonic resonance, it is desirable that the poles are placed not in the vicinity of harmonic frequencies produced by the polluting sources. For this, is possible to intervene on the network parameters, knowing the sensitivity of the poles position with the equivalent parameters value.

3. Application of the method in the study case

Numerical example for applying the state matrix method refers to the same network area that was analyzed in the first part of this paper, for which there is no longer reproduce the wiring diagram and its equivalent circuit. Choosing the same network was done with the aim of verifying the results. Real network equivalent circuit was designed so that its configuration conditions to properly apply state variable method are met. On this line, the connection points between the filters inductances and capacities were considered equivalent network buses (numbered 3 and 4), so that in the equivalent circuit, the filters inductance appear as longitudinal inductance and capacities become shunt elements. In each network bus is connected a power source and an electrical capacity. Current sources connected in buses 1, 3 and 4 are fictitious, but does not influence the form of the state matrix. The transverse capacity \( C_1 \) is fictional, but not to influence the calculations, its value is considered very low. The state equations of the automatic system associated to the network, obtained by applying Kirchhoff’s Laws in the equivalent circuit are in number of 11 (expressions 8 ÷ 18):

\[
\frac{d i_1}{dt} = -\frac{u_1}{L_4}
\]  
\[
\frac{d i_2}{dt} = -\frac{u_2}{L_2}
\]
The network state matrix is square, having 11x11 elements (expression (19)):

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{L_1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{L_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{L_3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{L_4} \\
0 & 0 & 0 & 0 & \frac{R_{12}}{L_{12}} & 0 & 0 & \frac{1}{L_{12}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{R_{23}}{L_{23}} & 0 & \frac{1}{L_{23}} & 0 & 0 \\
\frac{1}{C_1} & 0 & 0 & 0 & -\frac{1}{C_1} & 0 & 0 & -\frac{1}{C_1 \cdot R_1} & 0 & 0 \\
0 & \frac{1}{C_2} & 0 & 0 & \frac{1}{C_2} & -\frac{1}{C_2} & 0 & -\frac{1}{C_2 \cdot R_2} & 0 & 0 \\
0 & 0 & \frac{1}{C_3} & 0 & 0 & \frac{1}{C_3} & 0 & 0 & -\frac{1}{C_3 \cdot R_3} & 0 \\
0 & 0 & 0 & \frac{1}{C_4} & 0 & 0 & \frac{1}{C_4} & 0 & 0 & -\frac{1}{C_4 \cdot R_4}
\end{bmatrix}
\]

The index \(i\) of the matrix \(A_i\), used to Mathcad solving, corresponds to the index parameters \(R_{2i}, L_{2i}, C_{2i}\), and indicate the load, respectively capacitive compensation option (as defined in the cases \(a÷d\) defined in the first part).

For each of the four load cases, results a state matrix and four secondary matrixes, corresponding to the four buses. As a numerical example, it gives the state matrix corresponding to the conditions from case \(a\).
From the eigenvalues of secondary matrices, the zeros frequencies seen from buses 1÷4, a regime, are obtained.

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\[ \text{Pols\_frequencies} = \begin{pmatrix} 1006.78 \\ 344.97 \\ 246.3 \end{pmatrix} \text{ Hz} \]

**Figure 2** Frequencies on which the network poles occurred are obtained by dividing by $2\pi$ of the positive imaginary parts belonging to the complex conjugate pairs of the eigenvalues of the matrix $A$. For example, for $a$ regime, results:

\[ \text{eigvals}(A_1) = \begin{pmatrix} 1 \end{pmatrix} \]

\[ \text{Im\_eigvals}(A_1) = \begin{pmatrix} 2 \end{pmatrix} \]

\[ \text{Pols\_frequencies}_1 = \begin{pmatrix} 1006.78 \\ 344.97 \\ 246.3 \end{pmatrix} \text{ Hz} \]

**Figure 3**

The frequencies where zeros appears as seen from the four buses of the network are obtained by applying the same procedure as for the eigenvalues of the four matrices $A_{pr}$, obtained by removing by turn from the matrix $A$, the pairs row-column of index 8, 9, 10, 11. For example, the secondary matrix corresponding to $a$ regime and to bus 1 of the network is:

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\[ \text{Pols\_frequencies}_1 = \begin{pmatrix} 1006.78 \\ 344.97 \\ 246.3 \end{pmatrix} \text{ Hz} \]

**Figure 4**
The results corresponding to the four cases are presented in Table 2.

Table 2. The values of the frequencies corresponding to the network poles, respectively zeros (Hz).

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<tr>
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<th>pol</th>
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From the analysis results can be drawn the following conclusions:

- The values of parallel and series harmonic resonance frequencies that can occur in the network, obtained by the state matrix method, are very close to those obtained by the classical method (harmonic nodal admittance matrix method presented in the first part of the paper) for all the four load cases respectively capacitive compensation.
- For the analysis of risk amplification of non-sinusoidal regime due to the occurrence of a parallel resonance due to the operation of the capacitors installed on the consumer bus bars, the most relevant are the results for harmonic impedance seen at bus 2. The zeros seen from the bus 2, where a filter is installed were obtained from 250 Hz to 350 Hz, which means that the filter has been dimensioned in order to absorb harmonic currents 5 and 7 of the power injected into the network by the polluting source. In their vicinity there are the poles corresponding to the detuned frequencies.
- The poles are located at frequencies not dangerous from the point of view of non-sinusoidal regime amplification due to parallel harmonic resonances.
- The network presents a pole that corresponds to the fictitious capacity C1, so that because of its very low values, the pole frequency exceeds the area of interest.
4. Conclusions

This article represents the second part of a work for presentation, in a didactic manner, a numerical example of the application of two methods for determining the resonance frequencies of AC electrical distribution networks. This operation is essential if the network operates under the influence of accentuated harmonic pollution because of malfunctions and damage that may occur due to the proximity of the parallel resonance frequency values to the frequencies of the harmonic currents of high amplitude flowing in the network. The method implemented in the second part of the paper is based on the construction of the state matrix characterizing the automatic system associated to the distribution network harmonic polluted. The results are compared with those obtained by the classical method, consisting in determining the appropriate frequency peaks (poles) and minima (zeros) function with the frequency dependence of the diagonal elements of the inverse harmonic nodal matrix admittance - harmonic impedances, method which was presented in the first part of the paper. The similarity of results obtained by applying the two methods on the same network area is very good, making the state matrix method, by the reduced volume calculation, to help increase observability and controllability of electrical networks. It becomes a very useful tool for monitoring and diagnosing the distribution networks harmonic polluted. Through detailed numerical examples of two parts, the work can be considered as a useful didactic element for formation in electrical engineering, to study problems related to power quality in general, and especially non-sinusoidal regime.

References